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FIRST ORDER METHODS FOR NON-SMOOTH CONVEX OPTIMIZATION : PROXIMAL FORWARD-BACKWARD SPLITTING AND GENERALIZED GRADIENT METHODS

Dirk A. Lorenz[1]

[1]Technical University Carolo-Wilhelmina at Brunswick, Germany
d.lorenz@tu-braunschweig.de

Several problems in signal and image processing involve non-smooth convex optimization problems. Popular examples are TV methods in image processing or decoding by ℓ^1 minimiziation in compressed sensing and regularization of inverse problems [3]. In this talk we study first order methods for the minimization of functionals of the form $S + R$ where S is differentiable with Lipschitz continuous derivative and R is convex. In particular we deal with forward-backward splitting methods of the form

$$u^{n+1} = (I + \partial R)^{-1}(u^n - s_n S'(u^n))$$

which exploit that the operator $(I + \partial R)$ may be easily invertible [2]. We derive a resembling method as a generalized gradient projection method. With the help of this reformulation we are able to prove strong convergence of the iterates. Moreover, we will give conditions under which the algorithm converges with linear speed [1]. These conditons are fulfilled, for example, for the case of ℓ^1 minimization problems. The viewpoint as a generalized gradient projection method will enable us to extend some parts of the theory to non-convex functionals R.

References

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