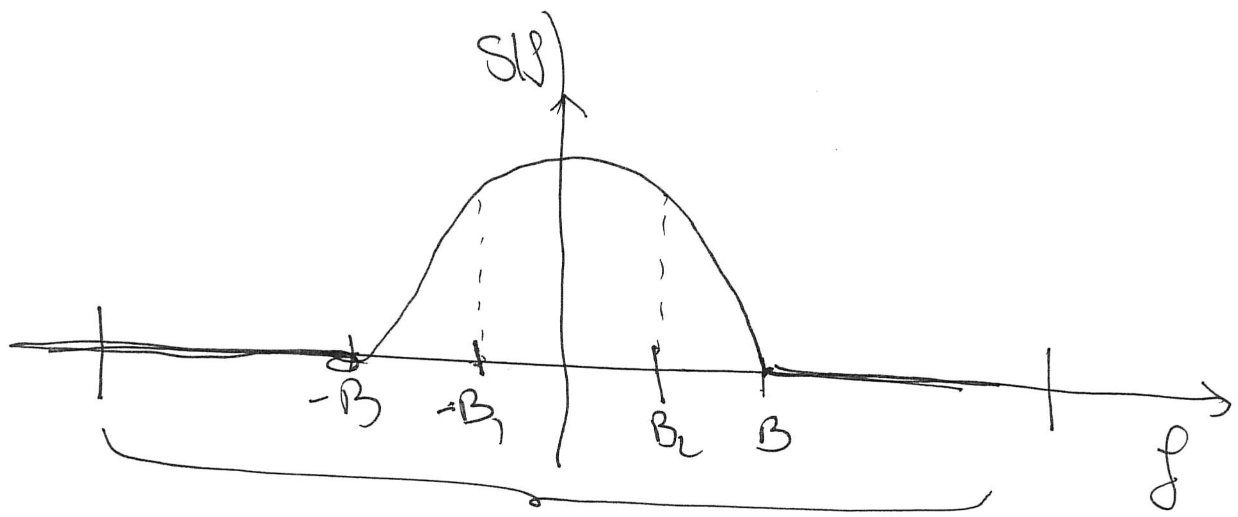
$$x(t) = \sum_m a_m\, g(t-mT)$$

$$R(\tau) = \frac{1}{T}\int_{-\infty}^{+\infty} g(t+\tau)\, g(t)\, dt$$

$$R(0) = \frac{1}{T}\int_{-\infty}^{+\infty} g^2(t)\, dt$$



La bande passante est $\boxed{\text{le plus petit}}$ intervalle en dehors duquel $S(f) = 0$

Exercice 1

$$y_a(t) = u_a(t) + b_a(t)$$

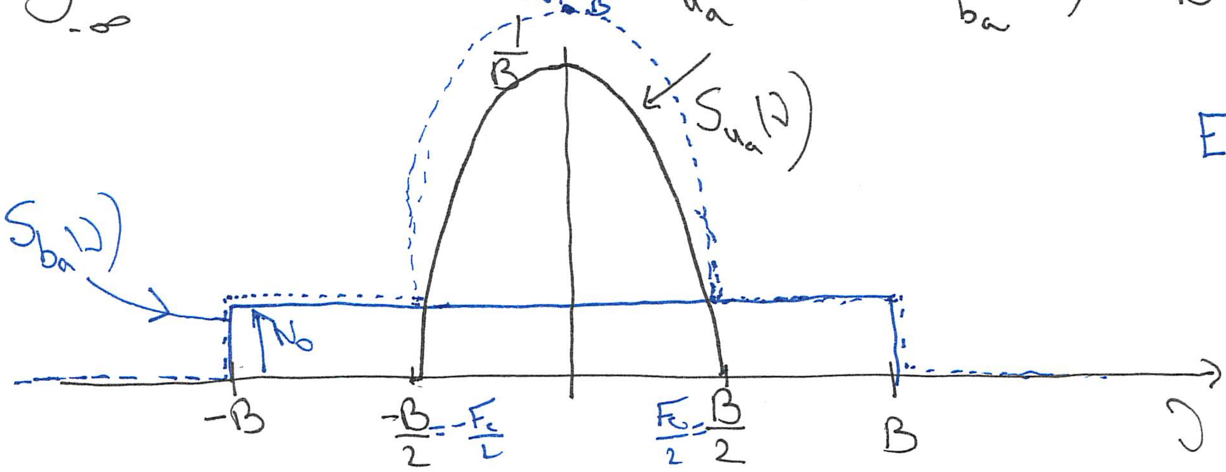
$$E\left(y_a(t+\tau)\, y_a^*(t)\right) = E\left[\left(u_a(t+\tau) + b_a(t+\tau)\right)\left(u_a^*(t) + b_a^*(t)\right)\right]$$

$$= E\left[u_a(t+\tau)u_a^*(t) + b_a(t+\tau)b_a^*(t) + u_a(t+\tau)b_a^*(t) + b_a(t+\tau)u_a^*(t)\right]$$

$$= \underbrace{E\left(u_a(t+\tau)u_a^*(t)\right)} + \underbrace{E\left(b_a(t+\tau)b_a^*(t)\right)} + \underbrace{E\left(u_a(t+\tau)b_a^*(t)\right)}_{= \underbrace{E(u_a(t+\tau))E(b_a^*(t))}_{0}} + \underbrace{E\left(b_a(t+\tau)u_a^*(t)\right)}_{0}$$

$$R_{y_a}(\tau) = R_{u_a}(\tau) + R_{b_a}(\tau)$$

$$S_{y_a}(\nu) = \int_{-\infty}^{+\infty} R_{y_a}(\tau)\, e^{-2i\pi\nu\tau}\, d\tau = S_{u_a}(\nu) + S_{b_a}(\nu) = \frac{1}{B}\cos\frac{\pi\nu}{B}\, \mathbb{1}_{[-\frac{B}{2},\frac{B}{2}]}(\nu) + N_0\, \mathbb{1}_{[-B,B]}(\nu)$$



$$E\left(b_a^2(t)\right) = R_{b_a}(0) = \int S_{b_a}(\nu)\, d\nu$$

$$= \int_{-B}^{B} N_0\, d\nu = 2N_0B$$

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$$R_{u_a}(0) = E(|u_a(t)|^2) = \int S_{u_a}(\nu)\,d\nu = \int_{-\frac{B}{2}}^{\frac{B}{2}} \frac{1}{B} \cos\frac{\pi\nu}{B}\,d\nu = \frac{1}{B} \underbrace{\int_{-\frac{B}{2}}^{\frac{B}{2}} \cos\frac{\pi\nu}{B}\,d\nu}$$

$$= \frac{1}{B} \frac{1}{\frac{\pi}{B}} \left(\sin\frac{\pi\nu}{B}\right)_{-\frac{B}{2}}^{\frac{B}{2}}$$

$$= \frac{1}{\pi}\left(\sin\frac{\pi}{2} - \sin-\frac{\pi}{2}\right) = \frac{2}{\pi}$$

$$\frac{E(u_a^2(t))}{E(b_a^2(t))} = \frac{\frac{2}{\pi}}{2N_0B} = \frac{1}{\pi N_0 B}$$

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$x_a(t)$ stationnaire, de fonction d'autocorrélation $R_{x_a}(\tau)$

$x_n = x_a(nT_e)$

(x_n) stationnaire, et $R_{x,n}$?

$$E\left(x_{n+k}\, x_k^*\right) = E\left(\underbrace{x_a\left((n+k)T_e\right)}_{\cancel{x_a(nT_e + kT_e)}}\, x_a^*(kT_e)\right) = R_{x_a}(nT_e) = R_{x,n}$$

$$E\left(x_a(nT_e + kT_e)\, x_a^*(kT_e)\right) = R_{x_a}(nT_e)$$

$$E(u_n^2) = R_{u,0} = R_{u_a}(0) = \frac{2}{\pi}$$

$$E(v_n^2) = R_{v,0} = R_{v_a}(0) = E(v_a^2(t)) = 2N_0B$$

$$\frac{E(u_n^2)}{E(v_n^2)} = \frac{1}{\pi N_0 B}$$

$x_a(t)$ de densité spectrale $S_{x_a}(\nu)$. On l'échantillonne

à une cadence T_e qui satisfait les conditions de Shannon.

Alors, si $x_n = x_a(nT_e)$, $S_x(f) = F_e \, S_{x_a}(f F_e)$

$R_{x_a}(\tau)$ la fonction d'autocorrélation de x_a :

$$R_{x,n} = E(x_{n+k} \, x_k^*) = R_{x_a}(nT_e)$$

$$R_{x,n} = \left(R_{x_a}(\tau)\right)_{\tau = nT_e}$$

Poisson appliquée à la fonction $\tau \longrightarrow R_{x_a}(\tau)$ (TF : $S_{x_a}(\nu)$)

$$T_e\left(\sum_{n\in\mathbb{Z}} R_{x_a}(nT_e)\, e^{-2i\pi n\nu T_e}\right) = \sum_k S_{x_a}\left(\nu - \frac{k}{T_e}\right) = S_{x_a}(\nu) \text{ si } \nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$$

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$$T_e \sum_m R_{x_a}(mT_e)\, e^{-2i\pi m \nu T_e} = S_{x_a}(\nu) \quad \text{si} \quad \nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$$

$$\nu T_e \in \left[-\frac{1}{2}, \frac{1}{2}\right], \quad f = \nu T_e \iff \nu = \frac{f}{T_e} = f F_e$$

$$T_e \sum_m \underbrace{R_{x_a}(mT_e)}\, e^{-2i\pi m f} = S_{x_a}(f F_e) \quad \text{si} \quad f \in \left[-\frac{1}{2}, \frac{1}{2}\right]$$

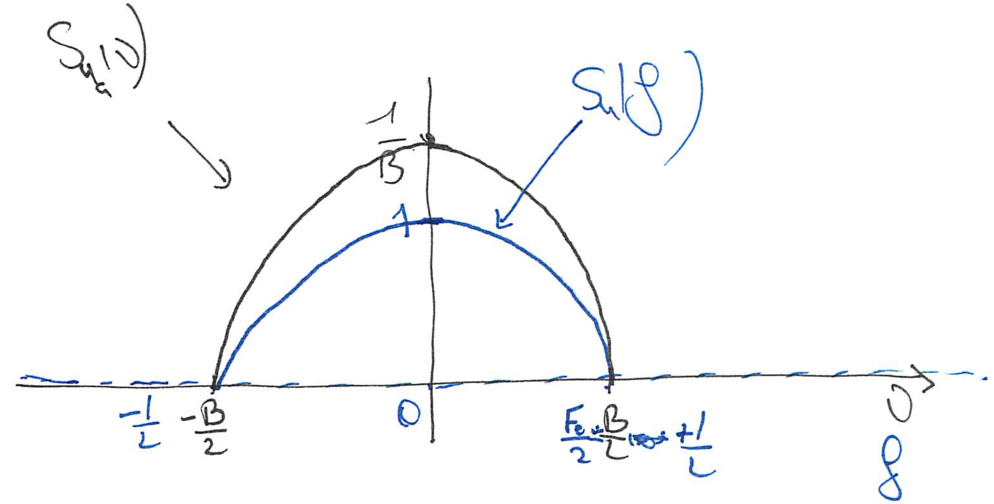
$$T_e \sum_m R_{x,m}\, e^{-2i\pi m f} = S_{x_a}(f F_e)$$

$$\underbrace{\sum_m R_{x,m}\, e^{-2i\pi m f}}_{S_x(f)} = F_e\, S_{x_a}(f F_e)$$

$$S_x(f) = F_e\, S_{x_a}(f F_e)$$

$$S_u(f) = F_e\, S_{u_a}(f F_e)$$

$$S_u(f) = B \cdot S_{u_a}(B f)$$



$$S_{b_a}(\nu) = N_0 \, \mathbb{1}_{[-B, B]}(\nu)$$

$$R_{b_a}(\tau) = \int S_{b_a}(\nu) \cdot e^{2i\pi\nu\tau} \, d\nu = \int_{-B}^{B} N_0 \cdot e^{2i\pi\nu\tau} \, d\nu$$

$$= N_0 \int_{-B}^{B} e^{2i\pi\nu\tau} \, d\nu = N_0 \frac{1}{2i\pi\tau} \left(e^{2i\pi\nu\tau} \right)_{-B}^{B}$$

$$= N_0 \frac{1}{2i\pi\tau} \underbrace{\left(e^{2i\pi B\tau} - e^{-2i\pi B\tau} \right)}_{2i \sin 2\pi B\tau}$$

$$= N_0 \frac{\sin 2\pi B\tau}{\pi\tau}$$

$$b_m = b_a(mT_e), \quad T_e = \frac{1}{B}$$

$$R_{b_a}\left(\frac{m}{B}\right) = N_0 \cdot \frac{\sin \frac{2\pi B m}{B}}{\pi \frac{m}{B}} = 0$$

si $m \neq 0$

$$S_b(f) = \sum_m R_{b,m} \, e^{-2i\pi m f}$$

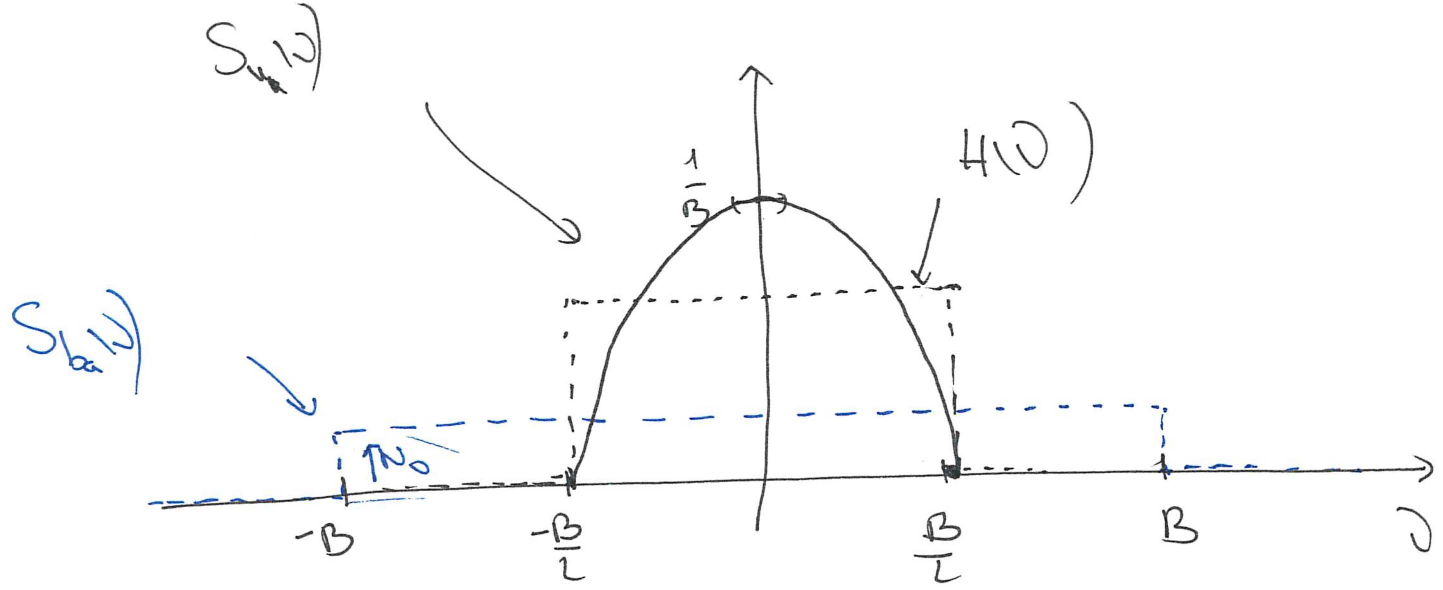
$$R_{b,m} = E\left(b_{m+k} \, b_k^* \right) = E\left(b_a((m+k)T_e) \, b_a^*(kT_e) \right) = R_{b_a}(mT_e) = 2N_0 B \cdot \delta_m$$

$S_b(f) = 2 N_0 B \quad \forall f \in [-\frac{1}{2}, \frac{1}{2}]$

Si on échantillonne b_a à la cadence $T_e = \frac{1}{2B}$, $F_e = 2B$,
la condition de Shannon est vérifiée :

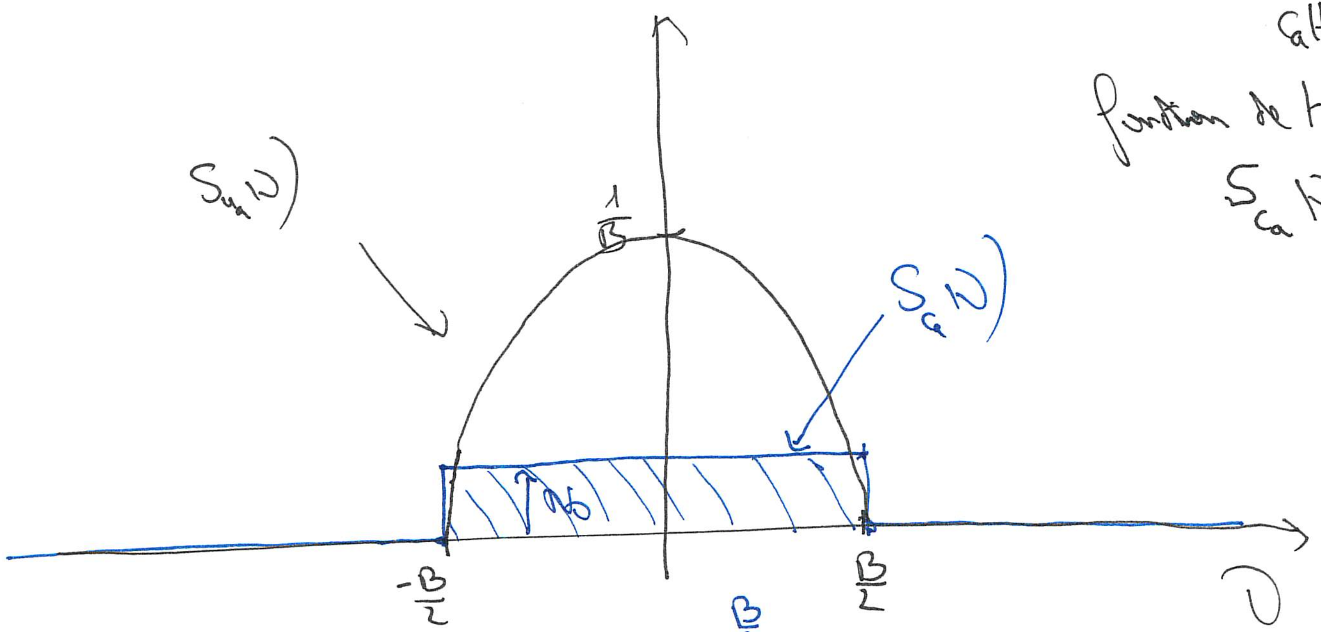
$S_b(f) = F_e \underbrace{S_{b_a}(f F_e)} \qquad f F_e = f\,2B \in [-B, B]$

$S_b(f) = 2 B N_0$



$c_a(t)$: la sortie du filtre de fonction de transfert $H(\nu)$ excité par $b_a(t)$

$$S_{c_a}(\nu) = |H(\nu)|^2 S_{b_a}(\nu) = S_{b_a}(\nu)\, \mathbb{1}_{[-\frac{B}{2}, \frac{B}{2}]}(\nu)$$



$$E(c_a^2(t)) = \int S_{c_a}(\nu)\, d\nu = \int_{-\frac{B}{2}}^{\frac{B}{2}} N_0 \, d\nu = N_0 B \qquad \rho = \frac{\frac{2}{\pi}}{N_0 B} = \frac{2}{\pi N_0 B}$$

$$E(u_a^2(t)) = \frac{2}{\pi}$$

$$\rho = \frac{E(u_m^2)}{E(c_m^2)} = \frac{2}{\pi N_0 B}$$