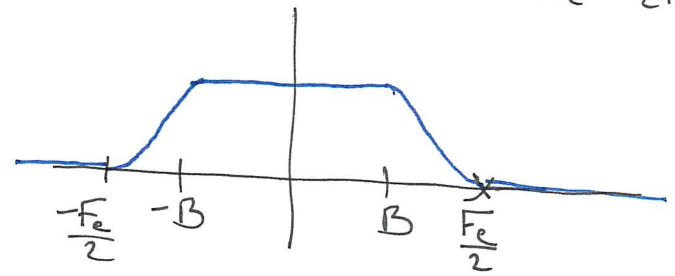
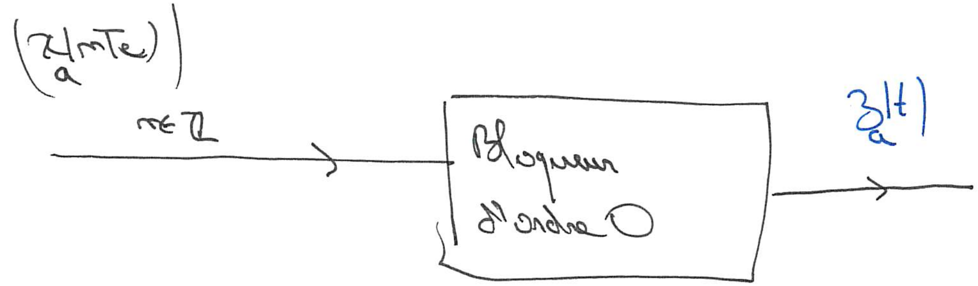
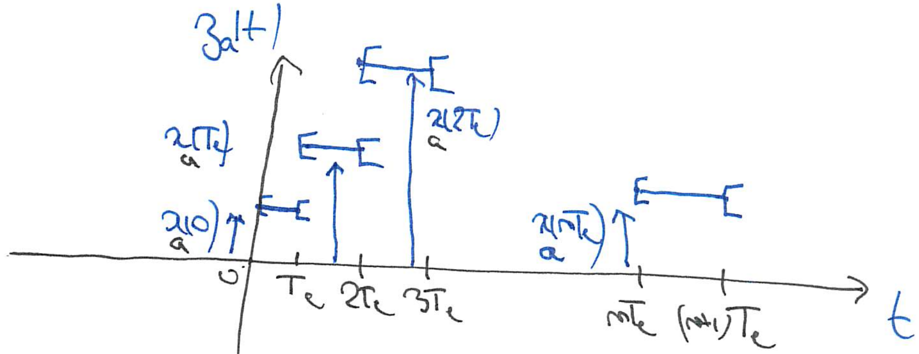
1

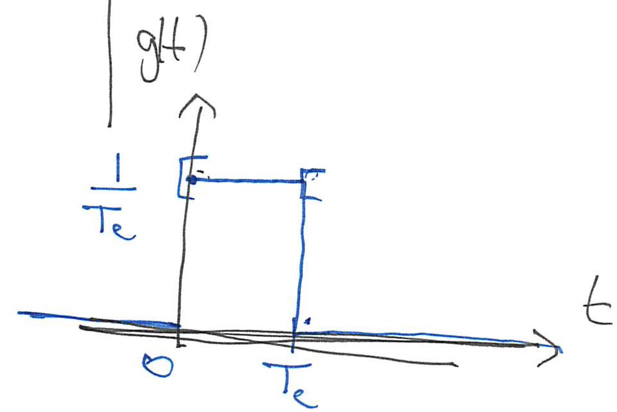
$$x_a(t) = T_e \sum_{n\in\mathbb{Z}} x_a(nT_e)\, h(t-nT_e)$$

$$H(f) = 1 \quad \text{si } f\in[-B, B]$$
$$= 0 \quad \text{si } f\notin\left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$$









$$z_a(t) = T_e \sum_{n\in\mathbb{Z}} x_a(nT_e)\, g(t-nT_e)$$

$$g(t) = \frac{1}{T_e} \quad \text{si } t\in[0, T_e[$$
$$= 0 \quad \text{ailleurs}$$

$t\in[0, T_e[$, $z_a(t) = x_a(0)$

$g(t-nT_e) = 0$ si $n\neq 0$

$g(t) = 1/T_e$, $T_e \cdot x_a(0)\frac{1}{T_e} = x_a(0)$

(2)

$$z_a(t) = T_e \sum_{m\in\mathbb{Z}} x_a(mT_e)\, g(t-mT_e)$$

$$g(t) = \frac{1}{T_e}, \quad \text{si } t\in[0,T_e[$$
$$= 0 \quad \text{ailleurs}$$

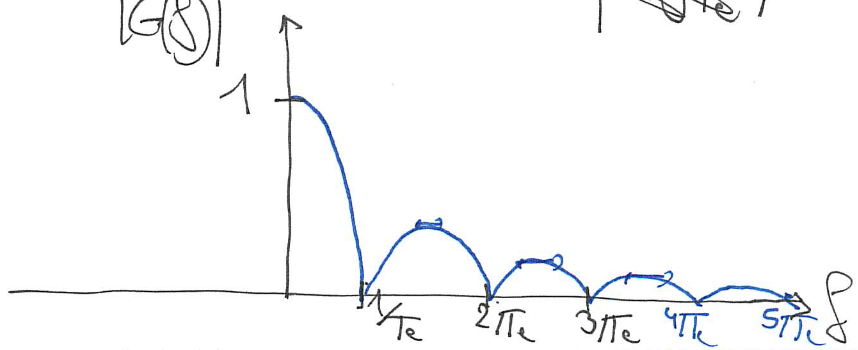
$$G(f) = \int_{-\infty}^{+\infty} g(t)\, e^{-2i\pi f t}\, dt = \int_0^{T_e} \frac{1}{T_e}\cdot e^{-2i\pi f t}\, dt$$

$$= \frac{1}{T_e} \times \frac{-1}{2i\pi f}\left[e^{-2i\pi f t}\right]_0^{T_e}$$

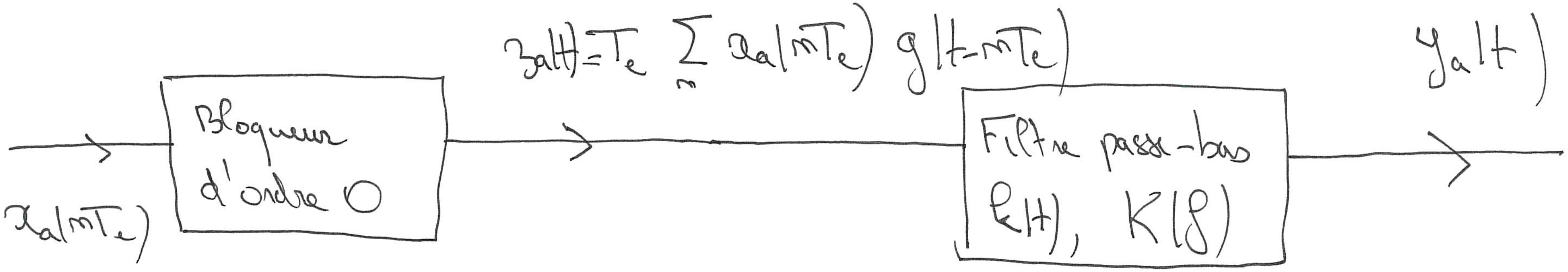
$$= \frac{1}{T_e} \times \frac{-1}{2i\pi f}\left(e^{-2i\pi f T_e} - 1\right) = \frac{1}{T_e}\,\frac{1}{2i\pi f}\underbrace{\left(1 - e^{-2i\pi f T_e}\right)}$$

$$G(f) = \frac{\sin \pi f T_e}{\pi f T_e}\cdot e^{-i\pi f T_e}, \quad e^{-i\pi f T_e}\underbrace{\left(e^{i\pi f T_e} - e^{-i\pi f T_e}\right)}_{2i\sin\pi f T_e}$$

$$|G(f)| = \left|\frac{\sin \pi f T_e}{\pi f T_e}\right|$$



3



On peut choisir $k(t)$, ou de façon équivalente $K(f)$, pour que $y_a(t) = x_a(t)$

Expression de $y_a(t)$?

$$y_a(t) = (k * z_a)(t), \quad \ell(t) = (g * k)(t)$$

$$y_a(t) = T_e \sum_m x_a(mT_e)\, \ell(t-mT_e)$$

$$y_a(t) = k * \Big[T_e \sum_m x_a(mT_e)\, g(t-mT_e) \Big](t) \qquad \text{On pose } \tilde{g}_m(t) = g(t-mT_e)$$

$$y_a(t) = \Big(k * \Big[T_e \sum_m x_a(mT_e)\, \tilde{g}_m \Big]\Big)(t) = T_e \sum_m x_a(mT_e) (k * \tilde{g}_m)(t)$$

$(k * \tilde{g}_m)(t)$? $\quad (k * \tilde{g}_m)(t) = (k * g)(t-mT_e) = \ell(t-mT_e)$

4

$$(k * \tilde{g}_m)(t) = (k * g)(t - mT_e) \qquad \tilde{g}_m(t) = g(t - mT_e)$$

$$* \quad (k * \tilde{g}_m)(t) = (\tilde{g}_m * k)(t) = \int \tilde{g}_m(s)\, k(t-s)\, ds$$

$$\tilde{g}_m(s) = g(s - mT_e) \quad , \quad (k * \tilde{g}_m)(t) = \int g(s - mT_e)\, k(t-s)\, ds$$

$$u = s - mT_e \quad , \quad s = u + mT_e$$

$$\int g(s - mT_e)\, k(t-s)\, ds = \int g(u)\, k(\overbrace{t - mT_e} - u)\, du$$

$$= (g * k)(t - mT_e)$$

(5)

$$\underbrace{(k * \tilde{g}_m)(t)} = \underbrace{(k * g)(t - mT_e)}$$

$$K(f)\,\tilde{G}_m(f) \qquad\qquad K(f)\,G(f)\,e^{-2i\pi f m T_e}$$

$$\tilde{g}_m(t) = g(t - mT_e)$$

$$\tilde{G}_m(f) = G(f)\,e^{-2i\pi f m T_e}$$

$$K(f)\,\tilde{G}_m(f) = K(f)\,G(f)\,e^{-2i\pi f m T_e}$$

On a donc établi que $y_a(t) = (k * z_a)(t) = T_e \sum_m x_a(mT_e)\, p(t - mT_e)$

où $p(t) = (k * g)(t)$

$$x_a(t) = T_e \sum_n x_a(nT_e)\, h(t-nT_e), \qquad H(f) = 1 \text{ si } f \in [-B, B], \quad = 0 \text{ si } |f| > \frac{1}{2T_e}$$

$$y_a(t) = T_e \sum_n x_a(nT_e)\, p(t-nT_e), \qquad p(t) = (k * g)(t)$$

Pour que $y_a(t)$ coïncide avec $x_a(t)$, il suffit que la fonction $p(t) = (k * g)(t)$ vérifie $L(f) = 1$ si $f \in [-B, B]$, $= 0$ si $|f| > \frac{1}{2T_e}$

$$L(f) = K(f)G(f) = e^{-i\pi f T_e} \frac{\sin \pi f T_e}{\pi f T_e} \cdot K(f) = \begin{cases} 1 & \text{si } f \in [-B, B] \\ 0 & \text{si } |f| > \frac{1}{2T_e} \end{cases}$$

$$K(f) = 0 \text{ si } |f| > \frac{1}{2T_e}, \qquad K(f) = \frac{1}{G(f)} = e^{i\pi f T_e} \frac{\pi f T_e}{\sin \pi f T_e} \text{ si } f \in [-B, B]$$

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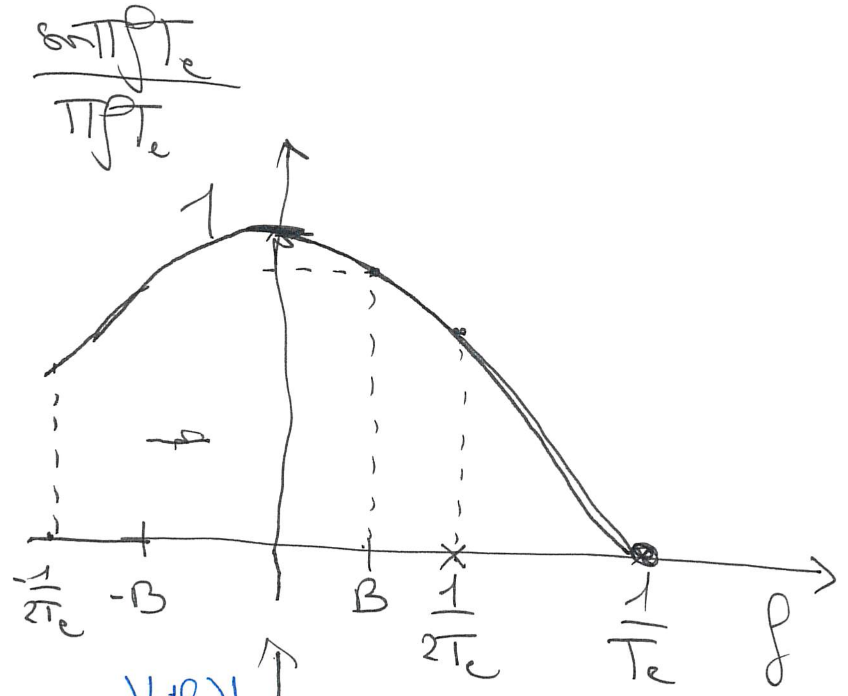
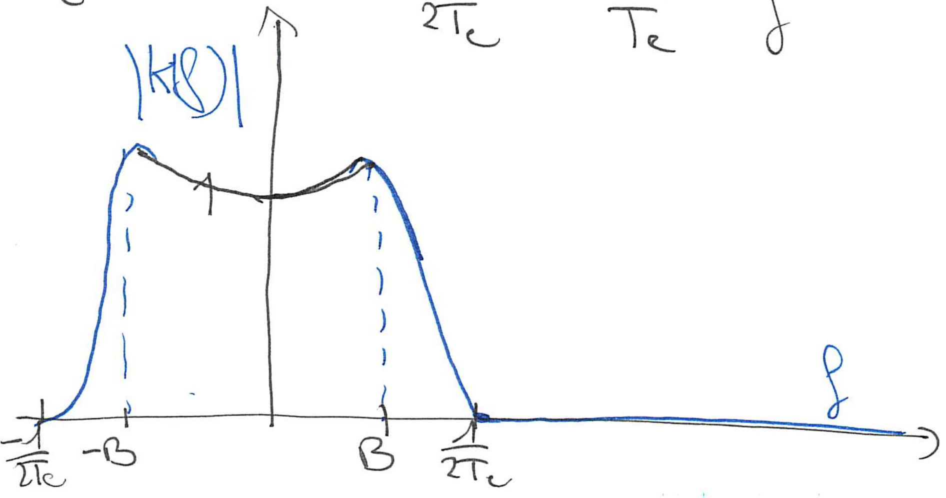
$$K(f) = 0 \text{ si } |f| > \frac{1}{2T_e}, \quad K(f) = e^{i\pi f T_e} \frac{\pi f T_e}{\sin \pi f T_e}$$

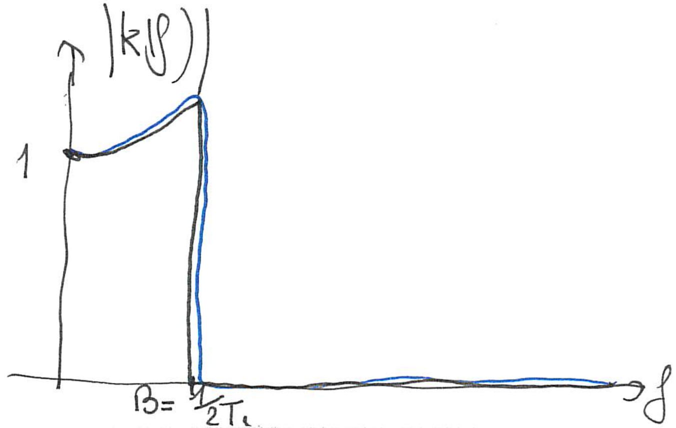
$$|K(f)| = \frac{1}{\left|\frac{\sin \pi f T_e}{\pi f T_e}\right|} \quad \text{si } f \in [-B, B]$$



$$\left(\frac{\sin \pi f T_e}{\pi f T_e}\right)_{f = \frac{1}{2T_e}} = \frac{\sin \frac{\pi}{2}}{\frac{\pi}{2}} = \frac{2}{\pi}$$



Si $\frac{1}{2T_e} = B$



(8)

$$\underline{H(f) = H_a(f\,F_e)}$$

$H_a(\nu)$ fonction de transfert du filtre analogique

$h_a(t)$ la réponse impulsionnelle.

$$H(f) = \sum_n h_n\, e^{-2i\pi n f}$$

Que vaut h_m pour tout m ?

Si m est fixé, $h_m = \displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} H(f)\, e^{2i\pi m f}\, df$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\sum_n h_n\, e^{-2i\pi n f}\right) \cdot e^{2i\pi m f}\, df$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\sum_n h_n\, e^{-2i\pi(n-m) f}\right) df = \sum_n h_n \underbrace{\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{-2i\pi(n-m) f}\, df}_{0 \text{ si } n \neq m,\ 1 \text{ si } n = m}$$

$$= h_m$$

$$h_m = \int_{-\frac{1}{2}}^{\frac{1}{2}} H(f)\, e^{2i\pi m f}\, df = \int_{-\frac{1}{2}}^{\frac{1}{2}} H_a(f F_e)\, e^{2i\pi m f}\, df$$

$$\nu = f F_e$$

$$f = \frac{\nu}{F_e} = \nu T_e$$

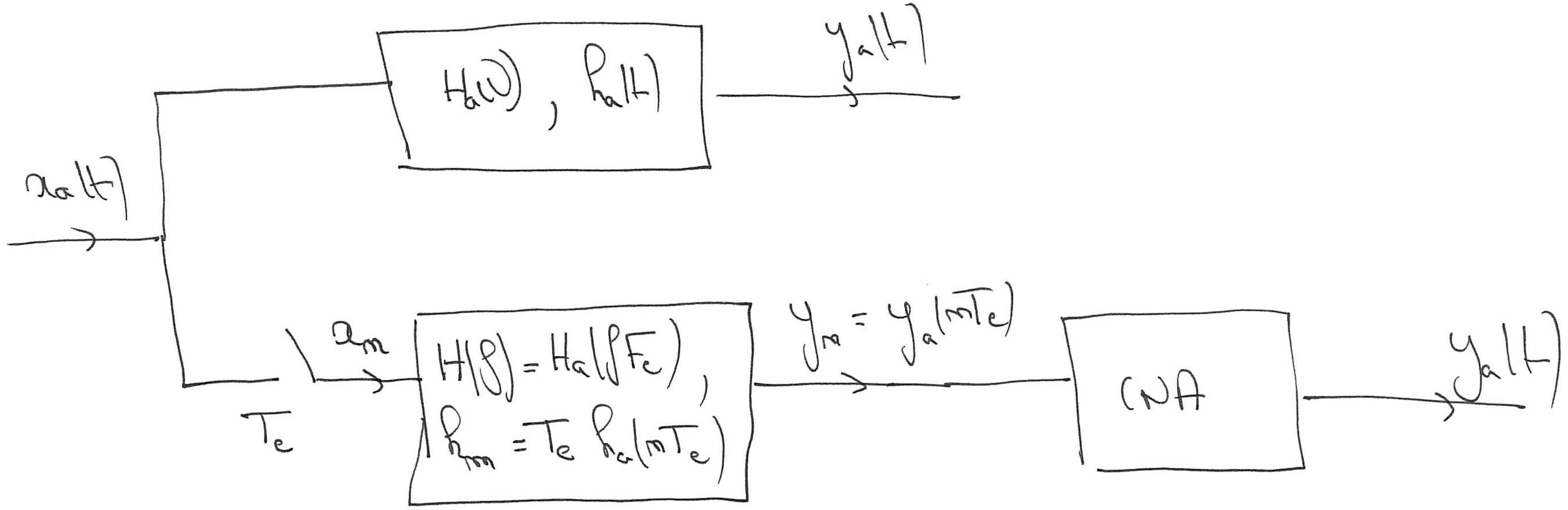
$$df = T_e\, d\nu$$

$$= T_e \int_{-\frac{F_e}{2}}^{\frac{F_e}{2}} H_a(\nu)\, e^{2i\pi m T_e \nu}\, d\nu$$

$$= T_e \underbrace{\int_{-\infty}^{+\infty} H_a(\nu)\, e^{2i\pi m T_e \nu}\, d\nu}_{h_a(mT_e)}$$

$$H(f) = H_a(f T_e)$$

$$\Longleftrightarrow \quad h_m = T_e h_a(mT_e)$$


$x_a(t)$
$H_a(\omega)$, $h_a(t)$
$y_a(t)$
T_e
x_n
$H(f) = H_a(f F_e)$,
$h_n = T_e\, h_a(n T_e)$
$y_n = y_a(n T_e)$
CNA
$y_a(t)$