Parameter estimation for fluorescence images deconvolution

Anna Jezierska[1] Caroline Chaux[1]
Jean-Christophe Pesquet[1] Hugues Talbot[1]
Gilbert Engler[2]
[1] *Université Paris-Est, Laboratoire d'informatique Gaspard-Monge, UMR CNRS 8049*
[2] *IBSV Unit, INRA - 06903 Sophia Antipolis*

December 12, 2011

CONTENTS

Physical acquisition process

Fluorescence imaging systems:

- Diverse noise sources
  - Some are signal dependent, e.g. photon noise
  - Some are signal independent, e.g. electric noise, thermal noise etc.
- Too complex for either Poisson or additive Gaussian noise
- Next simplest model: sum of Poisson and Gaussian models for signal dependent and signal independent components, respectively.
- Widely used in the literature [Paul *et al.* 2010] [Delibretti *et al.* 2008]

DEGRADATION MODEL

$$\boldsymbol{y} = \alpha \boldsymbol{z}(\boldsymbol{x}) + \boldsymbol{w}$$

Observations

$\boldsymbol{y} \in \mathbb{R}^Q$

Degradation model

Poisson noise

$Z_i(\boldsymbol{x}) \sim \mathcal{P}([\boldsymbol{H}\boldsymbol{x}]_i))$

$\boldsymbol{z}(\boldsymbol{x}) = \big(z_i(\boldsymbol{x})\big)_{1\leq i\leq Q}$ - realization of $Z_i(\boldsymbol{x})$

$\boldsymbol{x} \in [0, +\infty)^N$ - original signal

$\alpha \in \mathbb{R}$ - scaling parameter

$$\boldsymbol{y} = \alpha\, \boldsymbol{z}(\boldsymbol{x}) + \boldsymbol{w}$$

Observations

$\boldsymbol{y} \in \mathbb{R}^Q$

DEGRADATION MODEL

Poisson noise

$Z_i(\boldsymbol{x}) \sim \mathcal{P}([\boldsymbol{H}\boldsymbol{x}]_i))$

$\boldsymbol{z}(\boldsymbol{x}) = \big(z_i(\boldsymbol{x})\big)_{1\leq i\leq Q}$ - realization of $Z_i(\boldsymbol{x})$

$\boldsymbol{x} \in [0, +\infty)^N$ - original signal

$\alpha \in \mathbb{R}$ - scaling parameter

$$\boldsymbol{y} = \alpha\, \boldsymbol{z}(\boldsymbol{x}) + \boldsymbol{w}$$

Gaussian noise

$W_i \sim \mathcal{N}(c, \sigma^2)$

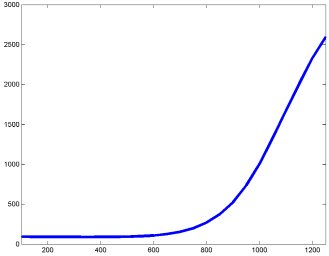
$\boldsymbol{w} = (w_i)_{1\leq i\leq Q}$ - realization of W_i

$c \in \mathbb{R}$ - mean $\quad \sigma^2$ - variance

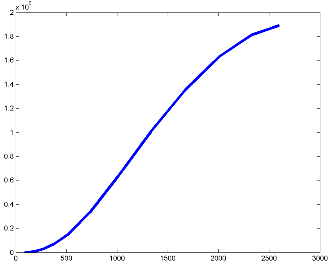
Observations

$\boldsymbol{y} \in \mathbb{R}^Q$

MACROSCOPE GAIN VERSUS NOISE PARAMETERS



(a)



(b)

Figure: (a) mean as a function of gain and (b) variance as a function of mean.

According to the model, measured signal variance should be a linear function of its mean

MACROSCOPE GAIN VERSUS NOISE PARAMETERS

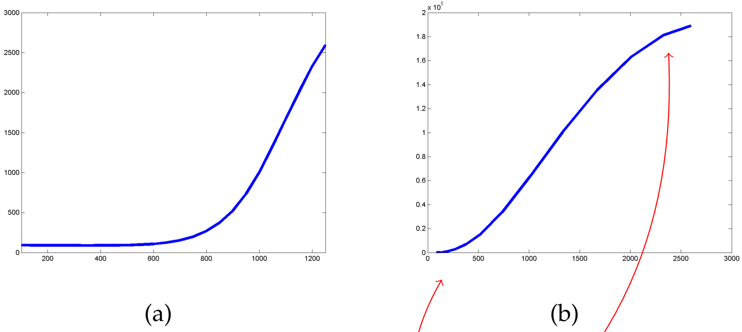


Figure: (a) mean as a function of gain and (b) variance as a function of mean.

NOTE: Poisson-Gaussian model is not valid for very low and very high GAIN settings

ALGORITHMS FOR RESTORING DATA CORRUPTED BY POISSON-GAUSSIAN NOISE: BENVENUTO *et al.* 2008

- Authors propose scaled gradient method for restoration
- Authors propose following approximation of its gradient:

$$\exp\left(-\frac{1+2([\boldsymbol{H}\boldsymbol{x}+c]_i)-y_i}{2([\boldsymbol{H}\boldsymbol{x}+c]_i)+\sigma^2}\right)$$

The noise parameters are assumed to be known.

ALGORITHMS FOR RESTORING DATA CORRUPTED BY POISSON-GAUSSIAN NOISE: BENVENUTO *et al.* 2008

- Authors propose scaled gradient method for restoration
- Authors propose following approximation of its gradient:

$$\exp\left(-\frac{1+2([\boldsymbol{H}\boldsymbol{x}+c]_i)-y_i}{2([\boldsymbol{H}\boldsymbol{x}+c]_i)+\sigma^2}\right)$$

The noise parameters are assumed to be known.

Algorithms for restoring data corrupted by Poisson-Gaussian noise: Gil-Rodrigo *et al.* 2011

- Authors propose alternating-minimization method for restoration
- The mean of Gaussian noise is assumed to be 0
- Authors propose to approximate the corresponding negative log likelihood with:

$$\frac{1}{2}\left\|\left(D_{\widetilde{\sigma}^2}(\boldsymbol{H}\boldsymbol{x})\right)^{-1/2}(\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x})\right\|_2^2$$

Covariance matrix depends on noise parameters which are assumed to be known.

ALGORITHMS FOR RESTORING DATA CORRUPTED BY POISSON-GAUSSIAN NOISE: GIL-RODRIGO *et al.* 2011

- Authors propose alternating-minimization method for restoration
- The mean of Gaussian noise is assumed to be 0
- Authors propose to approximate the corresponding negative log likelihood with:

$$\frac{1}{2}\left\|\left(D_{\widetilde{\sigma}^2}(\boldsymbol{H}\boldsymbol{x})\right)^{-1/2}(\boldsymbol{y}-\boldsymbol{H}\boldsymbol{x})\right\|_2^2$$

Covariance matrix depends on noise parameters which are assumed to be known.

Algorithms for restoring data corrupted by Poisson-Gaussian noise: Luisier *et al.* 2011

- Authors propose PURE-LET method for denoising
- Authors derive new unbiased MSE estimate corresponding to Poisson-Gaussian likelihood
- Authors show that proposed approach is faster and more reliable than Anscombe variance stabilizing transform.

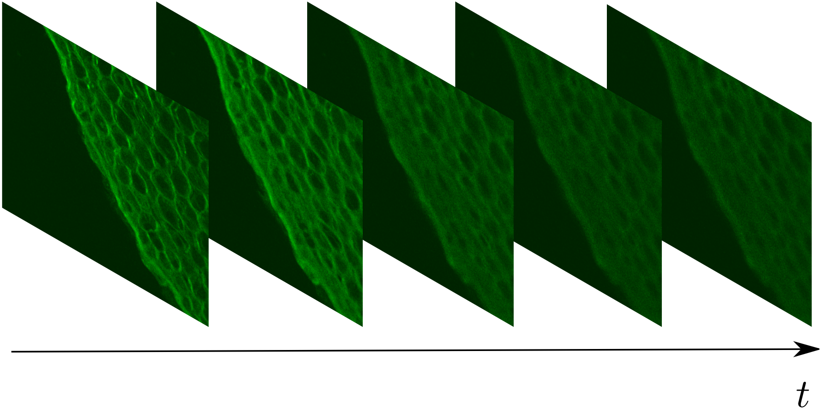
Unbiased MSE estimate depends on noise parameters which are assumed to be known.

METADATA VERSUS NOISE PARAMETERS

Metadata of fluorescence imaging systems (e.g. PMT Gain, PMT Offset) does not correspond to noise parameters (α, c) [Paul *et al.* 2010].

Noise parameters have to be estimated statistically.

Time samples - Bleaching effect



Bleaching effect - process of intensity time decay usually modeled with an exponentially decreasing function

Dependence between noise model and photobleaching

We adopted the same model as the one presented in [Rodrigues *et al.* 2009]

- Signal independent noise sources remain the same
- Intrinsic luminous intensity of fluorophore decays exponentially with time
- Fluorophore behaviour is not uniform across a sample.

As a result:

- The Gaussian part of the model remains time-independent
- The exponential decay is put inside the Poisson model.
- We do not assume a unique decay rate for all pixels

NOISE IDENTIFICATION - MODEL

$\forall s \in \{1, \ldots, S\}$
$\forall t \in \{1, \ldots, T\}$

$$R_{s,t} = \alpha Q_{s,t} + N_{s,t}$$

Observations

$r = (r_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$
$R = (R_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$

NOISE IDENTIFICATION - MODEL

Poisson noise -

$\forall s \in \{1, \dots, S\}$
$\forall t \in \{1, \dots, T\}$

$Q_{s,t} \sim \mathcal{P}(u_s e^{-k_s t})$
$\alpha \in \mathbb{R}$ - scaling par.
$(u_s)_{1 \leq s \leq S} \geq 0$ - "clean" image
$(k_s)_{1 \leq s \leq S} \geq 0$ - bleaching decay

$$R_{s,t} = \alpha Q_{s,t} + N_{s,t}$$

Observations

$r = (r_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$
$R = (R_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$

NOISE IDENTIFICATION - MODEL

$\forall s \in \{1, \ldots, S\}$
$\forall t \in \{1, \ldots, T\}$

Poisson noise -

$Q_{s,t} \sim \mathcal{P}(u_s e^{-k_s t})$
$\alpha \in \mathbb{R}$ - scaling par.
$(u_s)_{1 \leq s \leq S} \geq 0$ - "clean" image
$(k_s)_{1 \leq s \leq S} \geq 0$ - bleaching decay

$$R_{s,t} = \alpha Q_{s,t} + N_{s,t}$$

Observations

$r = (r_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$
$R = (R_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$

Gaussian noise

$N_{s,t} \sim \mathcal{N}(c, \sigma^2)$
$c \in \mathbb{R}$ - mean σ^2 - variance

NOISE IDENTIFICATION - MODEL

$\forall s \in \{1, \ldots, S\}$
$\forall t \in \{1, \ldots, T\}$

Poisson noise -

$Q_{s,t} \sim \mathcal{P}(u_s e^{-k_s t})$
$\alpha \in \mathbb{R}$ - scaling par.
$(u_s)_{1 \leq s \leq S} \geq 0$ - "clean" image
$(k_s)_{1 \leq s \leq S} \geq 0$ - bleaching decay

$$R_{s,t} = \alpha Q_{s,t} + N_{s,t}$$

Observations

$r = (r_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$
$R = (R_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$

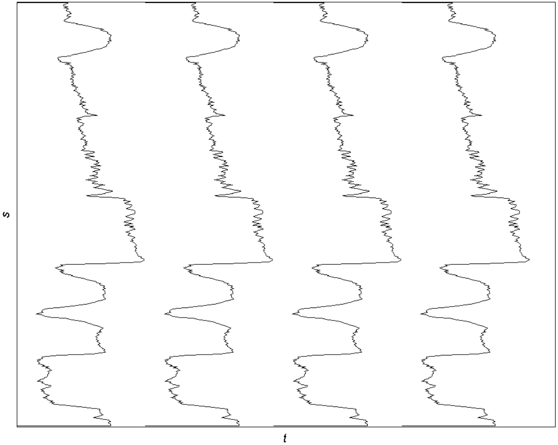
Gaussian noise

$N_{s,t} \sim \mathcal{N}(c, \sigma^2)$
$c \in \mathbb{R}$ - mean σ^2 - variance

Problem

Find $\theta = (u, k, \alpha, c, \sigma^2)$

EXAMPLE

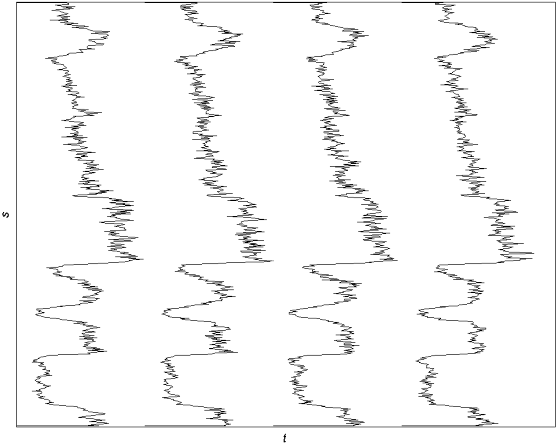


$U_{s,t}$

$T = 4$,
$S = 512$

Original signal

EXAMPLE

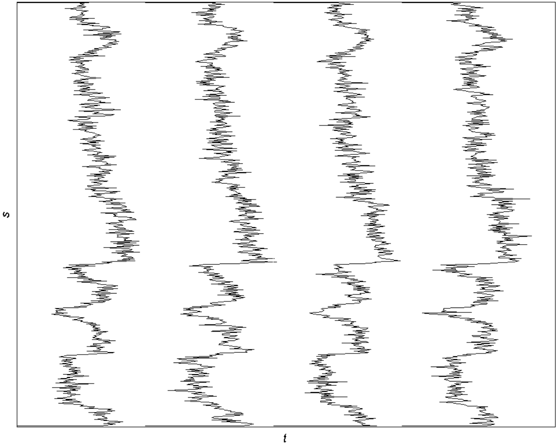


$Q_{s,t}$

$T = 4$,
$S = 512$

Signal corrupted by Poisson noise

Example

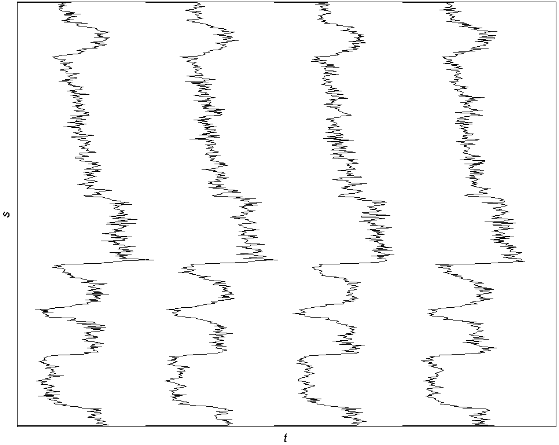


$U_{s,t} + N_{s,t}$

$T = 4$,
$S = 512$

Signal corrupted by Gaussian noise

EXAMPLE



$\alpha Q_{s,t} + N_{s,t}$

$T = 4$,
$S = 512$

Signal corrupted by Poisson and Gaussian noises

CUMULANTS

$$\kappa_n[R_{s,t}] = \alpha^n \kappa_n[Q_{s,t}] + \kappa_n[N_{s,t}]$$

Cumulants

$$\kappa_n[R_{s,t}] = \alpha^n \kappa_n[Q_{s,t}] + \kappa_n[N_{s,t}]$$

Cumulant of order n

CUMULANTS

$$\kappa_n[R_{s,t}] \;=\; \alpha^n \kappa_n[Q_{s,t}] \;+\; \kappa_n[N_{s,t}]$$

Then:

- mean value

$$\kappa_1[R_{s,t}] = \mathsf{E}[R_{s,t}] = \alpha e^{-k_s t} u_s + c$$

- variance

$$\kappa_2[R_{s,t}] = \mathsf{Var}[R_{s,t}] = \alpha^2 e^{-k_s t} u_s + \sigma^2$$

- higher-order cumulants

$$\kappa_n[R_{s,t}] = \alpha^n e^{-k_s t} u_s, \qquad n \geq 3$$

PROBLEM FORMULATION

Using $\mathsf{E}[R_{s,t}] = \alpha e^{-k_s t} u_s + c$

we have: $R_{s,t} = a_s \, e^{-k_s t} + c + E_{s,t}$

$(a_s = \alpha u_s)_{1 \leq s \leq S}$

$(E_{s,t})_{1 \leq s \leq S, 1 \leq t \leq T}$ - independent zero-mean random variables.

Optimization criteria

$$(\widehat{a}, \widehat{k}, \widehat{c}) = \underset{a,k,c}{\operatorname{argmin}} \sum_{s=1}^{S} \sum_{t=1}^{T} \omega_{s,t} \left(r_{s,t} - c - a_s e^{-k_s t} \right)^2$$

ALGORITHM - STEP 1

Initialization:

$c^{(0)} = \min\{\frac{1}{T}\sum_{t=1}^{T} r_{s,t}, 1 \leq s \leq S\}$

Main Loop:

for $n \leftarrow 1$ to N **do**

for $s \leftarrow 1$ to S **do**

$$(a_s^{(n)}, k_s^{(n)}) = \underset{a_s, k_s \geq 0}{\operatorname{argmin}} \sum_{t=1}^{T} \omega_{s,t}\left(r_{s,t} - c^{(n-1)} - a_s e^{-k_s t}\right)^2$$

end for

$$c^{(n)} = \sum_{s=1}^{S}\sum_{t=1}^{T}\omega_{s,t}(r_{s,t} - a_s^{(n)} e^{-k_s^{(n)} t}) / \sum_{s=1}^{S}\sum_{t=1}^{T}\omega_{s,t}$$

end for

$\widehat{a} = a^{(N)}, \widehat{k} = k^{(N)}, \widehat{c} = c^{(N)}$

Algorithm - Step 1

Initialization:

$c^{(0)} = \min\{\frac{1}{T}\sum_{t=1}^{T} r_{s,t}, 1 \leq s \leq S\}$

Main Loop:

for $n \leftarrow 1$ to N **do**

for $s \leftarrow 1$ to S **do**

$$(a_s^{(n)}, k_s^{(n)}) = \underset{a_s, k_s \geq 0}{\operatorname{argmin}} \sum_{t=1}^{T} \omega_{s,t}\left(r_{s,t} - c^{(n-1)} - a_s e^{-k_s t}\right)^2$$

end for

$$c^{(n)} = \sum_{s=1}^{S}\sum_{t=1}^{T} \omega_{s,t}(r_{s,t} - a_s^{(n)} e^{-k_s^{(n)} t}) / \sum_{s=1}^{S}\sum_{t=1}^{T} \omega_{s,t}$$

end for

$\widehat{a} = a^{(N)}, \widehat{k} = k^{(N)}, \widehat{c} = c^{(N)}$

Algorithm - Step 1

Initialization:

$c^{(0)} = \min\{\frac{1}{T}\sum_{t=1}^{T} r_{s,t}, 1 \leq s \leq S\}$

Main Loop:

for $n \leftarrow 1$ to N **do**

  for $s \leftarrow 1$ to S **do**

$$(a_s^{(n)}, k_s^{(n)}) = \operatorname*{argmin}_{a_s, k_s \geq 0} \sum_{t=1}^{T} \omega_{s,t} \left(r_{s,t} - c^{(n-1)} - a_s e^{-k_s t}\right)^2$$

  end for

$$c^{(n)} = \sum_{s=1}^{S}\sum_{t=1}^{T} \omega_{s,t}(r_{s,t} - a_s^{(n)} e^{-k_s^{(n)} t}) / \sum_{s=1}^{S}\sum_{t=1}^{T} \omega_{s,t}$$

end for

$\widehat{a} = a^{(N)}, \widehat{k} = k^{(N)}, \widehat{c} = c^{(N)}$

ALGORITHM - STEP 1

Initialization:

$c^{(0)} = \min\{\frac{1}{T}\sum_{t=1}^{T} r_{s,t}, 1 \leq s \leq S\}$

Main Loop:

for $n \leftarrow 1$ to N **do**

  for $s \leftarrow 1$ to S **do**

   $(a_s^{(n)}, k_s^{(n)}) = \underset{a_s, k_s \geq 0}{\text{argmin}} \sum_{t=1}^{T} \omega_{s,t} \left(r_{s,t} - c^{(n-1)} - a_s e^{-k_s t}\right)^2$

  end for

  $c^{(n)} = \sum_{s=1}^{S} \sum_{t=1}^{T} \omega_{s,t}(r_{s,t} - a_s^{(n)} e^{-k_s^{(n)} t}) / \sum_{s=1}^{S} \sum_{t=1}^{T} \omega_{s,t}$

end for

$\widehat{a} = a^{(N)}, \widehat{k} = k^{(N)}, \widehat{c} = c^{(N)}$

Remaining parameters

Closed-form expression

$$\widehat{\alpha} = \frac{ST\sum_{s=1}^{S} m_s v_s - \sum_{s=1}^{S} m_s \sum_{s=1}^{S} v_s}{ST\sum_{s=1}^{S} m_s^2 - \left(\sum_{s=1}^{S} m_s\right)^2}$$

$$\forall s \in \{1,\ldots,S\}) \quad \widehat{u}_s = \frac{\widehat{a}_s}{\widehat{\alpha}}$$

$$\widehat{\sigma}^2 = \frac{\sum_{(s,t)\in\mathbb{I}} \left(v_{s,t} - \widehat{\alpha}\widehat{a}_s e^{-\widehat{k}_s t}\right)}{\mathrm{card}(\mathbb{I})}$$

Where

$(\forall s \in \{1,\ldots,S\})$

$$m_s = \widehat{a}_s e^{-\widehat{k}_s} \frac{1 - e^{-\widehat{k}_s T}}{1 - e^{-\widehat{k}_s}}$$

$$v_s = \sum_{t=1}^{T} v_{s,t}$$

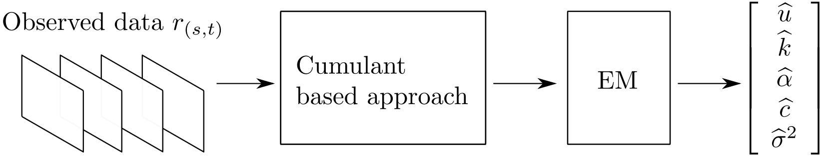
$\mathbb{I}$ - set imposing positivity

REFINED ESTIMATION

Cumulant based approach -summary

- Cumulant approach estimates vector of unknown parameters θ
- Unknown parameters are not jointly estimated
- Possible large error propagation

Refined estimation



EM ALGORITHM [JEZIERSKA *et al.* 2011]

Joint density function $f_R(r \mid u, k, \alpha, c, \sigma)$ for Poisson + Gaussian:

$$\frac{1}{(\sqrt{2\pi}\sigma)^{ST}} \prod_{s=1}^{S} e^{-Tu_s} \prod_{t=1}^{T} \sum_{q_{s,t}=1}^{+\infty} e^{-\frac{(r_{s,t}-\alpha q_{s,t}-c)^2}{2\sigma^2}} \frac{(u_s e^{-k_s t})^{q_{s,t}}}{q_{s,t}!}$$

Expectation step

$$J(\theta \mid \theta^{(n)}) = \mathsf{E}_{Q|R=r,\theta^{(n)}}[\ln p_{R,Q}(R , Q \mid \theta)]$$

R - observations, Q - missing data ; θ - vector of parameters

Maximization step

$$(\forall n \in \mathbb{N}) \qquad \theta^{(n+1)} = \underset{\theta}{\operatorname{argmin}} -J(\theta \mid \theta^{(n)})$$

EM MAXIMIZATION STEP

Update of $(\forall s \in \{1, \dots, S\}) \quad k_s^{(n+1)} = -\ln x^{(n+1)}$ **where** $x^{(n+1)}$ **is the solution in** $(0, 1)$ **of the polynomial equation:**

$$\left(1 + Tx^{T+1} - (T+1)x^T\right) \sum_{t=1}^{T} \mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}]$$
$$= \left(1 - x - x^T + x^{T+1}\right) \sum_{t=1}^{T} t\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}]$$

solved with Halley's algorithm

Update in a closed form: $u_s^{(n+1)}$, $\alpha^{(n+1)}$, $c^{(n+1)}$, $\sigma^{(n+1)}$ assuming known conditional mean $\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}]$ and $\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2]$ computed in EM Expectation Step.

EM Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \sum_{q_{s,t}=1}^{+\infty} q_{s,t} \mathsf{P}(Q_{s,t} = q_{s,t} \mid R = r, \theta^{(n)})$$

EM EXPECTATION STEP

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \sum_{q_{s,t}=1}^{+\infty} q_{s,t} \mathsf{P}(Q_{s,t} = q_{s,t} \mid R = r, \theta^{(n)})$$

After some calculations, we have:

EM Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \sum_{q_{s,t}=1}^{+\infty} q_{s,t} \mathsf{P}(Q_{s,t} = q_{s,t} \mid R = r, \theta^{(n)})$$

After some calculations, we have:

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

EM EXPECTATION STEP

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \sum_{q_{s,t}=1}^{+\infty} q_{s,t}\mathsf{P}(Q_{s,t} = q_{s,t} \mid R = r, \theta^{(n)})$$

After some calculations, we have:

$$\sum_{q_{s,t}=1}^{+\infty} e^{-\frac{(r_{s,t}-\alpha^{(n)}q_{s,t}-c^{(n)})^2}{2(\sigma^2)^{(n)}}} \frac{(u_s^{(n)}e^{-k_s^{(n)}t})^{q_{s,t}}}{(q_{s,t}-1)!}$$

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

EM Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \sum_{q_{s,t}=1}^{+\infty} q_{s,t} \mathsf{P}(Q_{s,t} = q_{s,t} \mid R = r, \theta^{(n)})$$

After some calculations, we have:

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

$$\zeta_{s,t}^{(n)} = \sum_{q_{s,t}=1}^{+\infty} e^{-\frac{(r_{s,t}-\alpha^{(n)} q_{s,t} - c^{(n)})^2}{2(\sigma^2)^{(n)}}} \frac{(u_s^{(n)} e^{-k_s^{(n)} t})^{q_{s,t}}}{(q_{s,t}-1)!}$$

$$\eta_{s,t}^{(n)} = \sum_{q_{s,t}=0}^{+\infty} e^{-\frac{(r_{s,t}-\alpha^{(n)} q_{s,t} - c^{(n)})^2}{2(\sigma^2)^{(n)}}} \frac{(u_s^{(n)} e^{-k_s^{(n)} t})^{q_{s,t}}}{q_{s,t}!}$$

EM EXPECTATION STEP

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \sum_{q_{s,t}=1}^{+\infty} q_{s,t}\mathsf{P}(Q_{s,t} = q_{s,t} \mid R = r, \theta^{(n)})$$

After some calculations, we have:

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] = \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

$$\sum_{q_{s,t}=1}^{+\infty} e^{-\frac{(r_{s,t}-\alpha^{(n)}q_{s,t}-c^{(n)})^2}{2(\sigma^2)^{(n)}}} \frac{(u_s^{(n)}e^{-k_s^{(n)}t})^{q_{s,t}}}{(q_{s,t}-1)!}$$

$$\sum_{q_{s,t}=0}^{+\infty} e^{-\frac{(r_{s,t}-\alpha^{(n)}q_{s,t}-c^{(n)})^2}{2(\sigma^2)^{(n)}}} \frac{(u_s^{(n)}e^{-k_s^{(n)}t})^{q_{s,t}}}{q_{s,t}!}$$

Problem: Infinite sums

Solution: Adaptive truncation technique

$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2]$ is computed similarly

PROPOSED ALGORITHM

Step 1: Cumulant Method

$\theta^{(1)} \leftarrow \left(u^{(1)}, k^{(1)}, \alpha^{(1)}, c^{(1)}, (\sigma^2)^{(1)}\right)$

Step 2: EM Algorithm

for $n \leftarrow 1$ to N **do**

Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] \leftarrow \frac{\zeta^{(n)}_{s,t}}{\eta^{(n)}_{s,t}}$$

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q^2_{s,t}] \leftarrow \frac{\xi^{(n)}_{s,t}}{\eta^{(n)}_{s,t}}$$

Maximization step

$$\theta^{(n+1)} \leftarrow \left(u^{(n+1)}, k^{(n+1)}, \alpha^{(n+1)}, c^{(n+1)}, (\sigma^2)^{(n+1)}\right)$$

end for

PROPOSED ALGORITHM

Step 1: Cumulant Method

$\theta^{(1)} \leftarrow \left(u^{(1)}, k^{(1)}, \alpha^{(1)}, c^{(1)}, (\sigma^2)^{(1)}\right)$

Step 2: EM Algorithm

for $n \leftarrow 1$ to N **do**

Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] \leftarrow \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2] \leftarrow \frac{\xi_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

Maximization step

$$\theta^{(n+1)} \leftarrow \left(u^{(n+1)}, k^{(n+1)}, \alpha^{(n+1)}, c^{(n+1)}, (\sigma^2)^{(n+1)}\right)$$

end for

Proposed Algorithm

Step 1: Cumulant Method

$\theta^{(1)} \leftarrow \left(u^{(1)}, k^{(1)}, \alpha^{(1)}, c^{(1)}, (\sigma^2)^{(1)}\right)$

Step 2: EM Algorithm

for $n \leftarrow 1$ to N **do**

Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] \leftarrow \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2] \leftarrow \frac{\xi_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

Maximization step

$$\theta^{(n+1)} \leftarrow \left(u^{(n+1)}, k^{(n+1)}, \alpha^{(n+1)}, c^{(n+1)}, (\sigma^2)^{(n+1)}\right)$$

end for

PROPOSED ALGORITHM

Step 1: Cumulant Method

$\theta^{(1)} \leftarrow \left(u^{(1)}, k^{(1)}, \alpha^{(1)}, c^{(1)}, (\sigma^2)^{(1)}\right)$

Step 2: EM Algorithm

for $n \leftarrow 1$ to N **do**

Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] \leftarrow \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2] \leftarrow \frac{\xi_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

Maximization step

$$\theta^{(n+1)} \leftarrow \left(u^{(n+1)}, k^{(n+1)}, \alpha^{(n+1)}, c^{(n+1)}, (\sigma^2)^{(n+1)}\right)$$

end for

Proposed Algorithm

Step 1: Cumulant Method

$\theta^{(1)} \leftarrow \left(u^{(1)}, k^{(1)}, \alpha^{(1)}, c^{(1)}, (\sigma^2)^{(1)}\right)$

Step 2: EM Algorithm

for $n \leftarrow 1$ to N **do**

Expectation step

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}] \leftarrow \frac{\zeta_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

$$\mathsf{E}_{Q|R=r,\theta^{(n)}}[Q_{s,t}^2] \leftarrow \frac{\xi_{s,t}^{(n)}}{\eta_{s,t}^{(n)}}$$

Maximization step

$$\theta^{(n+1)} \leftarrow \left(u^{(n+1)}, k^{(n+1)}, \alpha^{(n+1)}, c^{(n+1)}, (\sigma^2)^{(n+1)}\right)$$

end for

Synthetic data results

Evaluation criteria

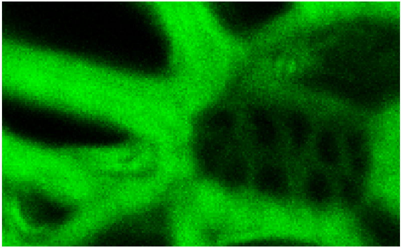
$$\mathrm{SNR} = 10\log_{10}\left(\frac{(ST)^{-1}\sum_{(t,s)}\left(a_s e^{-k_s t}\right)^2}{\sum_{(t,s)}\left(a_s e^{-k_s t} - \widehat{a}_s e^{-\widehat{k}_s t}\right)^2}\right)$$

Results

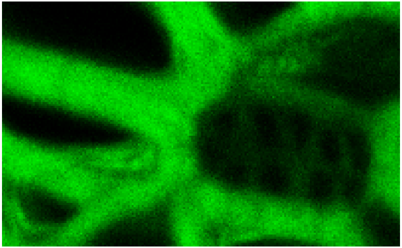
Statistics over 50 noise realizations.

Method.	$\hat{\sigma}$		$\hat{c}$		$\hat{\alpha}$		$\overline{\mathrm{SNR}}$
	bias	std	bias	std	bias	std	
Init.	357.5	3.1	1.9	1.0	−0.3	0.4	39.5
EM	2.9	0.9	1.4	0.8	−0.3	0.4	39.7

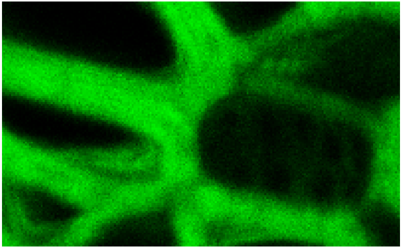
Table: Experiment conditions: $T = 180$, $S = 200$, $\alpha = 30$, $\sigma^2 = 100$, $c = 10$, $u_s \in [0, 150]$, $k_s \in [10^{-4}, 10^{-3}]$



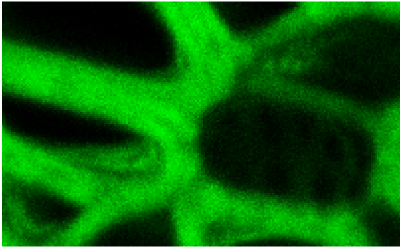
(a) $R_{s,1}$



(b) $R_{s,45}$

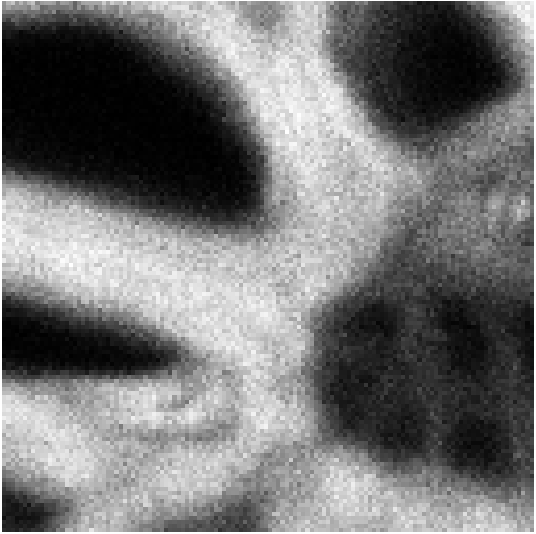


(c) $R_{s,90}$



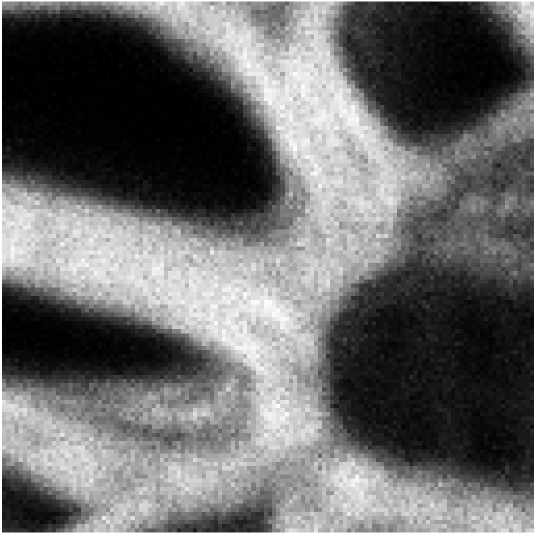
(d) $R_{s,180}$

VISUAL RESULTS



Original image $r_{s,1}$

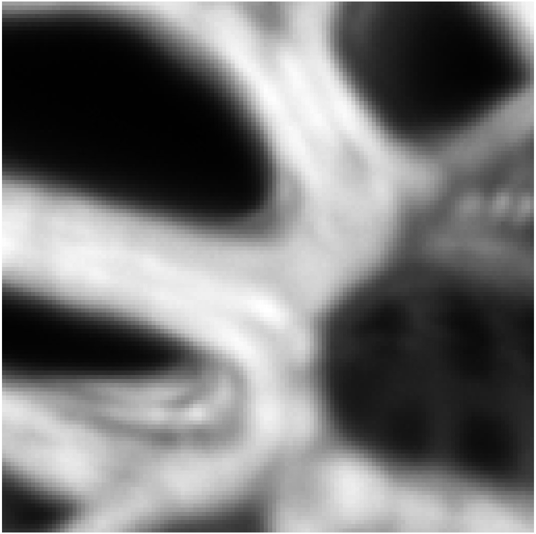
Visual results



Original image $r_{s,1}$

Original image $r_{s,180}$

Visual results

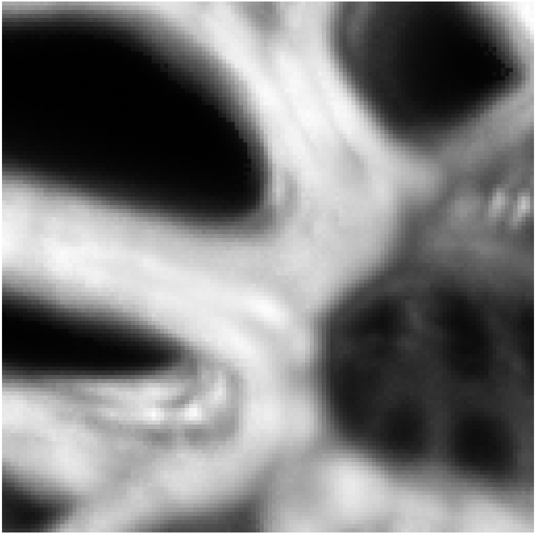


Original image $r_{s,1}$

Original image $r_{s,180}$

Mean over $T = 180$ realizations

VISUAL RESULTS



Original image $r_{s,1}$

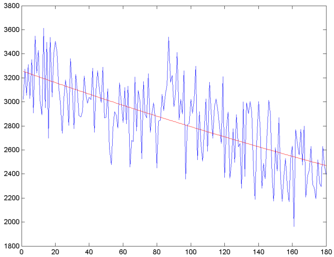
Original image $r_{s,180}$

Mean over $T = 180$ realizations

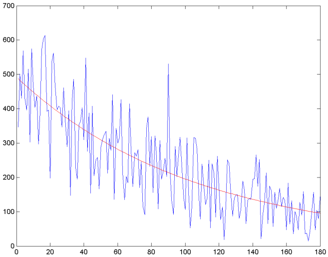
Reconstructed image

Parameters:
$\widehat{\alpha} = 25.8$
$\widehat{c} = 8$
$\widehat{\sigma}^2 = 119$
$\widehat{u}_s \in [0, 147]$
$\widehat{k}_s \in [0, 3.9 \times 10^{-6}]$

Time variations



(a) $25.8 \times 126\, e^{-5.7 \times 10^{-7} t} + 8$



(b) $25.8 \times 19\, e^{-3.6 \times 10^{-6} t} + 8$

Figure: (a,b) illustrate time variations for fixed s. True data $R_{s,t}$ plotted in blue and estimated time curve (using formula $\widehat{\alpha}\widehat{u}_s e^{-\widehat{k}_s t} + \widehat{c}$) plotted in red.

Summary

- We have proposed a new EM-based approach, which jointly estimates Poisson and Gaussian noise parameters.

SUMMARY

- We have proposed a new EM-based approach, which jointly estimates Poisson and Gaussian noise parameters.
- These algorithms can compute unknown noise parameters for denoising or restoration procedures, which normally are assumed to be known [Benvenuto *et al.* 2008, Luisier *et al.* 2011]

Summary

- We have proposed a new EM-based approach, which jointly estimates Poisson and Gaussian noise parameters.
- These algorithms can compute unknown noise parameters for denoising or restoration procedures, which normally are assumed to be known [Benvenuto *et al.* 2008, Luisier *et al.* 2011]
- Apart from noise parameters, we can also estimate the rate of bleaching.

FUTURE WORK

- The study of noise parameters in MACROscope, confocal and widefield case.
- Image restoration method with proposed noise model and space varying PSF

Summary

Our work concerning image restoration / denoising, noise identification, and image quantization in the presence of noise

- A. Jezierska, C. Chaux , H. Talbot and J.-C. Pesquet.
  Image quantization under spatial smoothness constraints,
  Sep. 26-29 Hong Kong, ICIP 2010
- C. Chaux , A. Jezierska, H. Talbot and J.-C. Pesquet.
  JMIV, 2010, A spatial regularization approach for vector quantization,
- A. Jezierska, H. Talbot, O. Veksler, and D. Wesierski.
  A fast solver for truncated-convex priors: quantized-convex split moves
  EMMCVPR 2011, Saint Petersburg, 25-27 July 2011
- A. Jezierska, C. Chaux , J.-C. Pesquet and H. Talbot .
  An EM approach for Poisson-Gaussian noise modeling
  EUSIPCO 2011, Barcelona, 29 August - 2 September 2011
- E. Chouzenoux, J.-C. Pesquet, H. Talbot, and A. Jezierska
  A memory gradient algorithm for l2 - l0 regularization with applications to image restoration,
  ICIP 2011, Brussels, 11-14 September 2011.