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DECODING IN COMPRESSED SENSING

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Compressed sensing is a new paradigm for signal and image processing. In discrete compressed sensing, we want to capture a vector $x \in \mathbb{R}^N$ (where N is very large) by using a much smaller number n of measurements. These new measurements are to take the form of inner products of x with prescribed vectors $r_1, \ldots, r_n \in \mathbb{R}^N$. Thus the information we extract from x consists of the n numbers $r_j \cdot x$. This can be represented in matrix form as $y = \Phi x$ where the rows of Φ are the vectors $r_1, \ldots, r_n$ and the output y is the information we record about x.

Compressed sensing is successful if the signal x is either sparse (i.e. has relatively few nonzero components) or compressible (can be well approximated by sparse vectors). The two main questions in compressed sensing are : (i) what are the best sensing matrices Φ, (ii) how do we decode (approximate) x from the information y that we have about x.

This talk will say a little about (i) but will concentrate on item (ii) since it entails optimization techniques. We shall discuss the three main methods used for decoding in (ii), namely, ℓ_1 minimization, greedy algorithms, and iterative reweighted least squares. We shall formulate a criteria called 'instance optimality' for measuring performance of the encoding-decoding and then discuss which encoder-decoder pairs achieve the highest range of instance optimality.