

# Vector Lifting Schemes for Stereo Images Coding

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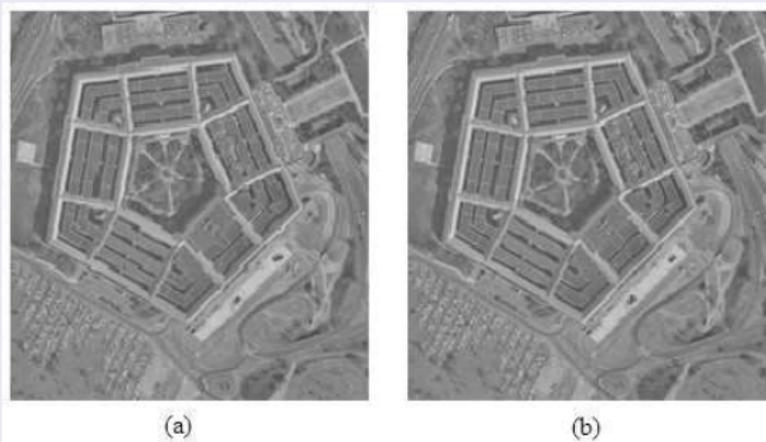
- 1 Context of study
- 2 Basic approach for joint coding of stereo image
- 3 Novel joint SI coding
- 4 Performances evaluation
- 5 Conclusions and perspectives

# Part I

## Context of study

## Type of data

**Stereo Image (SI):** Two images, captured from two viewpoints, corresponding to the same scene



The SI pair “pentagon”: (a) left image, (b) right image

## Interest of SI:

3D shape reconstruction in remote sensing, medical imaging



3D reconstruction of the SI pair “pentagon”

## Problem:

Constitution of enormous amounts of data.

## Our objectives: design a Stereo Image (SI) coding scheme with

- **Lossless** reconstruction  
⇒ exact decoding of SI (required for remote sensing imaging applications)
- **Progressive reconstruction**  
⇒ gradual decoding that generates two compact multiresolution representations of SI (suitable for telebrowsing applications)

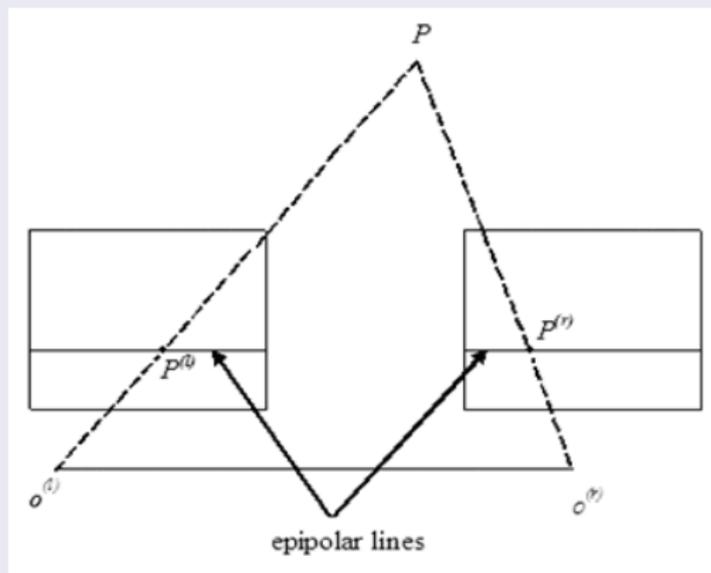
## Part II

# Basic approach for joint coding of stereo image

# Cross-view redundancies

Binocular imaging system:

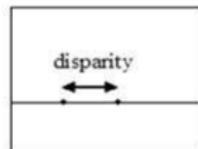
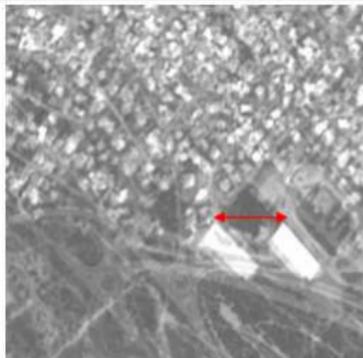
Assumption: rectified images.



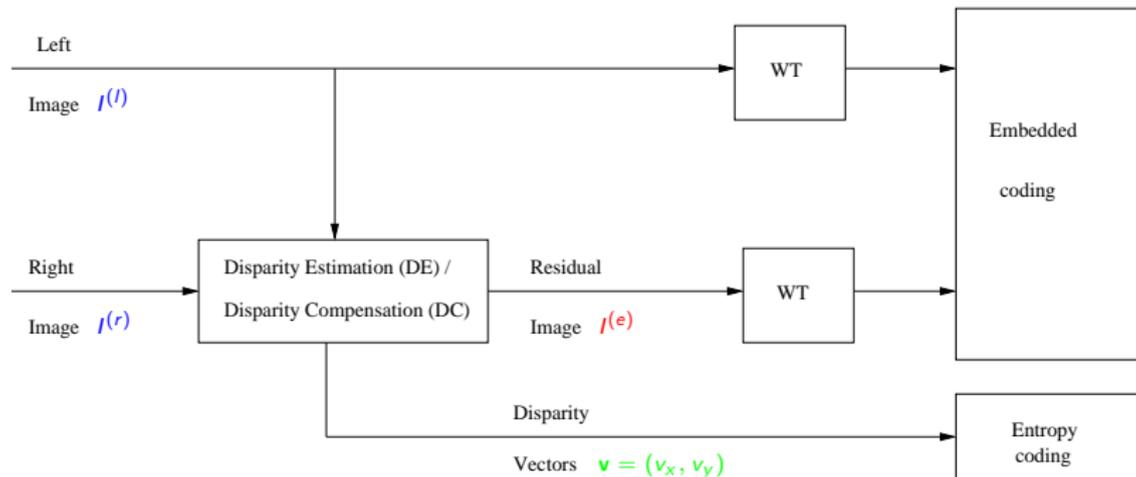
Binocular imaging system



If the two images are superimposed



# Generic decomposition scheme for SI progressive coding



$$\hat{I}^{(r)}(m, n) = I^{(l)}(m + v_x, n + v_y). \quad (1)$$

$$I^{(e)}(m, n) = I^{(r)}(m, n) - \hat{I}^{(r)}(m, n) \quad (2)$$

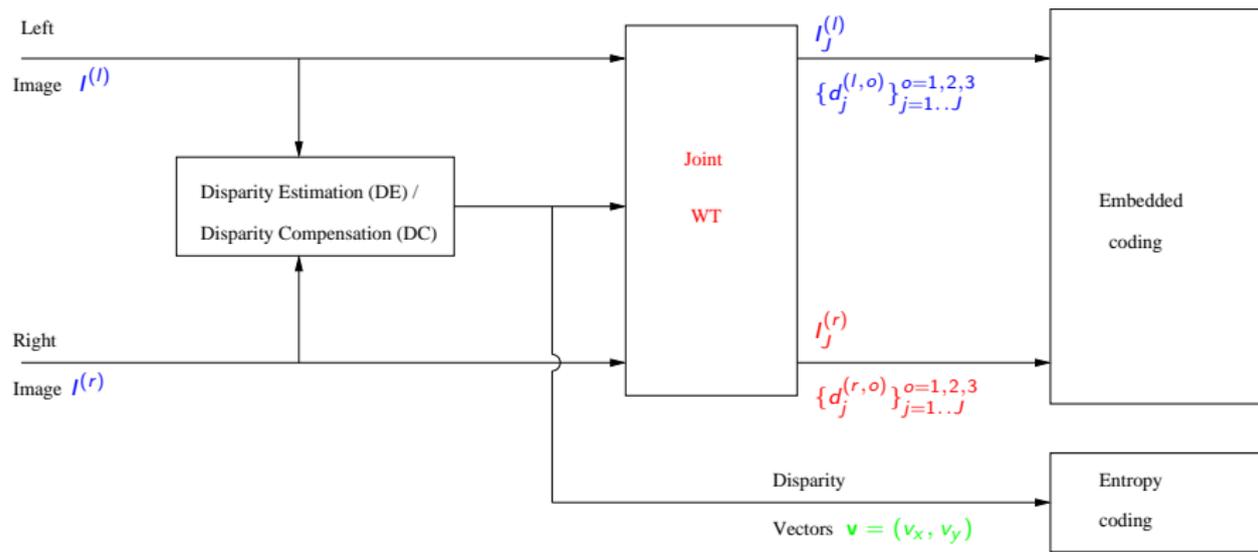
## Part III

# Novel joint SI coding

# Novel joint SI coding

## Motivation:

A new approach based on the Vector Lifting Scheme (VLS).



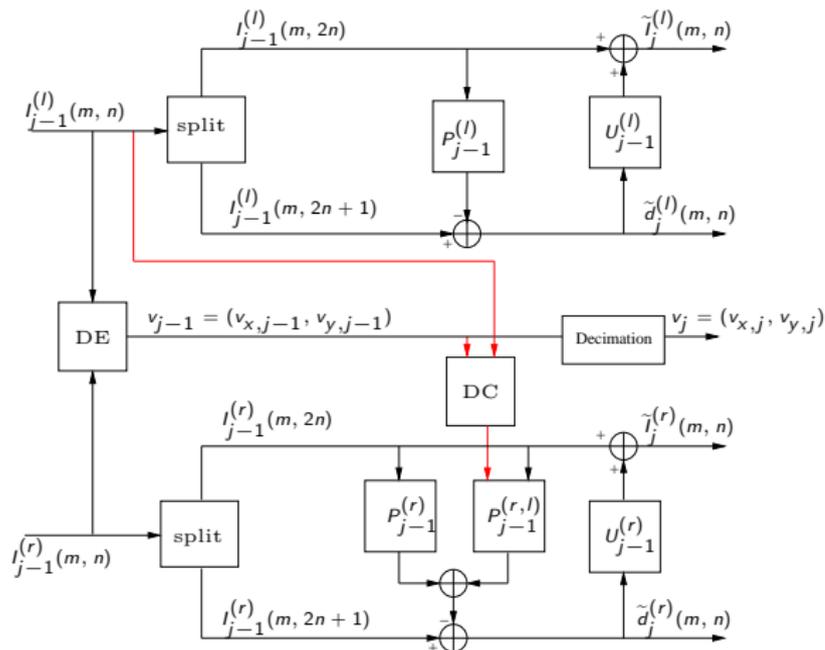
## Advantages:

- No generation of a residual image, but two multiresolution representations of  $I^{(l)}$  and  $I^{(r)}$ .
- Separable decompositions  $\implies$  Simplicity of their implementation

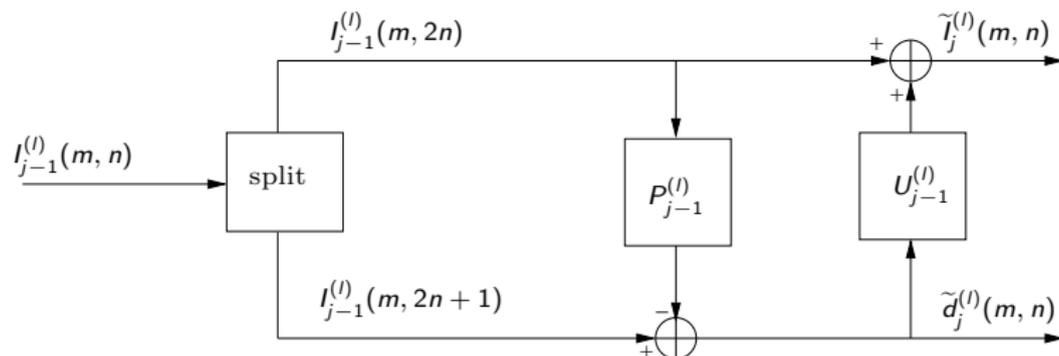
## Two decomposition examples:

- VLS-I
- VLS-II

## Principle of VLS-I decomposition



# Equations of VLS-I

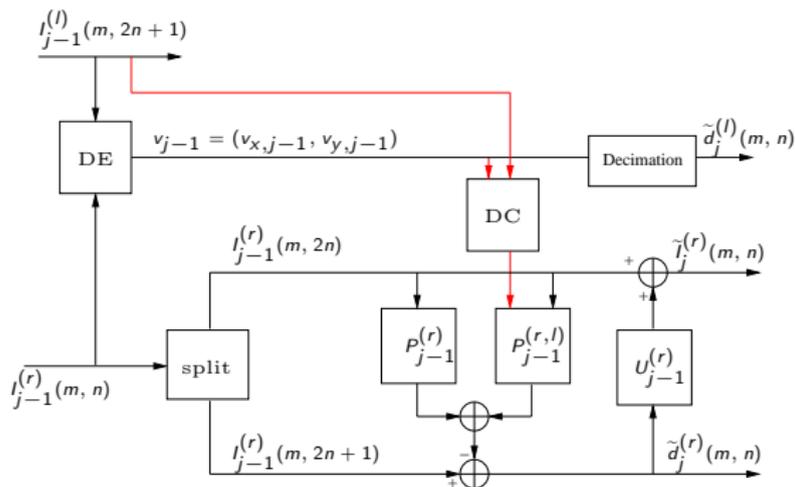


For the reference image  $I^{(l)}$ :

$$\tilde{d}_j^{(l)}(m, n) = I_{j-1}^{(l)}(m, 2n+1) - \lfloor \frac{1}{2}(I_{j-1}^{(l)}(m, 2n) + I_{j-1}^{(l)}(m, 2n+2)) \rfloor \quad (3)$$

$$\tilde{I}_j^{(l)}(m, n) = I_{j-1}^{(l)}(m, 2n) + \lfloor \frac{1}{4}(\tilde{d}_j^{(l)}(m, n-1) + \tilde{d}_j^{(l)}(m, n)) \rfloor \quad (4)$$

# Equations of VLS-I



For the right image  $I^{(r)}$ :

$$\begin{aligned} \tilde{d}_j^{(r)}(m, n) = & I_{j-1}^{(r)}(m, 2n+1) - [p_{j-1,1}^{(r)} I_{j-1}^{(r)}(m, 2n) + p_{j-1,2}^{(r)} I_{j-1}^{(r)}(m, 2n+2) \\ & + p_{j-1,3}^{(r,l)} I_{j-1}^{(l)}(m + v_{x,j-1}(m, 2n+1), 2n+1 + v_{y,j-1}(m, 2n+1))] \end{aligned} \quad (5)$$

$$\tilde{I}_j^{(r)}(m, n) = I_{j-1}^{(r)}(m, 2n) + \left[ \frac{1}{4} (\tilde{d}_j^{(r)}(m, n-1) + \tilde{d}_j^{(r)}(m, n)) \right] \quad (6)$$

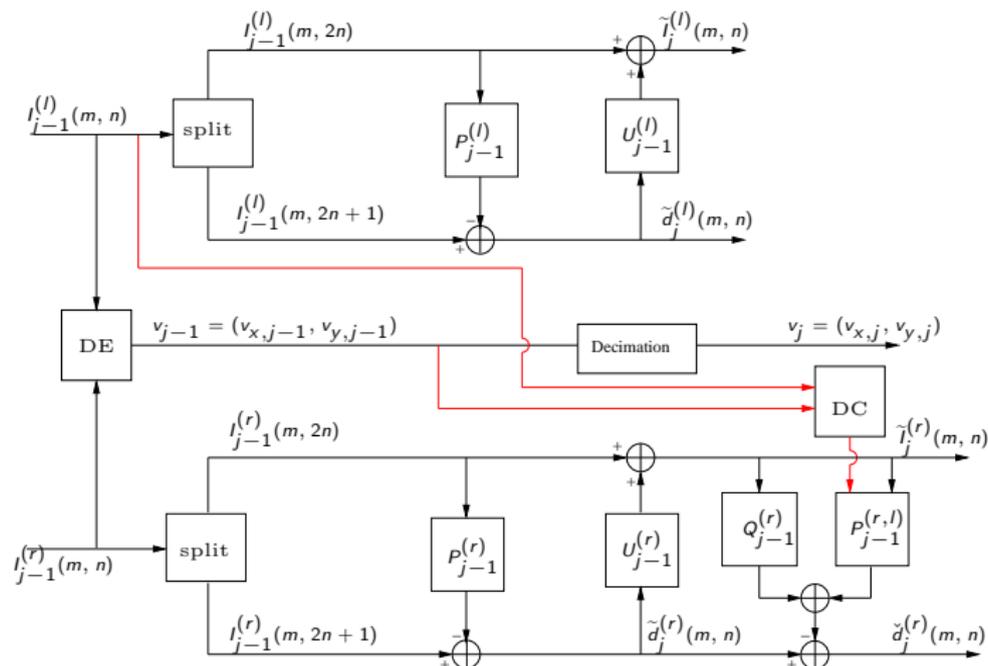
## Motivation:

VLS-I: P-U structure  $\Rightarrow$  approximation coefficients  $\tilde{l}_j^{(l)}(m, n)$  inserted into  $\tilde{d}_j^{(r)}(m, n)$  and then into  $\tilde{l}_j^{(r)}(m, n) \Rightarrow$  an update leakage effect.

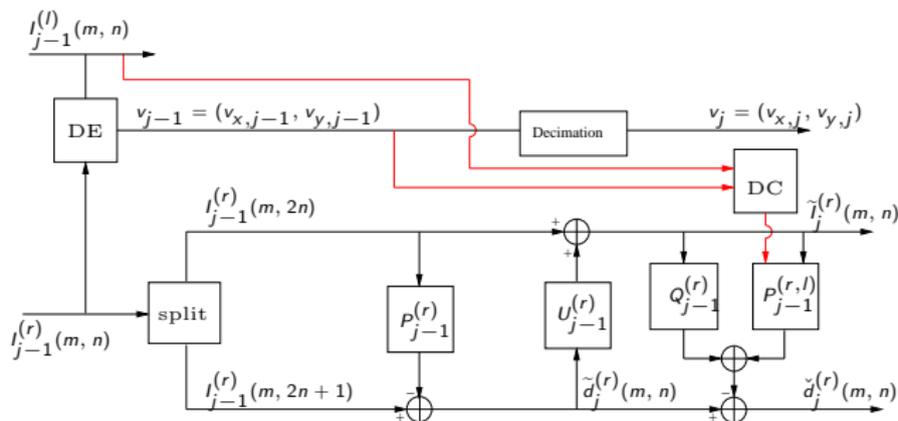
Proposed solution VLS-II: Another lifting with a  $P_1$ -U- $P_2$  structure.

- $P_1$  step: compute an intermediate detail signal by exploiting only the intra-image redundancies.
- U step: compute the approximation signal based on this intermediate detail signal.
- $P_2$  step: compute the final detail signal by exploiting the intra and inter-images redundancies.

## Principle of VLS-II decomposition



# Equations of VLS-II



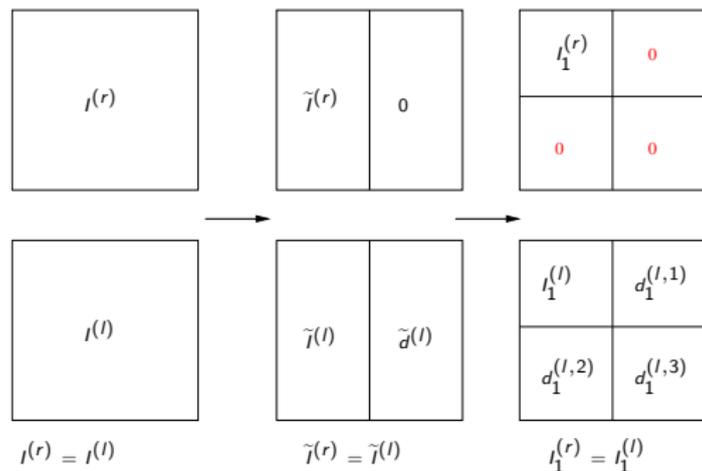
$$\tilde{d}_j^{(r)}(m, n) = I_{j-1}^{(r)}(m, 2n+1) - \lfloor \frac{1}{2}(I_{j-1}^{(r)}(m, 2n) + I_{j-1}^{(r)}(m, 2n+2)) \rfloor, \quad (7)$$

$$\tilde{l}_j^{(r)}(m, n) = I_{j-1}^{(r)}(m, 2n) + \lfloor \frac{1}{4}(\tilde{d}_j^{(r)}(m, n-1) + \tilde{d}_j^{(r)}(m, n)) \rfloor, \quad (8)$$

$$\check{d}_j^{(r)}(m, n) = \tilde{d}_j^{(r)}(m, n) - \lfloor q_{j-1}(\tilde{l}_j^{(r)}(m, n) + \tilde{l}_j^{(r)}(m, n+1)) + \sum_{k=-3}^3 p_{j-1,k}^{(r,l)} s_{j-1}^{(l)}(m + v_{x,j-1}(m, 2n+1), 2n+1 + v_{y,j-1}(m, 2n+1) - k) \rfloor, \quad (9)$$

## Advantage of VLS-II:

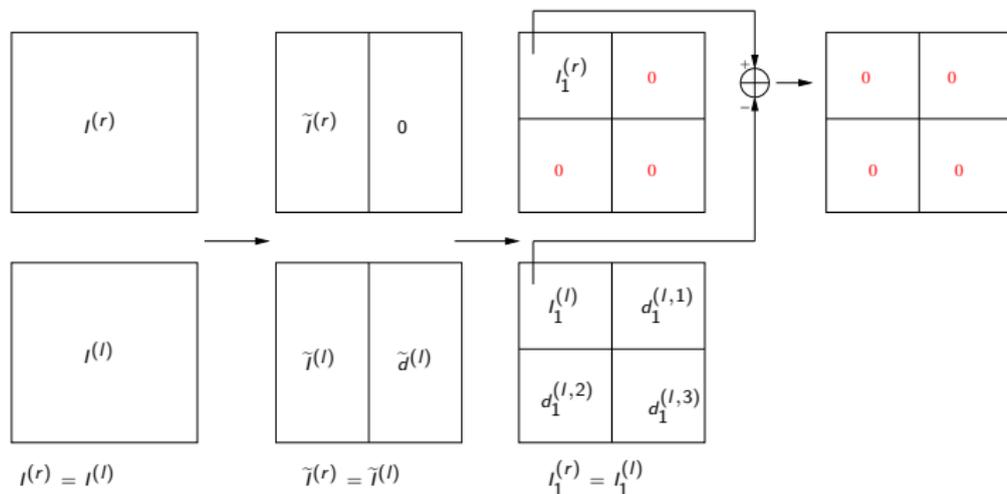
If  $I^{(r)} = I^{(l)}$  and  $\{0, 1, -1\} \subseteq \mathcal{P}_j^{(r,l)} \Rightarrow \tilde{I}^{(r)} = \tilde{I}^{(l)}$  and  $\check{d}^{(r)} = 0 \Rightarrow I_1^{(r)} = I_1^{(l)}$   
 and  $d_1^{(r,o)} = 0, \quad o \in \{1, 2, 3\}$



## Advantage of VLS-II:

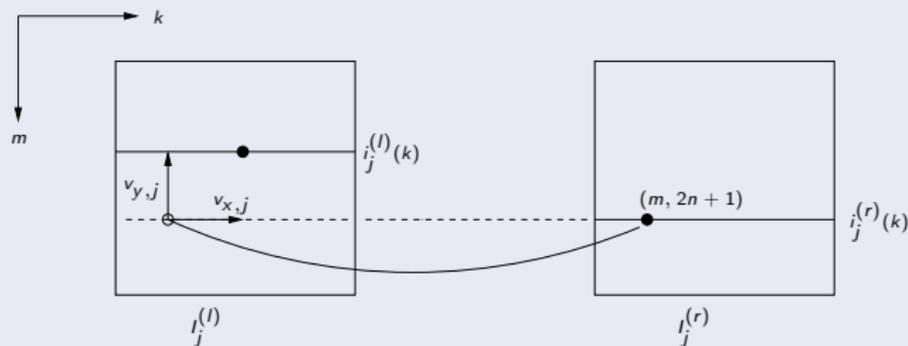
Finally, at the last resolution level  $j = J$ , instead of coding  $I_j^{(r)}$ , we code the residual subimage:

$$e_j^{(r)}(m, n) = I_j^{(r)}(m, n) - I_j^{(l)}(m + v_{x,J}(m, n), n + v_{y,J}(m, n)) \quad (10)$$



# Theoretical analysis of VLS-II:

Let  $(m, 2n + 1)$  be a given pixel to be predicted,



$$\begin{cases} i_j^{(r)}(k) = I_j^{(r)}(m, k), \\ i_j^{(l)}(k) = I_j^{(l)}(m + v_{x,j}(m, 2n + 1), k + v_{y,j}(m, 2n + 1)) \end{cases}, \quad (11)$$

## Assumption

$$\begin{cases} i_j^{(r)}(k) = \sin(\theta_j)x_j(k) + \cos(\theta_j)y_j(k) \\ i_j^{(l)}(k) = \cos(\theta_j)x_j(k) + \sin(\theta_j)y_j(k) \end{cases}, \quad (12)$$

where

- $x_j, y_j$ : AR(1), independent.
- $E[x_j(k)] = E[y_j(k)] = 0$ .
- $E[\{x_j(k)\}^2] = E[\{y_j(k)\}^2] = \sigma_j^2$ .

## Some properties

- $E[x_j(k)x_j(k-l)] = E[y_j(k)y_j(k-l)] = \sigma_j^2 \rho_j^{|l|}$ .
- $E[i_j^{(r)}(n)i_j^{(r)}(n-k)] = E[i_j^{(l)}(n)i_j^{(l)}(n-k)] = \rho_j^{|k|}$
- $E[i_j^{(r)}(n)i_j^{(l)}(n-k)] = s_j \rho_j^{|k|}$   
where  $s_j = \sin(2\theta_j)$ .
- The factor  $\theta_j$  controls the cross-redundancies between the samples  $i_j^{(l)}(k)$  and  $i_j^{(r)}(k)$ .

# Minimum prediction error variance

- independent scheme:

$$E[\{\tilde{d}_j^{(r)}(k)\}^2] = \frac{1}{2}(1 - \rho_j)(3 - \rho_j) \quad (13)$$

- VLS-I:

$$\varepsilon_{1,j}(\rho_j, \theta_j) = \sigma_j^2 \gamma_{1,j} \cos^2(2\theta_j)(\rho_j^2 - 1) \quad (14)$$

where

$$\gamma_{1,j} = 2 \sin^2(2\theta_j)(\rho_j^2 - \rho_j^2 - 1)^{-1}$$

- VLS-II:

$$\varepsilon_{2,j}(\rho_j, \theta_j) = \frac{1}{2} \sigma_j^2 \gamma_{2,j} \cos^2(2\theta_j)(1 - \rho_j)(3\rho_j^4 - 16\rho_j^3 + 4\rho_j^2 + 24\rho_j + 113) \quad (15)$$

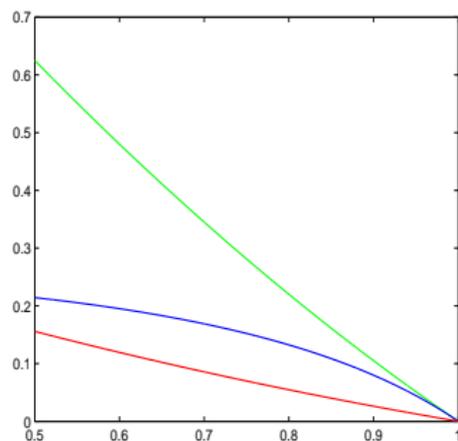
where  $\gamma_{2,j} = (\rho_j^5 - 5\rho_j^4 - \rho_j^3 + 13\rho_j^2 + 18\rho_j + 38)^{-1}$ .

# Performances of independent scheme, VLS-I and VLS-II in terms of prediction efficiency

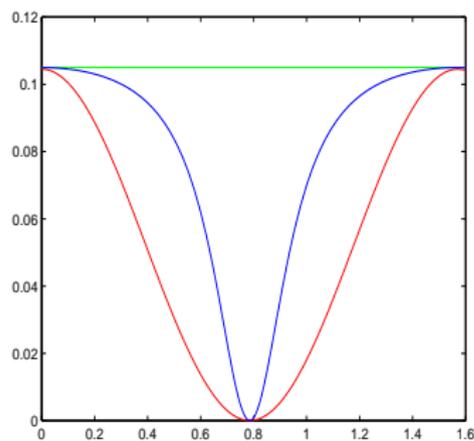
$$E[\{\tilde{d}_j^r(n)\}^2]$$

$$\varepsilon_{1,j}(\rho_j, \theta_j)$$

$$\varepsilon_{2,j}(\rho_j, \theta_j)$$



(a): Variations w.r.t  $\rho_j$



(b): Variations w.r.t  $\theta_j$

## Part IV

# Performances evaluation

## Experiments setup:

- Test images: natural and satellite (SPOT5) stereo images.
- Block size:  $8 \times 8$ .
- Search area  $S$ :  $[50, \pm 2]$  for SPOT5 stereo images and  $[30, \pm 4]$  for natural ones.
- Decomposition depth:  $J = 2$ .
- Performances evaluation in terms of bit rate, PSNR and SSIM (Structural SIMilarity)

## Methods used for comparison:

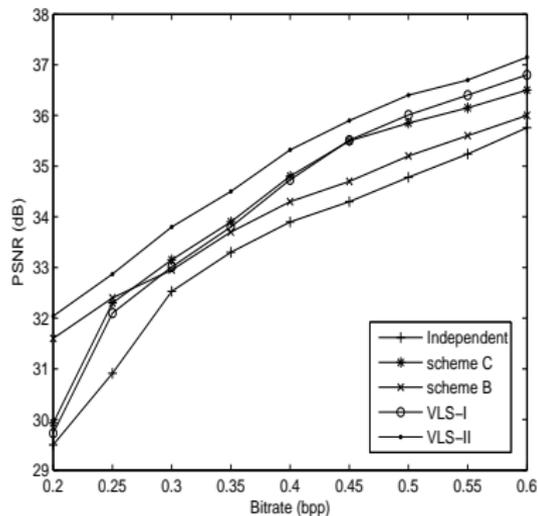
- Independent scheme : Applying the 5/3 transform separately to original images  $I^{(l)}$  and  $I^{(r)}$ .
- Scheme B : Applying the 5/3 transform to the reference and residual images  $I^{(l)}$  and  $I^{(e)}$ .
- Scheme C (version of JPEG2000 scheme): Applying the 5/3 transform to  $\tilde{I}^{(l)}$  and  $I^{(e)}$ , where

$$\begin{cases} I^{(e)}(m_x, m_y) & = I^{(r)}(m_x, m_y) - I^{(l)}(m_x + v_x, m_y + v_y) \\ \tilde{I}(m_x + v_x, m_y + v_y) & = [(I^{(r)}(m_x, m_y) + I^{(l)}(m_x + v_x, m_y + v_y))/2] \\ \quad \text{if } (m_x + v_x, m_y + v_y) \in \mathcal{S} \\ \tilde{I}(m_x, m_y) & = I^{(l)}(m_x, m_y) \quad \text{if } (m_x, m_y) \notin \mathcal{S} \end{cases} \quad (16)$$

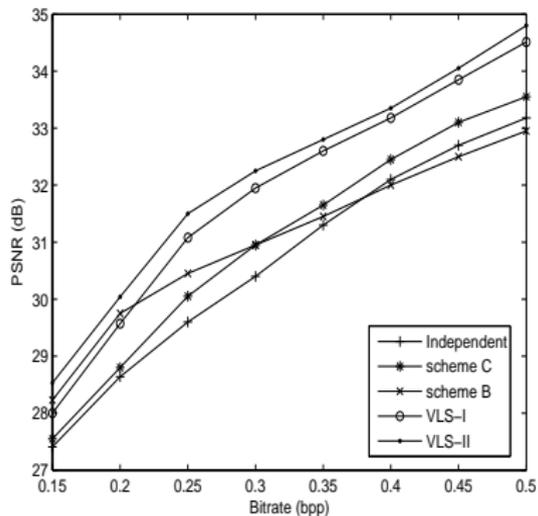
with  $\mathcal{S}$  is the set of connected pixels in the left image.

For all methods, wavelets coefficients are encoded by applying the JPEG2000 codec.

## PSNR curve



(a)



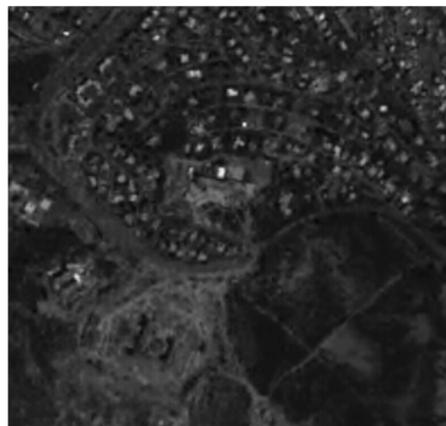
(b)

Figure: PSNR (in dB) versus the bitrate (bpp) after JPEG 2000 encoding for the SI pair “shrub” (a) and “spot5-6” (b).

## Image visual quality (PSNR and SSIM)



(a): PSNR=30.03 dB, SSIM=0.80



(b): PSNR=31.48 dB, SSIM=0.83

**Figure:** Reconstructed target image  $I^{(r)}$  of the “spot5-5” SI at 0.13 bpp: (a) scheme B; (b) VLS-II.

## Final bitrate

Transform	scheme B	scheme C	VLS-I	VLS-II
spot5-1	3.63	3.58	<b>3.49</b>	<b>3.35</b>
spot5-2	3.85	3.78	<b>3.67</b>	<b>3.53</b>
spot5-3	4.27	4.24	<b>4.03</b>	<b>3.93</b>
spot5-4	4.22	4.21	<b>4.05</b>	<b>3.92</b>
spot5-5	3.91	3.89	<b>3.80</b>	<b>3.73</b>
spot5-6	3.89	3.81	<b>3.73</b>	<b>3.63</b>
fruit	4.05	3.97	<b>3.78</b>	<b>3.72</b>
shrub	3.73	3.69	<b>3.81</b>	<b>3.63</b>
birch	4.52	4.47	<b>4.44</b>	<b>4.37</b>
pentagon	5.37	5.2	<b>5.12</b>	<b>5.04</b>
Average	4.14	4.08	<b>3.99</b>	<b>3.88</b>

Presentation of two versions of a novel joint coding methods for stereo pairs.



Improvement by designing more sophisticated prediction/update operators.