

**AVERAGE PERFORMANCE OF THE SPARSEST
APPROXIMATION USING A DICTIONARY****François Malgouyres¹ and Mila Nikolova²**¹Université Paris 13, CNRS UMR 7539 LAGA
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We consider the minimization of the number of non-zero coefficients (the ℓ_0 “norm”) of the representation of a data set in a general dictionary under a fidelity constraint. (The dictionary and the norm defining the constraint are arbitrary.) This (nonconvex) optimization problem leads to the sparsest approximation.

Our goal is to assess the average performance of this approximation (APA). To this end, we describe the geometry of the sets of data yielding a K sparse solution and then evaluate their Lebesgue measure. Under the assumption that data are uniformly distributed on a domain defined using a norm ν , we derive the probability to obtain a K -sparse solution. These probabilities are expressed in terms of the parameters of the model and the accuracy of the approximation. We comment the obtained formulas and give a simulation.

References

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