$$Y(p) = AK \frac{p}{(1+\tau p)(p-i\omega_0)(p+i\omega_0)} = AK\left[\frac{-\tau}{1+\omega_0^2\tau^2}\frac{1}{1+\tau p} + \frac{1}{2(1+i\omega_0\tau)}\frac{1}{p-i\omega_0} + \frac{1}{2(1-i\omega_0\tau)}\frac{1}{p+i\omega_0}\right]$$

1

$$=$$

$\cancel{Y(p)} =$

$\cancel{y(t)} =$

$$\frac{1}{p-i\omega_0} = \text{Transformée de Laplace de } e^{i\omega_0 t}\,\Upsilon(t)$$

$$\frac{1}{p-a} = \text{Transformée de Laplace de } e^{at}\,\Upsilon(t)$$

$$\frac{1}{p+i\omega_0} = \text{Transformée de Laplace de } e^{-i\omega_0 t}\,\Upsilon(t)$$

$$\frac{1}{1+\tau p} = \frac{1}{\tau\left(p+\frac{1}{\tau}\right)} \qquad \frac{1}{p+\frac{1}{\tau}} = \text{Transformée de Laplace de } e^{\frac{t}{\tau}}\,\Upsilon(t)$$

$$=$$

$$\text{Transformée de Laplace de } \frac{1}{\tau}e^{-\frac{t}{\tau}}\,\Upsilon(t)$$

2

$$y(t) = \frac{-AK\tau}{1+\omega_0^2\tau^2}\frac{1}{\tau}e^{-t/\tau}\Upsilon(t) + \frac{AK}{2(1+i\omega_0\tau)}e^{i\omega_0 t}\Upsilon(t) + \frac{AK}{2(1-i\omega_0\tau)}e^{-i\omega_0 t}\Upsilon(t)$$

$$y(t) = \left(\frac{-AK}{1+\omega_0^2\tau^2}e^{-t/\tau} + \frac{A}{2}\underbrace{\frac{K}{1+i\omega_0\tau}}_{H(i\omega_0)}e^{i\omega_0 t} + \frac{A}{2}\underbrace{\frac{K}{1-i\omega_0\tau}}_{H(i\omega_0)^*}e^{-i\omega_0 t}\right)\Upsilon(t)$$

$$H(p) = \frac{K}{1+\tau p}$$

$$\underbrace{\frac{A}{2}\left(H(i\omega_0)e^{i\omega_0 t} + H^*(i\omega_0)e^{-i\omega_0 t}\right)}$$

$$H(i\omega_0) = |H(i\omega_0)|e^{i\,\mathrm{Arg}\,H(i\omega_0)}$$

$$\frac{A}{2}|H(i\omega_0)|e^{i\,\mathrm{Arg}\,H(i\omega_0)}e^{i\omega_0 t} + \frac{A}{2}|H(i\omega_0)|e^{-i\,\mathrm{Arg}\,H(i\omega_0)}e^{-i\omega_0 t}$$

$$= \frac{A|H(i\omega_0)|}{2}\left(e^{i\omega_0 t}e^{i\,\mathrm{Arg}\,H(i\omega_0)} + e^{-i\omega_0 t}e^{-i\,\mathrm{Arg}\,H(i\omega_0)}\right) =$$

3

$$e^{i\omega_0 t}\, e^{i \operatorname{Arg} H(i\omega_0)} = e^{i(\omega_0 t + \operatorname{Arg} H(i\omega_0))}$$

$$e^{-i\omega_0 t}\, e^{-i \operatorname{Arg} H(i\omega_0)} = e^{-i(\omega_0 t + \operatorname{Arg} H(i\omega_0))}$$

$$e^{i(\omega_0 t + \operatorname{Arg} H(i\omega_0))} + e^{-i(\omega_0 t + \operatorname{Arg} H(i\omega_0))} = 2\cos(\omega_0 t + \operatorname{Arg} H(i\omega_0))$$

$$\frac{A}{2}\left(H(i\omega_0)\, e^{i\omega_0 t} + H^*(i\omega_0)\, e^{-i\omega_0 t}\right) \cancel{= A\cos(\omega_0 t + \operatorname{Arg} H(i\omega_0))}$$

$$= A\,|H(i\omega_0)| \cos(\omega_0 t + \operatorname{Arg} H(i\omega_0))$$

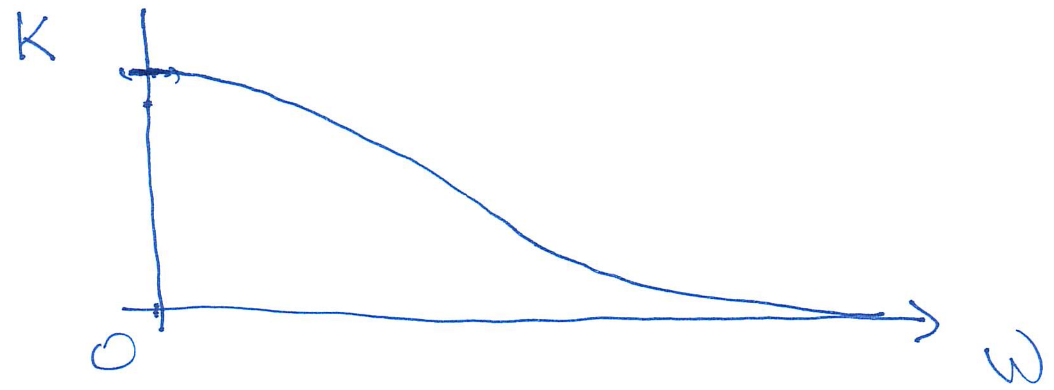
$$y(t) = \frac{-AK}{1+\omega_0^2\tau^2}\, e^{-\frac{t}{\tau}} + A\,|H(i\omega_0)| \cos(\omega_0 t + \operatorname{Arg} H(i\omega_0))$$

4

Représenter $\begin{cases} \omega \to |H(\omega)| \\ \omega \to \operatorname{Arg} H(i\omega) \end{cases}$

$$H(\omega) = \frac{K}{1+i\omega\tau}$$

$$|H(\omega)| = \frac{K}{\sqrt{1+(\omega\tau)^2}} = \frac{K}{\sqrt{1+\omega^2\tau^2}}$$



Si ω augmente, $1+\omega^2\tau^2$ augmente, $\sqrt{1+\omega^2\tau^2}$ augmente, $|H(\omega)|$ décroît

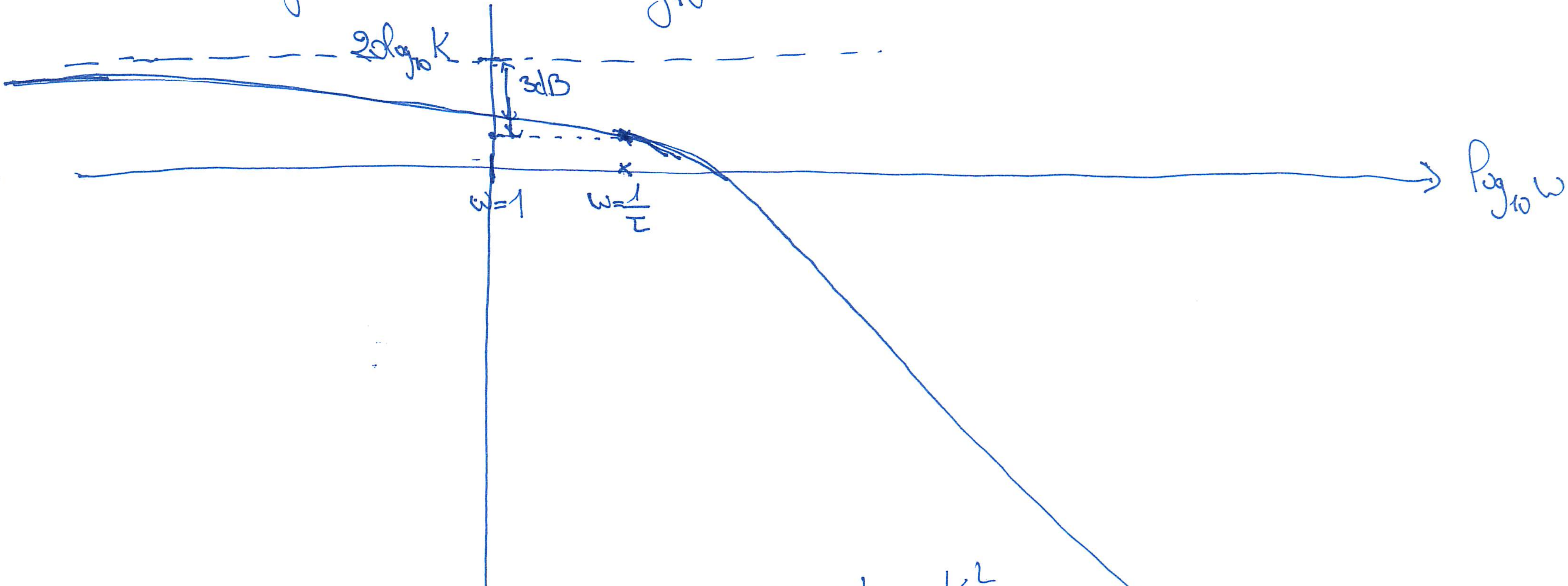
$$\lim_{\omega \to +\infty} |H(\omega)| = 0$$

(5) Diagramme de Bode des amplitudes : $|H(\omega)|^2 = \frac{K^2}{1+\omega^2\tau^2}$ $\left|H\left(i\frac{1}{\tau}\right)\right|^2 = \frac{K^2}{2}$

$10\log_{10}|H(\omega)|^2 = 10\log_{10} K^2/2 = 10\log_{10} K^2 - \underbrace{10\log_{10} 2}_{3\,dB}$

On représente $20\log|H(\omega)| = 10\log|H(\omega)|^2$

en fonction de $\log_{10}\omega$



Si $\omega \to +\infty$, $|H(\omega)|^2 \approx \frac{K^2}{\tau^2\omega^2}$

$10\log_{10}|H(\omega)|^2 \simeq 10\log_{10}\frac{K^2}{\tau^2} - 10\log_{10}\omega^2 = 10\log_{10}\frac{K^2}{\tau^2} - 20\log_{10}\omega$

$\omega \to 2\omega$, $10\log_{10}|H(\omega)|^2 \to 10\log_{10} K^2/\tau^2 - 20\log_{10}(2\omega) = 10\log_{10} K^2/\tau^2 - 20\log_{10}\omega - 20\log_{10} 2$ 6 dB

Diagramme de Bode des phases.

$$\omega \longrightarrow \operatorname{Arg} H(i\omega)$$

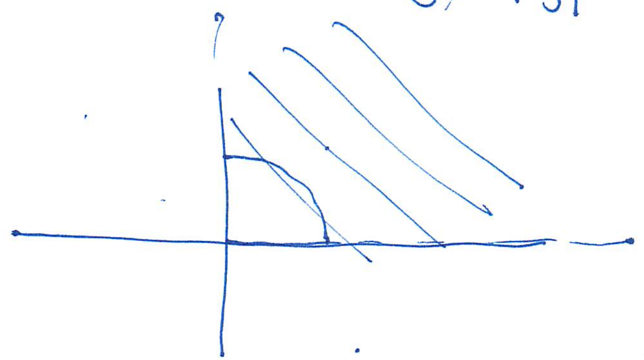
En fait, on représente $\operatorname{Arg} H(i\omega)$ en fonction de $\log_{10} \omega$

$$H(i\omega) = \frac{K}{1+i\omega\tau}$$

$$\operatorname{Arg} H(i\omega) = \operatorname{Arg} K + \operatorname{Arg} \frac{1}{1+i\omega\tau} = -\operatorname{Arg}(1+i\omega\tau)$$

$$\operatorname{Arg} \frac{1}{z} = -\operatorname{Arg} z \qquad z = |z| e^{i\theta} \qquad \frac{1}{z} = \frac{1}{|z|} e^{-i\theta}$$

$\operatorname{Arg}(\underline{1+i\omega\tau})$?

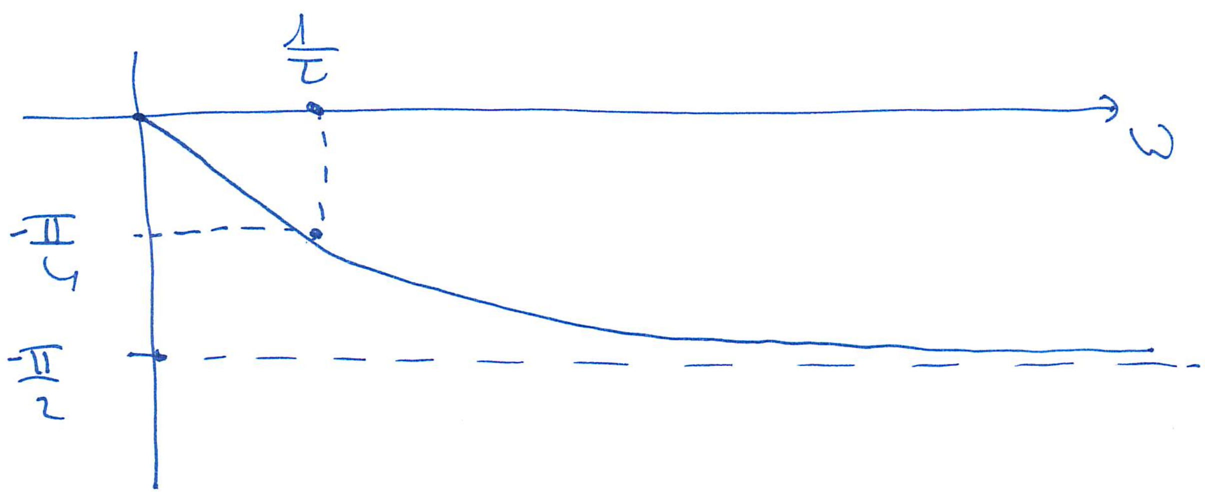


$$\operatorname{Re}(1+i\omega\tau) = 1$$

$$\operatorname{Im}(1+i\omega\tau) = \omega\tau$$

$$\Rightarrow \operatorname{Arg}(1+i\omega\tau) \in \left[0, \frac{\pi}{2}\right] \qquad \operatorname{tg}(\operatorname{Arg}(1+i\omega\tau)) = \omega\tau \Rightarrow \operatorname{Arg}(1+i\omega\tau) = \operatorname{Arctg} \omega\tau$$

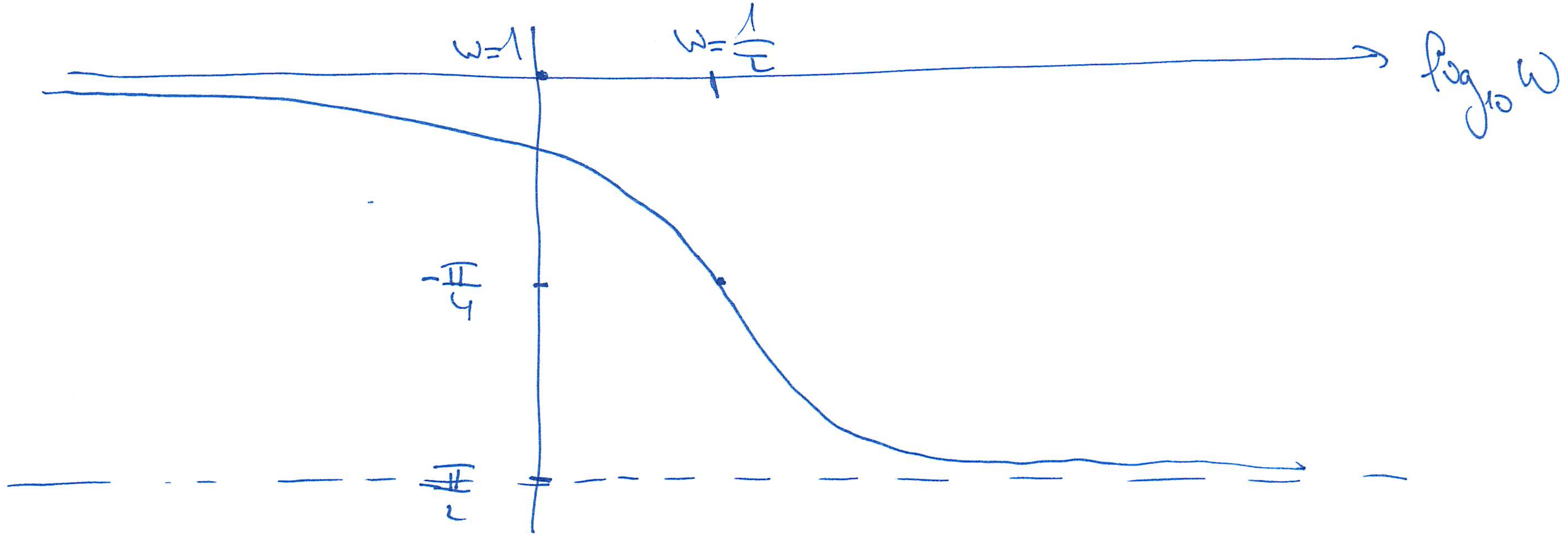
$\operatorname{Arg} H(i\omega) = -\operatorname{Arctg} \omega\tau$



Si ω croît de 0 à $+\infty$, $\operatorname{Arctg}(\omega\tau)$ croît de 0 à $\frac{\pi}{2}$

$\Rightarrow$ $\operatorname{Arg} H(i\omega)$ décroît de 0 à $-\frac{\pi}{2}$

$H\left(i\frac{1}{\tau}\right) = -\operatorname{Arctg} 1 =$



$\omega = 1$
$\omega = \frac{1}{\tau}$
$\log_{10} \omega$
$-\frac{\pi}{4}$
$-\frac{\pi}{2}$