Rappel du contexte de l'exercice 4

$x_a(t)$ de bande $[-B, B]$, on l'échantillonne à $T_e = \frac{1}{2B}$

$x(n)$ signal à temps discret $x(n) = x_a(nT_e)$

$$\begin{cases} y(2n) = x(n) \\ y(2n+1) = 0 \end{cases}$$

$$\begin{array}{ccccccc} y(0) & y(1) & y(2) & y(3) & y(4) & y(5) & y(6) \\ \| & \| & \| & \| & \| & & \\ x(0) & 0 & x(1) & 0 & x(2) & 0 & x(3) \end{array}$$

Il existe un unique signal à temps continu $y_a(t)$, de bande passante $[-2B, 2B]$, vérifiant $y_a\left(\frac{nT_e}{2}\right) = y(n)$

$$Y_a(\nu) = \frac{T_e}{2} \sum_n y_a\left(\frac{nT_e}{2}\right) e^{-2i\pi n\nu\frac{T_e}{2}} \quad \text{si } \nu \in \left[-\frac{1}{T_e}, \frac{1}{T_e}\right]$$

$$= 0 \quad \text{si } |\nu| > \frac{1}{T_e}$$

$$Y_a(\nu) = \frac{T_e}{2} \sum_n y(n)\, e^{-2i\pi n\nu\frac{T_e}{2}} \quad \text{si } \nu \in \left[-\frac{1}{T_e}, \frac{1}{T_e}\right], \quad Y_a(\nu) = 0 \text{ si } |\nu| > \frac{1}{T_e}$$

$$Y_a(\nu) = \frac{T_e}{2} \sum_n y(n)\, e^{-2i\pi n \nu \frac{T_e}{2}} \quad \text{si} \quad \nu \in \left[-\frac{1}{T_e}, \frac{1}{T_e}\right]$$

$$= 0 \quad \text{si} \quad |\nu| > \frac{1}{T_e}$$

$y(2n) = x(n)$

$y(2n+1) = 0$

$$Y_a(\nu) = \frac{T_e}{2}\left[\sum_{n \text{ pair}} y(n)\, e^{-2i\pi n \nu \frac{T_e}{2}} + \underbrace{\sum_{n \text{ impair}} y(n)\, e^{-2i\pi n \nu \frac{T_e}{2}}}_{0} \right]$$

n pair $\Rightarrow$ $n = 2p$

$$Y_a(\nu) = \frac{T_e}{2} \sum_p \underbrace{y(2p)}_{x(p)}\, e^{-2i\pi 2p \nu \frac{T_e}{2}} \quad \text{si} \quad \nu \in \left[-\frac{1}{T_e}, \frac{1}{T_e}\right]$$

$$Y_a(\nu) = \frac{T_e}{2} \sum_p x(p)\, e^{-2i\pi p \nu T_e} \quad \text{si} \quad \nu \in \left[-\frac{1}{T_e}, \frac{1}{T_e}\right]$$

$x(p) = x_a(pT_e)$

$Y_a(\nu) = \frac{T_e}{2} \sum_{n} x(n)\, e^{-2i\pi n \nu T_e}$ si $\nu \in \left[-\frac{1}{T_e}, \frac{1}{T_e}\right]$

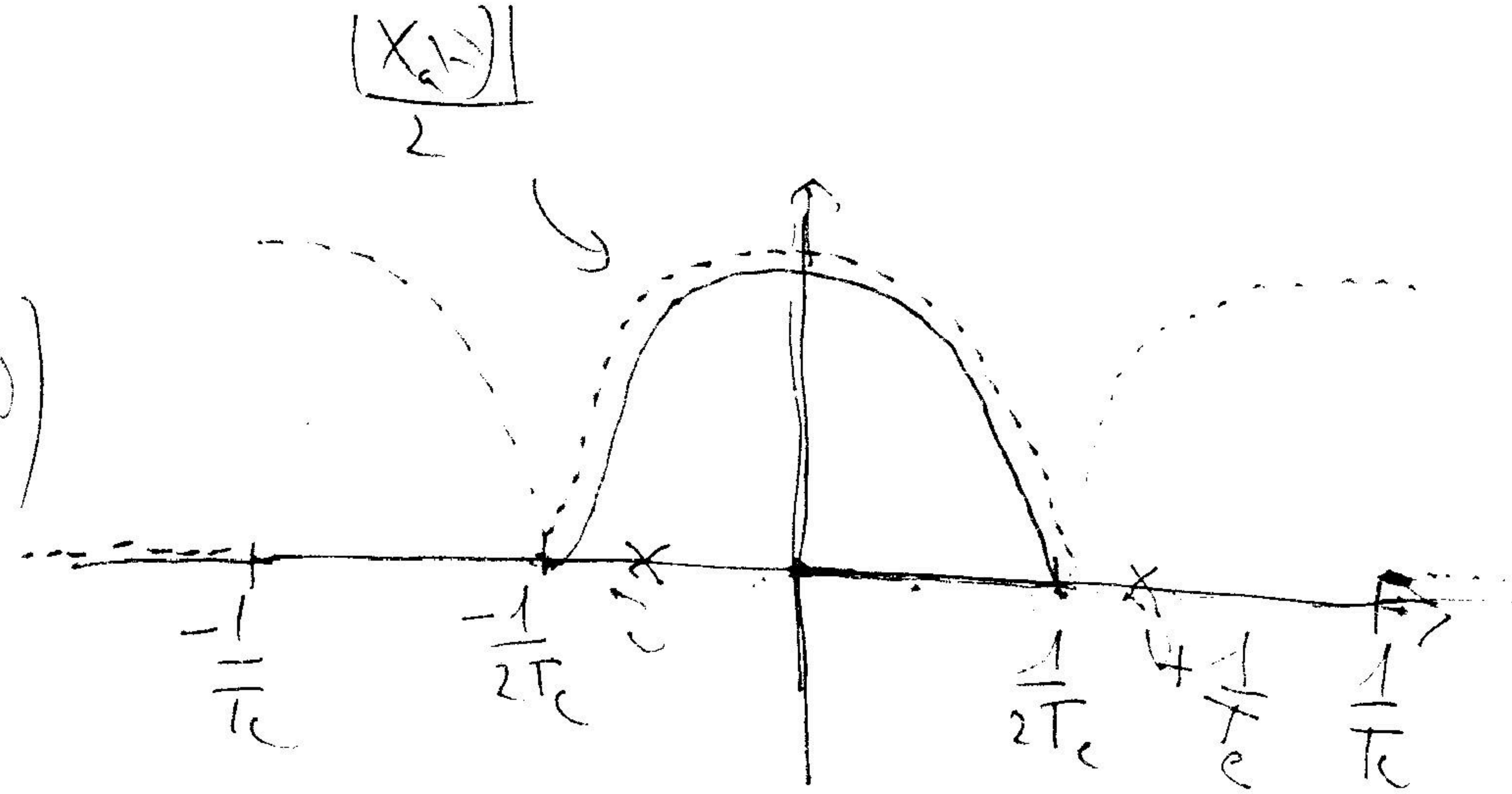
Formule de Poisson pour $x_a(t)$:

$$X_a(\nu) = T_e \left(\sum_n x_a(nT_e)\, e^{-2i\pi n\nu T_e} \right) = T_e \left(\sum_n x(n)\, e^{-2i\pi n \nu T_e} \right)$$

si $\nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$

$= 0$ si $|\nu| > \frac{1}{2T_e}$

Si $\nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$, $Y_a(\nu) = \frac{1}{2} X_a(\nu)$



~~Si $\nu \in \left[-\frac{1}{2T_e}\right.$~~

Si $\nu \in \left[-\frac{1}{2T_e}, 0\right]$, $Y_a(\nu) = \frac{1}{2} X_a(\nu)$

$\nu + \frac{1}{T_e} \in \left[\frac{1}{2T_e}, \frac{1}{T_e}\right]$ $Y_a\left(\nu + \frac{1}{T_e}\right)$ et $Y_a(\nu)$?

si $\nu \in \left[0, \frac{1}{2T_e}\right]$

$\nu - \frac{1}{T_e} \in \left[-\frac{1}{T_e}, -\frac{1}{2T_e}\right]$

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Si $\nu \in \left[-\frac{1}{2T_e}, 0\right]$ $\quad Y_a(\nu) = \frac{T_e}{2} \sum_n x(n)\, e^{-2i\pi n\nu T_e}$

$\nu + \frac{1}{T_e} \in \left[\frac{1}{2T_e}, \frac{1}{T_e}\right]$ $\quad Y_a\left(\nu + \frac{1}{T_e}\right) = \frac{T_e}{2} \sum_n x(n)\, \underbrace{e^{-2i\pi n\left(\nu + \frac{1}{T_e}\right) T_e}}_{\underbrace{e^{-2i\pi n(\nu T_e + 1)}}_{e^{-2i\pi n\nu T_e} \times \underbrace{e^{-2i\pi n}}_{1}}}$

$$e^{-2i\pi n\left(\nu + \frac{1}{T_e}\right) T_e} = e^{-2i\pi n\nu T_e}$$

$$Y_a\left(\nu + \frac{1}{T_e}\right) = \frac{T_e}{2} \sum_n x(n)\, e^{-2i\pi n\nu T_e} = Y_a(\nu)$$

Si $\nu \in \left[0, \frac{1}{2T_e}\right]$, $\quad Y_a(\nu) = \frac{T_e}{2} \sum_n x(n)\, e^{-2i\pi n\nu T_e}$

$\nu - \frac{1}{T_e} \in \left[-\frac{1}{T_e}, -\frac{1}{2T_e}\right]$, $\quad Y_a\left(\nu - \frac{1}{T_e}\right) = \frac{T_e}{2} \sum_n x(n)\, \underbrace{e^{-2i\pi n\left(\nu - \frac{1}{T_e}\right) T_e}}_{e^{-2i\pi n\nu T_e}} = Y_a(\nu)$

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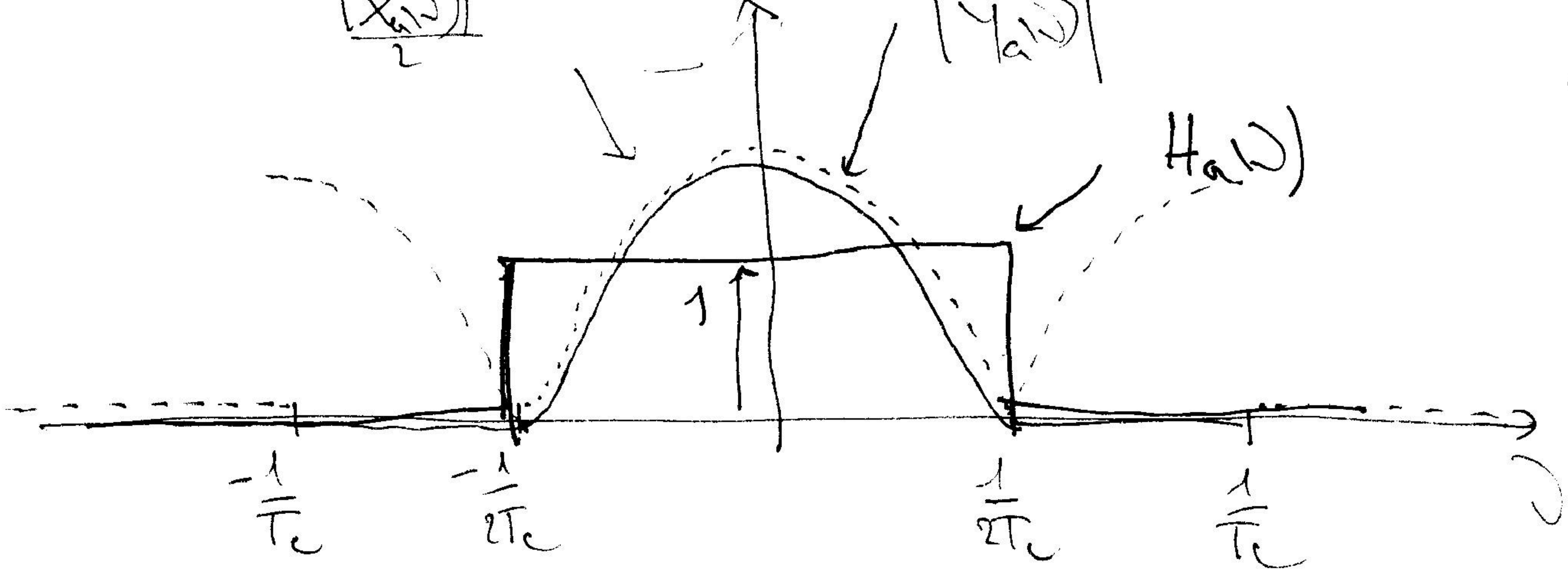
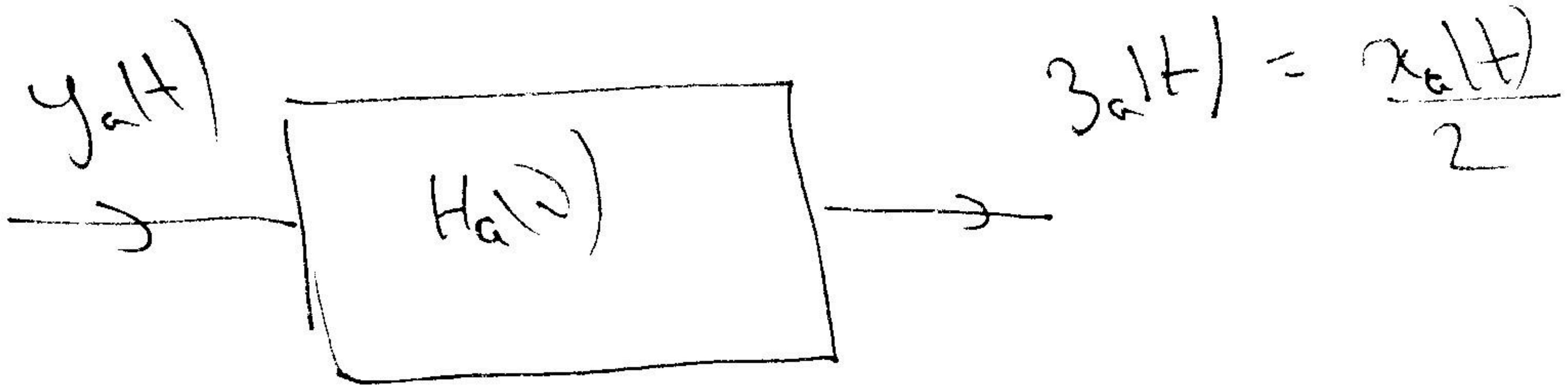


si $\nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$, $Y_a(\nu) = \frac{X_a(\nu)}{2}$

$\nu \in \left[-\frac{1}{T_e}, -\frac{1}{2T_e}\right]$, $Y_a(\nu) = \frac{X_a(\nu + \frac{1}{T_e})}{2}$

$\nu \in \left[\frac{1}{2T_e}, \frac{1}{T_e}\right]$, $Y_a(\nu) = \frac{X_a(\nu - \frac{1}{T_e})}{2}$



Que vaut $z_a(t)$?

$H_a(\nu) = 1$ si $\nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$

$H_a(\nu) = 0$ si $|\nu| > \frac{1}{2T_e}$

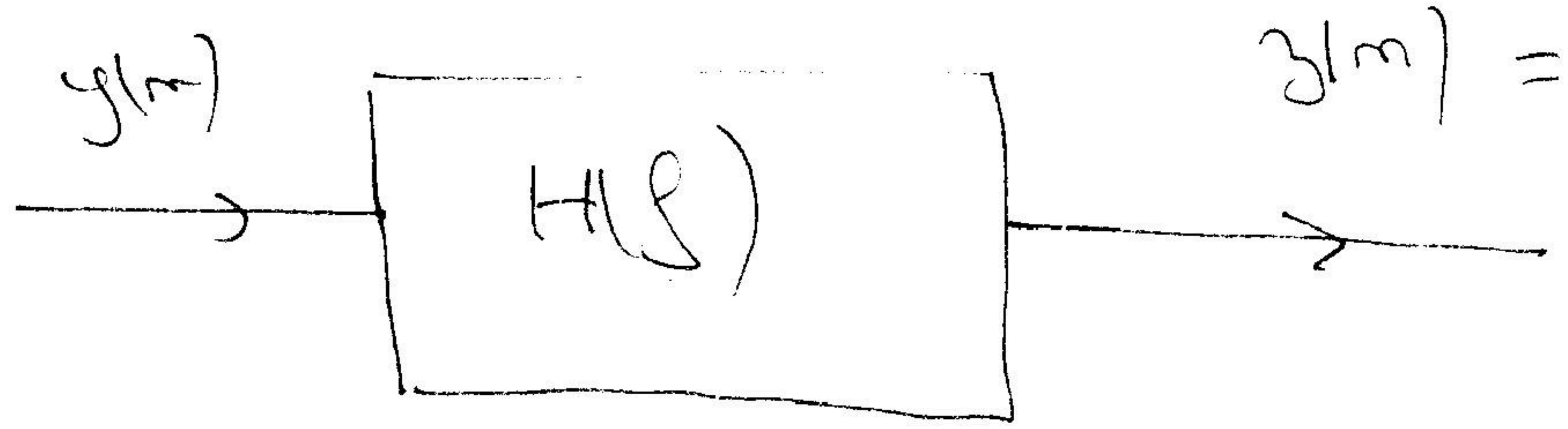
$$Z_a(\nu) = H_a(\nu)\, Y_a(\nu)$$

$\Rightarrow Z_a(\nu) = 0$ si $|\nu| > \frac{1}{2T_e}$

$Z_a(\nu) = Y_a(\nu)$ si $\nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$

$= \frac{X_a(\nu)}{2}$ si $\nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$

$\Rightarrow \forall \nu$, $Z_a(\nu) = \frac{X_a(\nu)}{2} \Rightarrow z_a(t) = \frac{x_a(t)}{2}$



$$z(n) = \frac{x_a\left(\frac{nT_e}{2}\right)}{2}$$

$$H(f) = 1 \text{ si } f \in \left[-\frac{1}{4}, \frac{1}{4}\right]$$
$$= 0 \text{ si } |f| > \frac{1}{4}$$

Alors, $z(n) = \cancel{\frac{x_a(nT_e)}{2}} \; \frac{x_a\left(\frac{nT_e}{2}\right)}{2}$

$$H_a(\nu) = 1 \text{ si } \nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$$
$$= 0 \text{ sinon}$$

$$H(f) = H_a\left(f\frac{2}{T_e}\right) = 1 \text{ si } f\frac{2}{T_e} \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$$
$$= 0 \text{ sinon}$$

$$\Leftrightarrow f \in \left[-\frac{1}{4}, \frac{1}{4}\right]$$

$$z(2n) = \frac{x_a(nT_e)}{2} = \frac{x(n)}{2}$$

$$z(2n+1) = \frac{x_a\left(nT_e + \frac{T_e}{2}\right)}{2} \qquad \frac{1}{T_e} = F_e$$

$$H(f) = \cancel{}\, H_a\left(f\frac{2}{T_e}\right)$$
$$= \cancel{}\, H_a(\nu\, 2F_e)$$