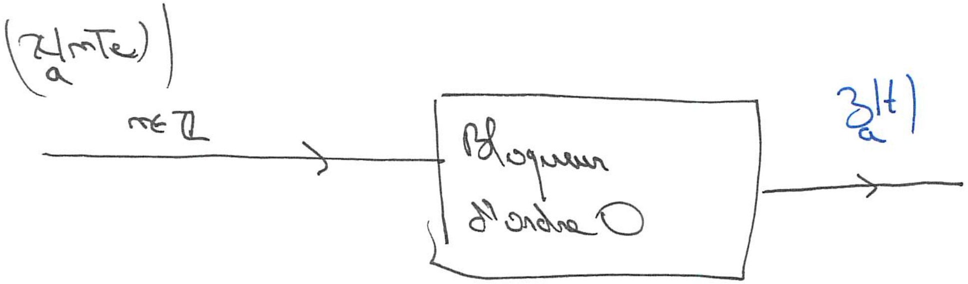
1

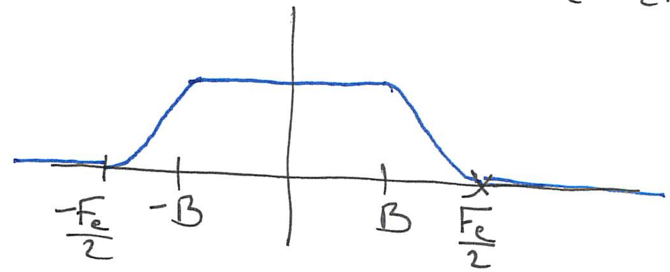
$$x_a(t) = T_e \sum_{m \in \mathbb{Z}} x_a(mT_e)\, h(t - mT_e)$$

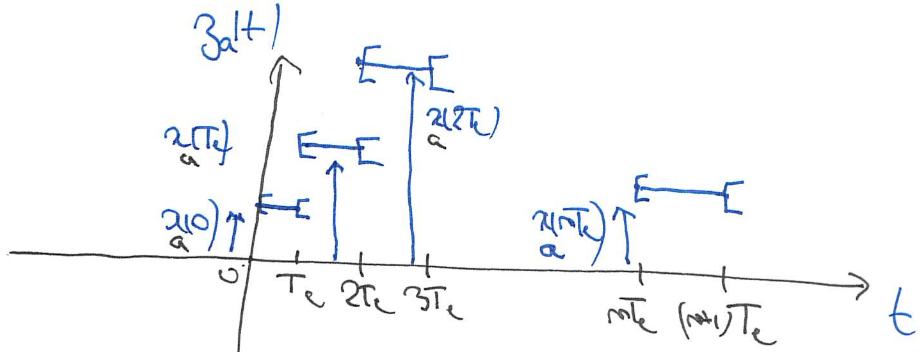


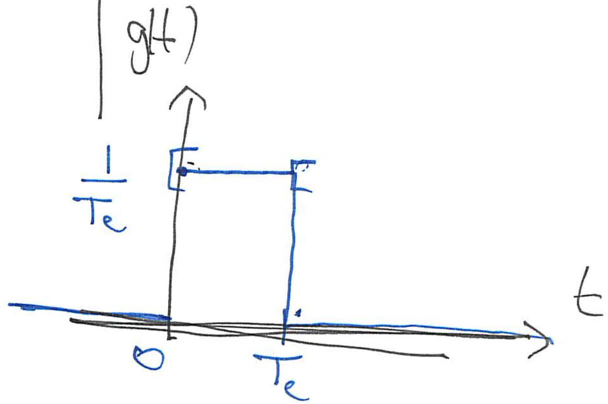
$$z_a(t) = T_e \sum_{m \in \mathbb{Z}} x_a(mT_e)\, g(t - mT_e)$$

$$g(t) = \frac{1}{T_e} \quad \text{si } t \in [0, T_e[$$
$$= 0 \quad \text{ailleurs}$$

$$H(f) = 1 \quad \text{si } f \in [-B, B]$$
$$= 0 \quad \text{si } f \notin \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$$







$t \in [0, T_e[$, $z_a(t) = x_a(0)$

$g(t - mT_e) = 0$ si $m \neq 0$

$g(t) = 1/T_e$, $T_e \cdot x_a(0) \frac{1}{T_e} = x_a(0)$

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$$z_a(t) = T_e \sum_{m\in\mathbb{Z}} x_a(mT_e)\, g(t-mT_e)$$

$$g(t) = \frac{1}{T_e}, \quad \text{si } t \in [0, T_e[$$
$$= 0 \quad \text{ailleurs}$$

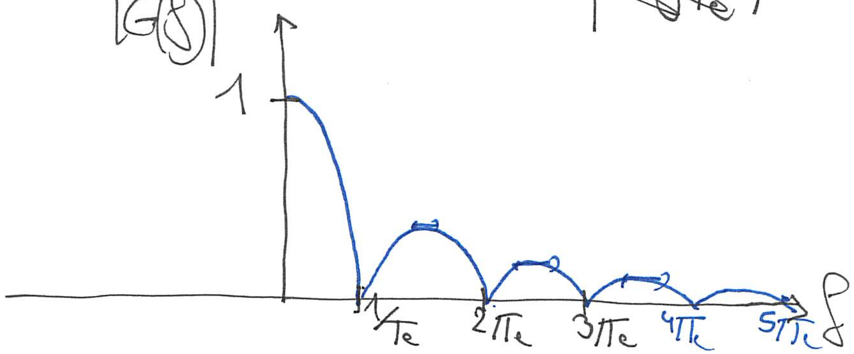
$$G(f) = \int_{-\infty}^{+\infty} g(t)\, e^{-2i\pi f t}\, dt = \int_0^{T_e} \frac{1}{T_e} \cdot e^{-2\pi f t}\, dt$$

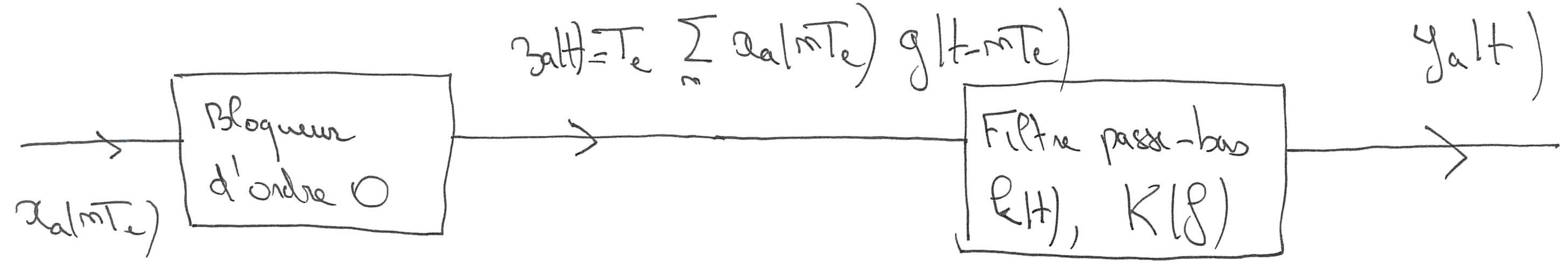
$$= \frac{1}{T_e} \times \frac{-1}{2i\pi f} \left[e^{-2i\pi f t} \right]_0^{T_e}$$

$$= \frac{1}{T_e} \times \frac{-1}{2i\pi f} \left(e^{-2i\pi f T_e} - 1 \right) = \frac{1}{T_e} \frac{1}{2i\pi f} \underbrace{\left(1 - e^{-2i\pi f T_e} \right)}$$

$$G(f) = \frac{\sin \pi f T_e}{\pi f T_e} \cdot e^{-i\pi f T_e}, \quad e^{-i\pi f T_e} \underbrace{\left(e^{i\pi f T_e} - e^{-i\pi f T_e} \right)}_{2i \sin \pi f T_e}$$

$$|G(f)| = \left| \frac{\sin \pi f T_e}{\pi f T_e} \right|$$





On peut choisir $k(t)$, ou de façon équivalente $K(f)$, pour que $y_a(t) = x_a(t)$

Expression de $y_a(t)$?

$$y_a(t) = (k * z_a)(t), \quad \ell(t) = (g * k)(t)$$

$$y_a(t) = T_e \sum_m x_a(mT_e)\, \ell(t - mT_e)$$

$$y_a(t) = k * \left[T_e \sum_m x_a(mT_e)\, g(t-mT_e)\right](t)$$

On pose $\tilde{g}_m(t) = g(t - mT_e)$

$$y_a(t) = \left(k * \left[T_e \sum_m x_a(mT_e)\, \tilde{g}_m\right]\right)(t) = T_e \sum_m x_a(mT_e)\, (k * \tilde{g}_m)(t)$$

$(k * \tilde{g}_m)(t)$?

$$(k * \tilde{g}_m)(t) = (k * g)(t - mT_e) = \ell(t - mT_e)$$

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$$(k * \tilde{g}_m)(t) = (k * g)(t - mT_e) \qquad \tilde{g}_m(t) = g(t - mT_e)$$

$$* \quad (k * \tilde{g}_m)(t) = (\tilde{g}_m * k)(t) = \int \tilde{g}_m(s)\, k(t-s)\, ds$$

$$\tilde{g}_m(s) = g(s - mT_e), \quad (k * \tilde{g}_m)(t) = \int g(s - mT_e)\, k(t-s)\, ds$$

$$u = s - mT_e, \quad s = u + mT_e$$

$$\int g(s - mT_e)\, k(t-s)\, ds = \int g(u)\, k(\overbrace{t - mT_e} - u)\, du$$

$$= (g * k)(t - mT_e)$$

$$(k * \tilde{g}_m)(t) = (k * g)(t - mT_e)$$

$$K(f)\,\tilde{G}_m(f) \qquad\qquad K(f)\,G(f)\,e^{-2i\pi f m T_e}$$

$$\tilde{g}_m(t) = g(t - mT_e)$$

$$\tilde{G}_m(f) = G(f)\,e^{-2i\pi f m T_e}$$

$$K(f)\,\tilde{G}_m(f) = K(f)\,G(f)\,e^{-2i\pi f m T_e}$$

On a donc établi que $y_a(t) = (k * z_a)(t) = T_e \sum_m x_a(mT_e)\, p(t - mT_e)$

où $p(t) = (k * g)(t)$

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$$x_a(t) = T_e \sum_m x_a(mT_e)\, h(t-mT_e), \qquad H(f) = 1 \text{ si } f\in[-B,B]$$
$$= 0 \text{ si } |f| > \frac{1}{2T_e}$$

$$y_a(t) = T_e \sum_m x_a(mT_e)\, p(t-mT_e), \qquad p(t) = (h * g)(t)$$

Pour que $y_a(t)$ coïncide avec $x_a(t)$, il suffit que la fonction $p(t) = (h*g)(t)$ vérifie $L(f) = 1$ si $f\in[-B,B]$

$= 0$ si $|f| > \frac{1}{2T_e}$

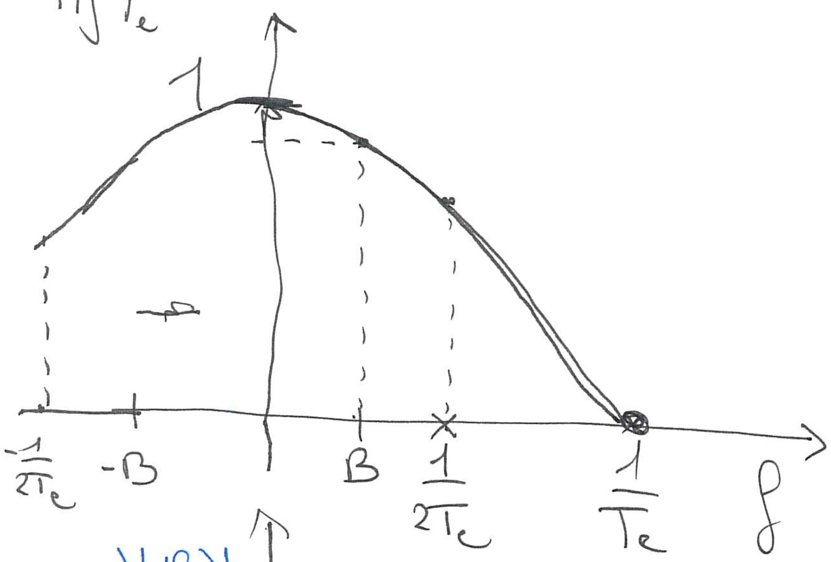
$$L(f) = K(f)G(f) = e^{-i\pi f T_e} \frac{\sin \pi f T_e}{\pi f T_e} \cdot K(f) = \begin{cases} 1 & \text{si } f\in[-B,B] \\ 0 & \text{si } |f| > \frac{1}{2T_e} \end{cases}$$

$$K(f) = 0 \text{ si } |f| > \frac{1}{2T_e}, \qquad K(f) = \frac{1}{G(f)} = e^{i\pi f T_e} \frac{\pi f T_e}{\sin \pi f T_e} \text{ si } f\in[-B,B]$$

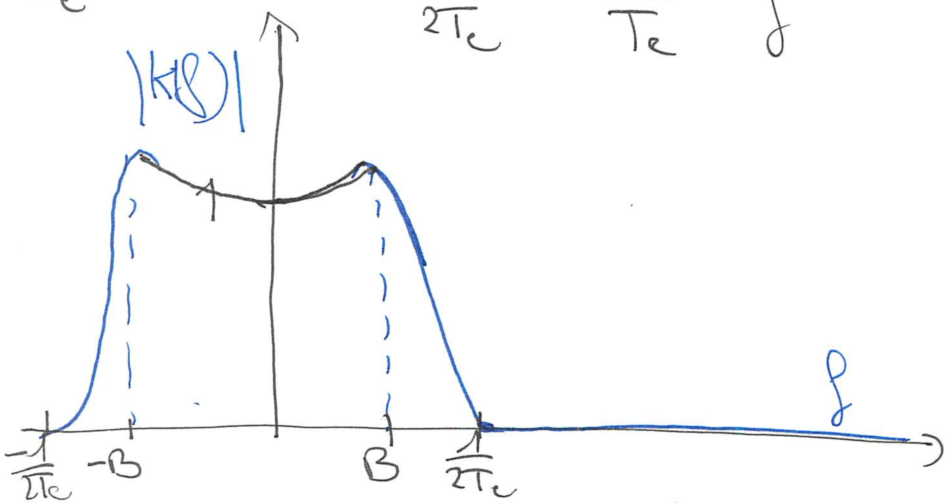
$$K(f) = 0 \text{ si } |f| > \frac{1}{2T_e}, \qquad K(f) = e^{i\pi f T_e} \frac{\pi f T_e}{\sin \pi f T_e}$$

$$|K(f)| = \frac{1}{\left| \frac{\sin \pi f T_e}{\pi f T_e} \right|} \quad \text{si } f \in [-B, B]$$

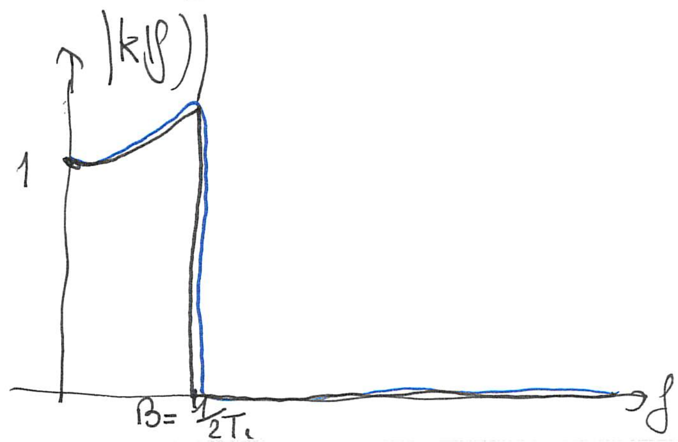
$\dfrac{\sin \pi f T_e}{\pi f T_e}$



$$\left(\frac{\sin \pi f T_e}{\pi f T_e} \right)_{f = \frac{1}{2T_e}} = \frac{\sin \frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2}{\pi}$$



Si $\dfrac{1}{2T_e} = B$



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