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FENCHEL-ROCKAFELLAR DUALIZATION OF SIGNAL RECOVERY PROBLEMS

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In [1,2] we have shown that many problems in signal recovery can be reduced to the minimization of the sum $f_1 + f_2$ of two proper lower semicontinuous convex functions in a Hilbert space $\mathcal{H}$ and provided reliable algorithms to solve this variational problem. To be numerically implementable, these algorithms require that the proximity operator of f_1 be tractable and that either f_2 be Lipschitz differentiable on $\mathcal{H}$ [1] or that its proximity operator be tractable [2]. Important formulations do not fall within these frameworks, which leaves their numerical solution an open problem. We show that a such problems can often be approached via Fenchel-Rockafellar duality. More specifically, we propose a duality framework that leads to a numerically implementable algorithm for solving both the primal problem and its dual. Convergence to primal and dual solutions is formally established. This framework captures and extends several existing duality schemes that were often devised without formal convergence proofs, and makes it possible to solve a variety of new structured signal recovery problems outside the scope of [1,2].

References

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