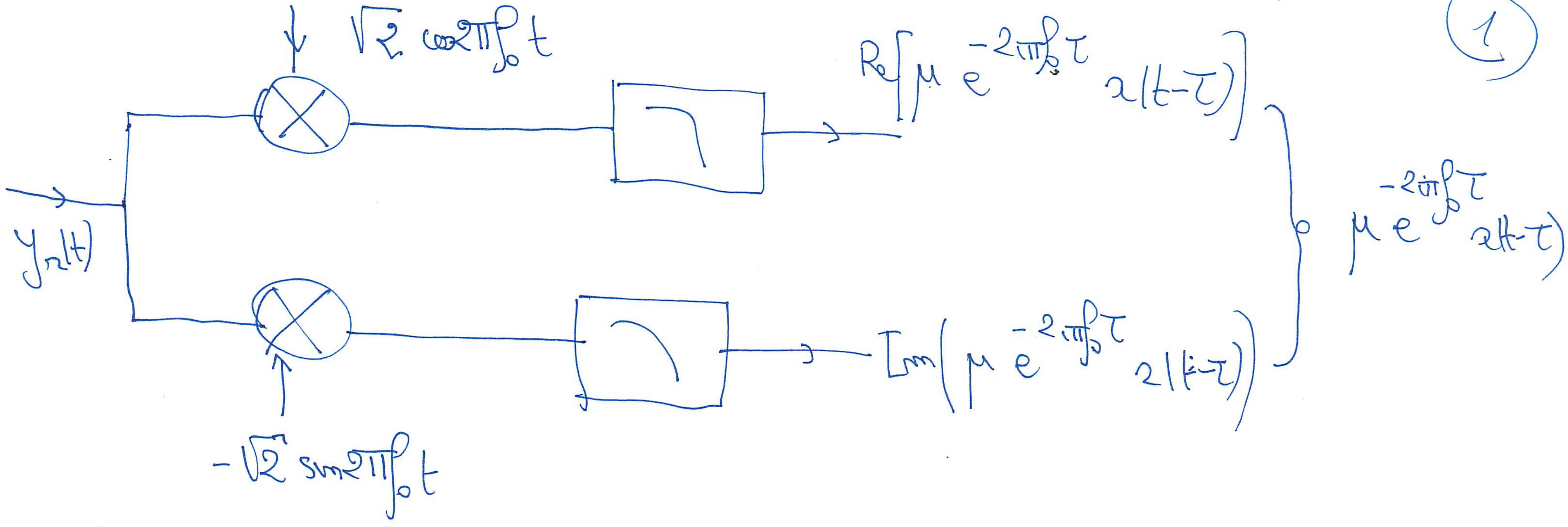
(1)



Preuve $\tau = 0$

$$y_r(t) = \underbrace{\sqrt{2}\,\mu\, x_r(t)} + b_r(t)$$

Sur la voie (I) $\sqrt{2}\,\mu\, x_r(t) \times \sqrt{2}\cos 2\pi f_0 t = 2\mu\, x_r(t)\cos 2\pi f_0 t$

$$x_r(t) = Re\left[x(t)\, e^{-2i\pi f_0 t}\right] = Re\left[(x_1(t) + i\, x_2(t))(\cos 2\pi f_0 t - i \sin 2\pi f_0 t)\right]$$

$$= x_1(t)\cos 2\pi f_0 t + x_2(t)\sin 2\pi f_0 t$$

Sur la voie (I) : $2\mu\,(x_1(t)\cos 2\pi f_0 t + x_2(t)\sin 2\pi f_0 t) \times \cos 2\pi f_0 t$

$$= 2\mu\, x_1(t) \cos^2 2\pi f_0 t + 2\mu\, x_2(t) \sin 2\pi f_0 t \cos 2\pi f_0 t$$

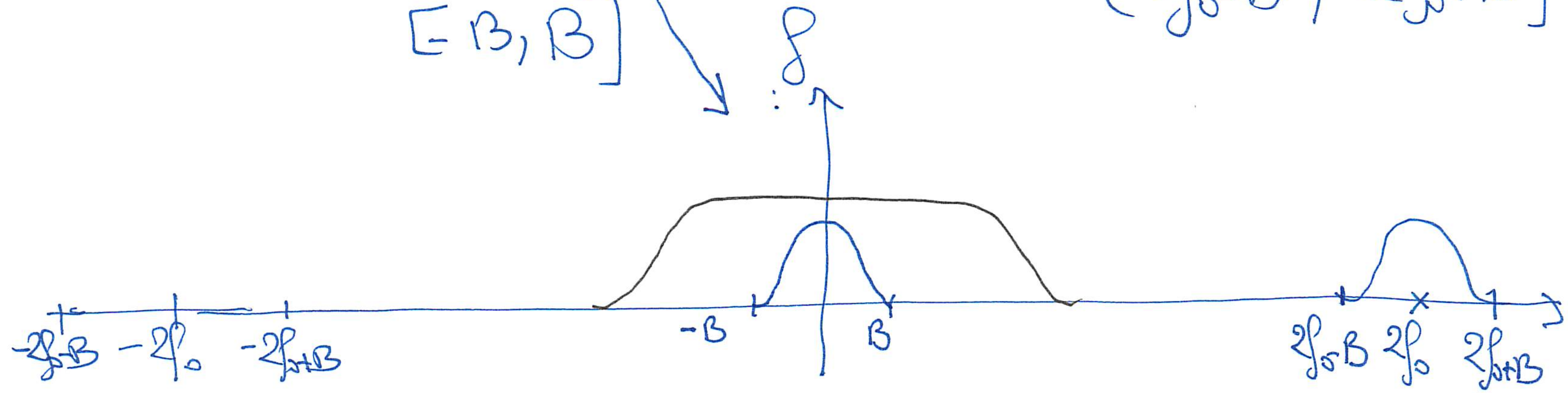
$2 \sin x \cos x = \sin 2x$

$2 \cos^2 x = 1 + \cos 2x$

$x = 2\pi f_0 t$

Sur la voie (I) : $\mu\, x_1(t)\left(1 + \cos 4\pi f_0 t\right) + \mu\, x_2(t) \sin 4\pi f_0 t$

$$= \underbrace{\mu\, x_1(t)}_{[-B,\,B]} + \underbrace{\mu\left(x_1(t) \cos 4\pi f_0 t + x_2(t) \sin 4\pi f_0 t\right)}_{[2f_0 - B,\, 2f_0 + B]}$$



$$y(t) = \mu e^{-2i\pi f_0 \tau} a_0 \, g(t-\tau) + b(t)$$

$$y = \frac{1}{T}\int y(t)\, g(t-\tau)\, dt = \underbrace{\frac{1}{T}\int b(t)\, g(t-\tau)\, dt}_{b} + \mu e^{-2i\pi f_0 \tau} a_0 \underbrace{\frac{1}{T}\int g^2(t-\tau)\,dt}_{}$$

Si τ est connu

$$\frac{1}{T}\int g^2(t)\,dt = 1$$

$$y = \mu e^{-2i\pi f_0 \tau} a_0 + b$$

$b = b_1 + i b_2$, (b_1, b_2) indépendantes de même loi : $\mathcal{N}\left(0, \frac{N_0}{2T}\right)$

$$E|b|^2 = E(b_1^2) + E(b_2^2) = \frac{N_0}{T}$$

$$\text{SNR} = \frac{\mu^2}{N_0/T} = \frac{\mu^2 T}{N_0}$$

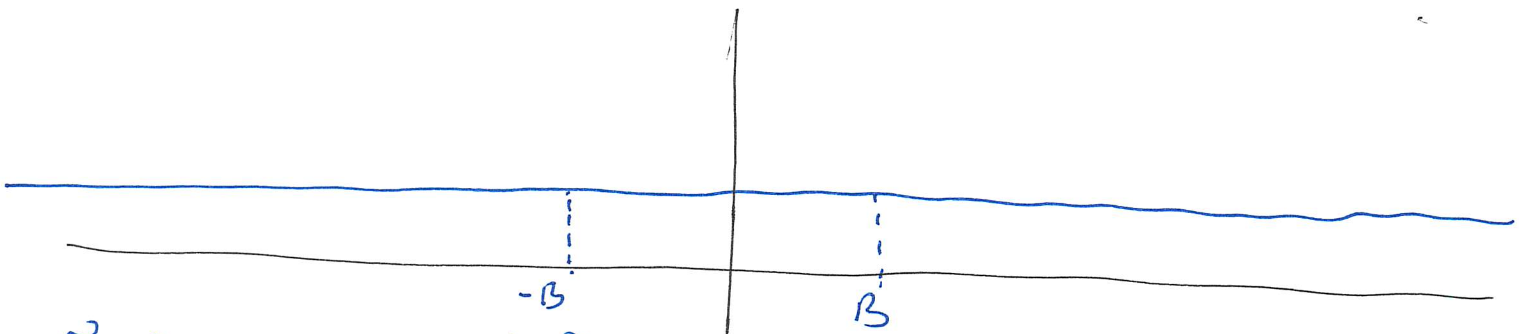
$$b = \frac{1}{T}\int b(t)\, g(t-\tau)\, dt$$

$\cancel{E|b(t)|^2 =}$ $\quad E|b|^2 = \frac{N_0}{T}$

$b(t)$: bruit blanc dans la bande $[-B, B]$

$b(t) = b_1(t) + i\, b_2(t)$

On peut remplacer $b(t)$ par un bruit blanc dans la bande $]-\infty, +\infty[$, $\tilde{b}(t)$ $\quad E\left(\tilde{b}(t)\,\tilde{b}^*(s)\right) = N_0\, \delta(t-s)$



$$b = \frac{1}{T}\int \tilde{b}(t)\, g(t-\tau)\, dt$$

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$$b = \frac{1}{T}\int \tilde{b}(t)\, g(t-\tau)\, dt$$

$$\left(\int h(t)\,dt\right)\left(\int h(s)\,ds\right) = \iint h(s)\, h(t)\, ds\, dt$$

$E|b|^2$

$$|b|^2 = \left|\frac{1}{T}\int \tilde{b}(t)\, g(t-\tau)\, dt\right|^2$$

$$= \frac{1}{T}\int \tilde{b}(t)\, g(t-\tau)\, dt \times \frac{1}{T}\int \tilde{b}^*(s)\, g(s-\tau)\, ds$$

$$= \frac{1}{T^2}\iint \tilde{b}(t)\,\tilde{b}^*(s)\, g(t-\tau)\, g(s-\tau)\, ds\, dt$$

$$E(|b|^2) = \frac{1}{T^2}\iint \underbrace{E\left(\tilde{b}(t)\,\tilde{b}^*(s)\right)}_{N_0\,\delta(t-s)} g(t-\tau)\, g(s-\tau)\, ds\, dt$$

$$= \frac{1}{T^2}\iint N_0\,\delta(t-s)\, g(t-\tau)\, g(s-\tau)\, ds\, dt = \frac{1}{T^2}\int N_0\, g^2(t-\tau)\, dt = \frac{N_0}{T}\cdot\underbrace{\frac{1}{T}\int g^2(t-\tau)\,dt}_{1}$$

$$E|b|^2 = N_0/T$$

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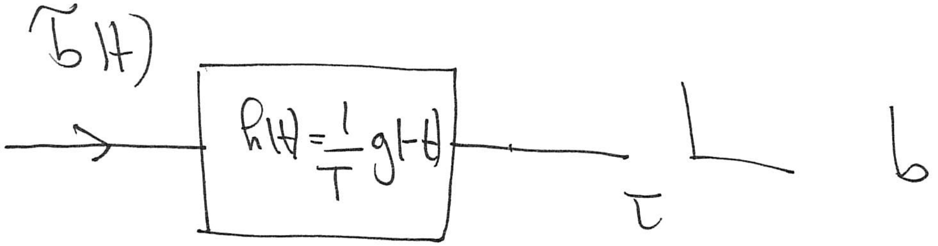
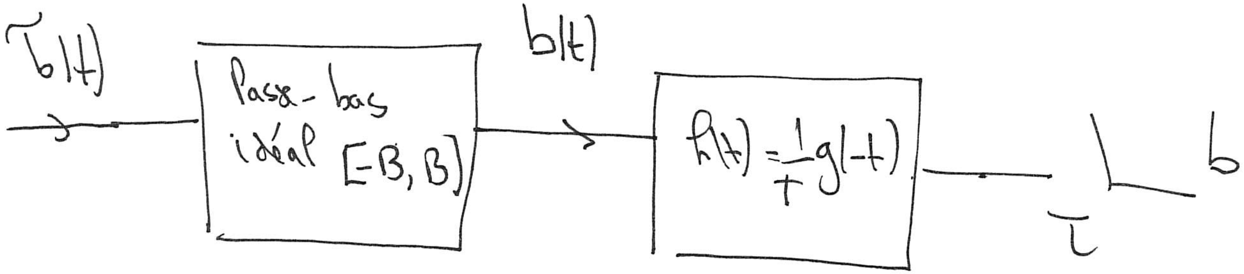
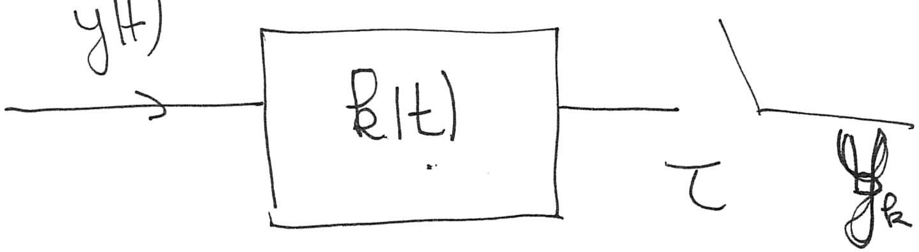




$\underbrace{H_B(f)}$ $\quad H(f) = H(f)$

1 si $f \in [-B, B]$
0 ailleurs

Interprétation de l'optimalité du filtre adapté



Quel est le rapport signal sur bruit ??

$k(f) = 0$ si $f \notin [-B, B]$

$$y_k = \int y(s)\, k(\tau - s)\, ds$$

$$y(s) = \mu e^{-2i\pi f_0 \tau} x(s-\tau) + b(s) = \mu e^{-2i\pi f_0 \tau} a_0\, g(s-\tau) + b(s)$$

$$\cancel{y_k = \mu e^{-2i\pi f_0 \tau} \int x(s-\tau)\, k(\tau - s)\, ds}$$

$$y_k = \mu e^{-2i\pi f_0 \tau} a_0 \int g(s-\tau)\, k(\tau-s)\, ds + \int b(s)\, k(\tau - s)\, ds$$

b_k

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$$b_k = \int b(s)\, k(\tau - s)\, ds$$

$$E|b_k|^2 = N_0 . \int k^2(t)\, ds$$

$$y_k = \mu e^{-2i\pi f_0 \tau} \int g(s-\tau)\, k(\tau-s)\, ds + b_k$$

$$SNR = \frac{\mu^2 \left(\int g(s-\tau)\, k(\tau-s)\, ds \right)^2}{N_0 . \int k^2(t)} \leq \frac{\mu^2 \left(\int g^2(s-\tau)\, ds \right) \left(\int k^2(\tau-s)\, ds \right)}{N_0 \int k^2(t)\, dt}$$

$$\underbrace{}$$

$$\frac{\mu^2 \int g^2(t)\, dt}{N_0} = \frac{\mu^2 T}{N_0}$$

égalité ssi $\boxed{k(\tau-s) = \alpha\, g(s-\tau) \quad \forall s}$

$\Updownarrow$

$k(t) = \alpha\, g(-t)$

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$$\int g(t+kT)\, g(t)\, dt = 0 \quad \text{si} \quad k \neq 0$$

$$y_m = \frac{1}{T}\int y(t)\, g(t-\tau-mT)\, dt$$

$$y(t) = \mu e^{-2i\pi f_0 \tau}\, x(t-\tau) + b(t)$$

$$= \mu e^{-2i\pi f_0 \tau} \sum_m a_m\, g(t-mT-\tau) + b(t)$$

$$\frac{1}{T}\int \mu e^{-2i\pi f_0 \tau}\left(\sum_m a_m\, g(t-mT-\tau)\right) g(t-mT-\tau)\, dt$$

$$b_m = \frac{1}{T}\int b(t)\, g(t-\tau-mT)\, dt$$

$$= \mu e^{-2i\pi f_0 \tau}\frac{1}{T}\int \left(\sum_m a_m\, g(t-mT-\tau)\, g(t-mT-\tau)\right) dt$$

$$b_m = b_m^1 + i b_m^2$$

b_m^1, b_m^2 : iid. $\mathcal{N}\left(0, \frac{1}{2}\frac{N_0}{T}\right)$

$$= \mu e^{-2i\pi f_0 \tau} \sum_m a_m \underbrace{\frac{1}{T}\int g(t-mT-\tau)\, g(t-mT-\tau)\, dt}_{u = t-mT-\tau} = \mu e^{-2i\pi f_0 \tau} a_m$$

$t = u + mT + \tau$

$$\underbrace{\frac{1}{T}\int g(u+(m-m)T)\; g(u)\, du}_{0 \text{ si } m \neq m,\quad m=m\quad \frac{1}{T}\int g^2(u)\,du = 1}$$