DECODING IN COMPRESSED SENSING

Ronald A. DeVore

Department of Mathematics
Texas A & M University

Compressed sensing is a new paradigm for signal and image processing. In discrete compressed sensing, we want to capture a vector \(x \in \mathbb{R}^N\) (where \(N\) is very large) by using a much smaller number \(n\) of measurements. These new measurements are to take the form of inner products of \(x\) with prescribed vectors \(r_1, \ldots, r_n \in \mathbb{R}^N\). Thus the information we extract from \(x\) consists of the \(n\) numbers \(r_j \cdot x\). This can be represented in matrix form as \(y = \Phi x\) where the rows of \(\Phi\) are the vectors \(r_1, \ldots, r_n\) and the output \(y\) is the information we record about \(x\).

Compressed sensing is successful if the signal \(x\) is either sparse (i.e. has relatively few nonzero components) or compressible (can be well approximated by sparse vectors).

The two main questions in compressed sensing are: (i) what are the best sensing matrices \(\Phi\), (ii) how do we decode (approximate) \(x\) from the information \(y\) that we have about \(x\).

This talk will say a little about (i) but will concentrate on item (ii) since it entails optimization techniques. We shall discuss the three main methods used for decoding in (ii), namely, \(\ell_1\) minimization, greedy algorithms, and iterative reweighted least squares. We shall formulate a criteria called ‘instance optimality’ for measuring performance of the encoding-decoding and then discuss which encoder-decoder pairs achieve the highest range of instance optimality.