Consistent Estimation of Rayleigh Fading Channel Second-Order Statistics in the Context of the Wideband CDMA Mode of the UMTS

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***Abstract*—In this paper, we address the problem of estimating the second-order statistics of a frequency-selective Rayleigh channel in the context of the wideband CDMA mode of the UMTS. The data to be transmitted are sent over slots on which the channel is assumed to remain constant. Each slot contains a pilot symbol sequence from which it is possible to estimate the current value of the channel. The covariance matrix of the channel is usually estimated by a denoised version of the empirical covariance matrix of the trained channel estimate. However, in the UMTS, this estimate is not consistent in the sense that if the number of slots used to estimate it tends to infinity, it does not converge to the true covariance matrix of the channel. In this paper, we propose a new consistent estimate of the covariance matrix and evaluate the performances of two Wiener-like channel estimation schemes based on the proposed estimate. The performances of the new approach are evaluated by means of the bit error rate provided by a RAKE receiver based on the proposed channel estimates. It is shown that our estimate of the covariance matrix allows significant improvement in the performance of the RAKE receiver.**

I. INTRODUCTION

IN the context of high-rate mobile communication systems, the received signal is often corrupted by a fading frequency-selective channel. In this case, the coefficients of the equivalent discrete-time channel can be considered as highly low-pass time-varying centered Gaussian random variables (see e.g., [10] and [11]), which must be estimated in order to retrieve the transmitted data. In practice, the data to be transmitted are sent over slots on which the channel coefficients can be considered to be constant. Each slot contains a training sequence from which the channel coefficients are estimated using a least-squares or a correlation procedure. The accuracy of these estimates, which depends, of course, on both the length of the training sequence and on the signal-to-interference plus noise ratio, may have an important influence on the global performance of the receiver. This turns out to be the case in the context of the wideband CDMA mode of the third mobile generation system (UMTS) [1]. In the downlink, the size of the training sequence is rather short, and the accuracy of the conventional channel estimate is very poor when the system is heavily loaded. This affects significantly the performances of most of the conventional receivers based on this channel estimate.

In order to improve the performances of the channel estimate, one can use semiblind approaches. These methods aim at estimating the channel not only from the training sequence but from the entire slot. However, the existing algorithms have a very high computational cost, especially in the context of multiusers systems (see e.g., [16], [19], and [22]).

This paper is devoted to a completely different approach based on Wiener estimates of the discrete-time equivalent channel. Due to the Rayleigh assumption, the vector coefficients of the discrete time equivalent channel on each slot can be represented as a slot-varying zero mean Gaussian random vector. Its probability distribution is slot invariant. If the covariance matrix of this distribution is known or consistently estimated, it is possible to improve significantly the classically trained estimate by using a simple Wiener estimate.

This idea seems to have been introduced by [2] in the context of channel estimation of a mono-user system (the GSM system) (see also [5], where this approach is briefly considered in multiuser systems). However, we remark that it can be considered as a simplification of Kalman procedures developed in the context of fast fading channel estimation [8], [17], [18] in which the channel coefficients cannot be assumed to be constant over the duration of a slot.

As the covariance matrix is, in practice, unknown, [2] proposed to estimate it by a denoised version of the empirical covariance matrix of the trained channel estimate, assuming that the estimation noise is white, which is relevant in the context of the GSM but not in the context of the UMTS. The purpose of this paper is twofold. We first propose a new consistent estimation scheme of the channel covariance matrix $\mathbf{\Gamma}$ in the context of the downlink of the wideband CDMA mode of the UMTS. Next, we study and compare the performances of two channel estimation algorithms (Wiener and rank reduction Wiener) using our consistent estimation scheme of $\mathbf{\Gamma}$.

This paper is organized as follows. In Section II, we make precise the structure of the signals that are transmitted and received in the downlink of the wideband CDMA mode of the UMTS and introduce the Wiener channel estimation. In Section III, we present our consistent estimate of $\mathbf{\Gamma}$. We study the corresponding estimation schemes in Section IV, and evaluate their performances by numerical simulations in Section V.

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II. Problem Statement and Signal Model

A. Downlink UMTS Signal Structure

We consider a mobile station that is supposed to receive slots of QPSK data symbols sequence $(b_{m,0}(l))_{l=0,K}$ transmitted by the base station of its closest cell. Here, the subscript m represents the index of the slot, l represents the index of the symbol of the slot m, and K is the number of symbols per slot. The base station transmits simultaneously Q other data symbol slots $(b_{m,q})_{q=1,...Q}$ to Q other users.

We first make precise the structure of the signal received by mobile station 0 (see [1] for more details). In the context of UMTS, different users may use different spreading factors. In order to simplify the notations, we assume that the same spreading factor N is assigned to the $(Q+1)$ users of the cell under consideration. The number of chips per slot is thus equal to NK. This assumption does not induce any restriction and that our results remain valid if different spreading factors are assigned to certain users. Each sequence of symbols $b_{m,q}(l)$ is spread by a BPSK periodic sequence $c_q(n)$ of period N. The corresponding $(Q+1)$ chip sequences are finally scrambled by the same long aperiodic code (this code characterizes the cell). We denote by $s_m(n)$ the value of the scrambling code on chip n of slot m. In the following, we denote by $d_{m,q}(n)$ the chip sequence corresponding to slot m of user q, which, according to the above specifications, is given by

$$d_{m,q}(lN+k) = b_{m,q}(l)c_q(k)s_m(lN+k) \tag{1}$$

for $0 \leq k \leq N-1$ and $0 \leq l \leq K-1$. It is also useful to introduce the sequence $v_{m,q}(n)$ defined by

$$v_{m,q}(lN+k) = b_{m,q}(l)$$

for $0 \leq k \leq N-1$ and $0 \leq l \leq K-1$.

The continuous-time signal $x_a(t)$ received by mobile station 0 and corresponding to the transmission of slot m of the various users is thus given by

$$x_{m,a}(t) = \sum_{q=0}^{Q} \mu_q \sum_n d_{m,q}(n) h_{m,a}(t - nT_c) + w_a(t). \tag{2}$$

Here, T_c represents the chip period, and $h_{m,a}(t)$ represents the (unknown) impulse response resulting from the shaping filter (i.e., a square root-raised cosine of roll-off 0.22), the propagation channel between the base station and the mobile station 0, and the reception filter. We assume without restriction that it is causal. Note that it depends on the slot m to take into account the time variations of the propagation channel. The coefficients $\mu_0, \ldots, \mu_Q$ are positive and represent the square roots of the powers of the contributions of each active user to the received signal. In the following, we assume without loss of generality that $\mu_0 = 1$; the coefficients $(\mu_q)_{q=1,\ldots,Q}$ thus represent the relative powers of the other users. Finally, $w_a(t)$ is an additive noise due to the signals transmitted by other interfering cells [which have a structure similar to the first term of the righthand-side of (2)] and to the background noise assumed to be white Gaussian with spectral density $N_0/2$.

We assume that the mobile station 0 has synchronized with the base station. This implies in particular that the mobile has a perfect knowledge of the scrambling code sequence. Each slot contains a pilot sequence of P symbols that can be used in order to estimate the channel. In other words, the mobile station 0 knows the first NP chips of each slot m transmitted by user 0 (i.e., the sequence $d_{m,0}(0), \ldots, d_{m,0}(NP-1)$). However, the mobile station 0 is not aware of the pilot sequences transmitted by the users $1, \ldots, Q$.

B. Discrete-Time Equivalent Model

The signal $x_a(t)$ is sampled at the period $T_c/2$. We denote by $\mathbf{x}_m(n)$ the two-dimensional (2-D) vector

$$\mathbf{x}_m(n) = (x_{m,a}(nT_c), x_{m,a}(nT_c + T_c/2))^T$$

and by $\mathbf{h}_m(k)$ the vector

$$\mathbf{h}_m(k) = (h_{m,a}(kT_c), h_{m,a}(kT_c + T_c/2))^T.$$

We also denote by $\mathbf{h}_m(z)$ the 2-D transfer function

$$\mathbf{h}_m(z) = \sum_{k=0}^{L} \mathbf{h}_m(k) z^{-k}$$

where LT_c represents the maximum duration of the channel. We finally put $\mathbf{h}_m = (\mathbf{h}_m(0)^T, \ldots, \mathbf{h}_m(L)^T)^T$. It is easily seen that the discrete-time signal $\mathbf{x}_m(n)$ can be written as

$$\mathbf{x}_m(n) = \sum_{q=0}^{Q} \mu_q [\mathbf{h}_m(z)] d_{m,q}(n) + \mathbf{w}_m(n) \tag{3}$$

where $\mathbf{w}_m(n)$ is defined as $\mathbf{x}_m(n)$. We now formulate the following assumptions:

- **A1**) For each m, $\mathbf{h}_m$ is a centered complex Gaussian random vector, and its covariance matrix is time invariant, i.e., it does not depend on m. In the following, we denote by $\mathbf{\Gamma} = E(\mathbf{h}_m \mathbf{h}_m^H)$ this covariance matrix.
- **A2**) The (known) pseudo-noise sequence $(s_m(n))$ for $n = 0, \ldots, NK-1, m \in \mathbb{Z}$ is assumed to coincide with a realization of an independent identically distributed centered QPSK sequence. In particular, for each p and each function Φ

$$\lim_{M \to +\infty} \frac{1}{M} \sum_{m=0}^{M-1} \Phi(s_m(n+\tau_1), \ldots, s_m(n+\tau_p)) = E_s(\Phi(s(\tau_1), \ldots, s(\tau_p)))$$

  where s represents a centered QPSK i.i.d. sequence.
- **A3**) For each $q \geq 0$, the symbol sequence transmitted by user q is i.i.d. The various sequences are also mutually independent.

Assumptions **A2** and **A3** are standard and do not need to be discussed. Let us discuss **A1**. The assumption that $\mathbf{h}_m$ is centered Gaussian is a direct consequence of the Rayleigh channel hypothesis [10], [11]. **A1** also implicitly implies that the channel can be considered as time invariant on the slot duration. The relevance of this hypothesis, of course, depends on the mobile speed. Although it is clearly not valid at high velocities, our estimation scheme shows good performances at velocities of 50 km/h (see Section V), which is a quite realistic value in an urban environment. Finally, the covariance matrix $\mathbf{\Gamma}$ essentially depends on the directions of arrival of the various paths of the channel [10]. The slot invariance of the probability distribution of $\mathbf{h}_m$ over a few hundred of slots is therefore realistic because the time variation of these spatial parameters is much slower than the variation of the complex amplitudes of the paths (see, e.g., [10]).

C. Wiener Channel Estimation

If the covariance matrix $\mathbf{\Gamma}$ of $\mathbf{h}_m$ was known, several schemes could be used to improve the accuracy of the conventional training sequence-based estimate $\hat{\mathbf{h}}_m$ of $\mathbf{h}_m$ on slot m (the detailed presentation of this estimate is shown later).

The first possible scheme consists of using a Wiener estimate of $\mathbf{h}_m$. In order to explain this, assume for the moment that the conventional estimate $\hat{\mathbf{h}}_m$ can be written as

$$\hat{\mathbf{h}}_m = \mathbf{h}_m + \epsilon_m \tag{4}$$

where ϵ_m is a random vector independent of $\mathbf{h}_m$ with known covariance matrix $\mathbf{\Sigma}$. In this case, the classical Wiener estimate of $\mathbf{h}_m$, which is given by

$$E\left(\mathbf{h}_m\hat{\mathbf{h}}_m^H\right)\left(E\left(\hat{\mathbf{h}}_m\hat{\mathbf{h}}_m^H\right)\right)^{-1}\hat{\mathbf{h}}_m = \mathbf{\Gamma}(\mathbf{\Gamma}+\mathbf{\Sigma})^{-1}\hat{\mathbf{h}}_m \tag{5}$$

may produce significant improvement. In the context of a GSM system considered in [2], relation (4) is satisfied, and the covariance matrix $\mathbf{\Sigma}$ of ϵ_m can be assumed to be a multiple $\sigma^2 I$ of the identity matrix. As $\mathbf{\Gamma}$ is, of course, unknown, [2] proposed, if σ^2 is known, to estimate it by

$$\hat{\mathbf{\Gamma}} = \frac{1}{M}\sum_{m=0}^{M-1}\hat{\mathbf{h}}_m\hat{\mathbf{h}}_m^H - \sigma^2 I. \tag{6}$$

If σ^2 is unknown, those authors propose to estimate σ^2 by the smallest eigenvalue of matrix $(1/M)\sum_{m=0}^{M-1}\hat{\mathbf{h}}_m\hat{\mathbf{h}}_m^H$. This estimate is consistent as soon as $\mathbf{\Gamma}$ is rank deficient, which is a condition that is often met in practice when the channel is supposed to be specular. This approach can, of course, be adapted if the covariance matrix of ϵ_m is a multiple of a known matrix possibly different from identity.

A second possible scheme can be used if the matrix $\mathbf{\Gamma}$ is rank deficient or close to being rank deficient. This means that vector $\mathbf{h}_m$ can be written as $\mathbf{h}_m = \mathbf{U}\mathbf{g}_m$, where the columns of $\mathbf{U}$ represent a basis of range of $\mathbf{\Gamma}$. The estimation of vector $\mathbf{h}_m$ is thus equivalent to the estimation of vector $\mathbf{g}_m$, which is an easier problem because the number of parameters to be estimated (the dimension of $\mathbf{g}_m$) is smaller than the number of components of $\mathbf{h}_m$. Of course, vector $\mathbf{g}_m$ may also be estimated by a Wiener procedure. Remark that thresholding approaches are connected to this scheme. Indeed, one can force certain components $\mathbf{h}_m(k)$ of vector $\mathbf{h}_m$ to 0 if $\mathbf{\Gamma}_{k,k} = E|\mathbf{h}_m(k)|^2$ is smaller than a certain threshold. In this case, $\mathbf{U}$ has to be replaced by a selection matrix whose particular elements depend on the chosen threshold. These kinds of approaches are widely used in the context of CDMA receivers based on Rake receivers. Only the most significant fingers are selected.

If (4) holds and if the covariance matrix of ϵ_m is known to within a scalar multiplicative factor, then the estimate (6) of $\mathbf{\Gamma}$ turns out to be consistent (in the sense that $\hat{\mathbf{\Gamma}}$ converges toward $\mathbf{\Gamma}$ when $M \to \infty$). These conditions are, however, not verified in the context of the wideband CDMA mode of the UMTS.

III. Estimation of the Channel Covariance Matrix

The conventional estimate is obtained by correlation of the received signal with delayed versions of the chip sequence corresponding to the pilot symbols sequence

$$\hat{\mathbf{h}}_m(k) = \frac{1}{NP}\sum_{n=0}^{NP-1}\mathbf{x}_m(n+k)d_{m,0}^*(n). \tag{7}$$

We put $\mathbf{x}_m = (\mathbf{x}_m(0)^T, \ldots, \mathbf{x}_m(NP+L-1)^T), \mathbf{w}_m = (\mathbf{w}_m(0)^T, \ldots, \mathbf{w}_m(NP+L-1)^T)$ and

$$\mathcal{D}_{m,q} = \begin{bmatrix} d_{m,q}(0) & d_{m,q}(-1) & \cdots & d_{m,q}(-L) \\ d_{m,q}(1) & d_{m,q}(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ d_{m,q}(NP+L-1) & \cdots & \cdots & d_{m,q}(NP-1) \end{bmatrix}.$$

Using (3), we get immediately that

$$\mathbf{x}_m = \sum_{q=0}^{Q}\mu_q(\mathcal{D}_{m,q}\otimes\mathbf{I}_2)\mathbf{h}_m + \mathbf{w}_m \tag{8}$$

where $\otimes$ is the Kronecker product and that $\mathbf{I}_2$ is the 2×2 identity matrix. Denoting by $\bar{\mathcal{D}}_{m,0}$ the matrix obtained from $\mathcal{D}_{m,0}$ by replacing by 0 the terms corresponding to indices smaller than 0 or greater than $NP-1$, we can express vector $\hat{\mathbf{h}}_m = (\hat{\mathbf{h}}_m(0)^T, \ldots, \hat{\mathbf{h}}_m(L)^T)^T$ as

$$\hat{\mathbf{h}}_m = \frac{1}{NP}(\bar{\mathcal{D}}_{m,0}\otimes\mathbf{I}_2)^H\mathbf{x}_m.$$

Developing this expression, we get

$$\begin{aligned}\hat{\mathbf{h}}_m = {} & \frac{1}{NP}\left(\left(\bar{\mathcal{D}}_{m,0}^H\mathcal{D}_{m,0}\right)\otimes\mathbf{I}_2\right)\mathbf{h}_m \\ & + \frac{1}{NP}\sum_{q=1}^{Q}\mu_q\left(\left(\bar{\mathcal{D}}_{m,0}^H\mathcal{D}_{m,q}\right)\otimes\mathbf{I}_2\right)\mathbf{h}_m \\ & + \frac{1}{NP}(\bar{\mathcal{D}}_{m,0}\otimes\mathbf{I}_2)^H\mathbf{w}_m.\end{aligned} \tag{9}$$

The estimation error $\epsilon_m = \hat{\mathbf{h}}_m - \mathbf{h}_m$ thus has three components. The first one is

$$\left[\frac{1}{NP}\left(\left(\bar{\mathcal{D}}_{m,0}^H\mathcal{D}_{m,0}\right)\otimes\mathbf{I}_2\right) - I_{2(L+1)}\right]\mathbf{h}_m$$

whereas the second and the third coincide, respectively, with the second and the third term of the right-hand side of (9). Our problem thus differs deeply from the context used by [2], where the first component is zero, and the second one does not exist [see (4) and the corresponding assumptions]. Moreover, we observe the following.

- Vectors $\mathbf{h}_m$ and ϵ_m are not statistically independent.

- The covariance matrix of ϵ_m is not known to within a constant multiplicative factor; it actually depends on $\mathbf{\Gamma}$ and on the unknown distribution of the co-cell interference. Moreover, due to the scrambling code, it is not time-invariant, i.e., it depends on the slot under consideration.

The standard estimate (6) is thus not consistent in the present context, and its performance may be very poor if the multiuser interference and the co-cell interference terms are dominant. A quite different approach is thus needed to estimate matrix $\mathbf{\Gamma}$.

In order to present our new estimation method, we need to introduce some notations. First, we denote by $\mathbf{\Delta}_m$ the covariance matrix $E(\hat{\mathbf{h}}_m\hat{\mathbf{h}}_m^H)$ of vector $\hat{\mathbf{h}}_m$, which, as shown later, depends on m. Next, we denote $\mathbf{\Delta}_\infty$ as the "temporal mean" of matrices $\mathbf{\Delta}_m$ defined by

$$\mathbf{\Delta}_\infty = \lim_{M\to\infty} \frac{1}{M}\sum_{m=0}^{M-1} \mathbf{\Delta}_m.$$

From now on, we denote by $\mathbf{x}_{m,L}(n)$ the $2(L+1)$-dimensional vector

$$\mathbf{x}_{m,L}(n) = (\mathbf{x}_m(n)^T, \ldots, \mathbf{x}_m(n+L)^T)^T$$

and put $\mathbf{R}_{x,m} = E(\mathbf{x}_{m,L}(n)\mathbf{x}_{m,L}(n)^H)$ and

$$\mathbf{R}_{x,\infty} = \lim_{M\to\infty} \frac{1}{M}\sum_{m=0}^{M-1} \mathbf{R}_{x,m}.$$

Vector $\mathbf{w}_{m,L}(n)$ and matrices $\mathbf{R}_{w,m}$ and $\mathbf{R}_{w,\infty}$ are defined similarly. Our approach is based on the following identities.

Proposition 1: The matrices $\mathbf{R}_{x,\infty}$ and $\mathbf{\Delta}_\infty$ can be written as

$$\mathbf{R}_{x,\infty} = \left(\sum_{q=0}^{Q}\mu_q^2\right)\mathcal{T}(\mathbf{\Gamma}) + \mathbf{R}_{w,\infty} \tag{10}$$

and

$$\mathbf{\Delta}_\infty = \mathbf{\Gamma} + \frac{1}{NP}\left(\sum_{q=0}^{Q}\mu_q^2\right)(\mathcal{T}(\mathbf{\Gamma}) - \mathbf{\Gamma}) + \frac{1}{NP}\mathbf{R}_{w,\infty} \tag{11}$$

where matrix $\mathcal{T}(\mathbf{\Gamma})$ represents the block Toeplitz matrix whose 2×2 blocks $\mathcal{T}(\mathbf{\Gamma})(k,l)$ are given by

$$\mathcal{T}(\mathbf{\Gamma})(k,l) = \sum_{(i,j), i-j=k-l} \mathbf{\Gamma}(i,j). \tag{12}$$

See Appendix A for the proof. It turns out that

$$\mathbf{\Delta}_\infty - \frac{1}{NP}\mathbf{R}_{x,\infty} = \left(1 - \frac{1}{NP}\left(\sum_{q=0}^{Q}\mu_q^2\right)\right)\mathbf{\Gamma}. \tag{13}$$

Under certain standard mixing assumptions on sequence $(\mathbf{h}_m)_{m\in\mathbf{z}}$ (see, e.g., [3] for more details), the matrices $\mathbf{\Delta}_\infty$ and $\mathbf{R}_{x,\infty}$ can be consistently estimated by

$$\hat{\mathbf{\Delta}}_\infty = \frac{1}{M}\sum_{m=0}^{M-1}\hat{\mathbf{h}}_m\hat{\mathbf{h}}_m^H \tag{14}$$

and by

$$\hat{\mathbf{R}}_{x,\infty} = \frac{1}{MNK}\sum_{m=0}^{M-1}\sum_{n=0}^{NK-1}\mathbf{x}_{m,L}(n)\mathbf{x}_{m,L}(n)^H. \tag{15}$$

Therefore, (13) provides a direct way to estimate $\mathbf{\Gamma}$ consistently up to a constant multiplicative factor. The estimate is noted $\hat{\mathbf{\Gamma}}$ and is defined by

$$\hat{\mathbf{\Gamma}} = \hat{\mathbf{\Delta}}_\infty - \frac{1}{NP}\hat{\mathbf{R}}_{x,\infty}. \tag{16}$$

We note that this approach is also able to provide an estimate of the matrix $\mathbf{R}_{w,\infty}$. This can be interesting in order to evaluate the quality of the transmission. Equation (11) also shows that the present estimate is probably not appropriate if the system is not heavily loaded and if the co-cell interference term is negligible w.r.t. the background noise. In this context, the matrix $\mathbf{\Delta}_\infty$ is nearly equal to $\mathbf{\Gamma}+(\sigma^2/NP)I$, where σ^2 represents the variance of the background noise, and the conventional estimate (6) is good enough. If, however, the system is heavily loaded or if the co-cell interference is powerful enough, our new estimate may potentially improve quite significantly the performances of the chosen receiver. This point will be illustrated in Section V, which is devoted to the numerical simulations.

We finally note that the result of Proposition 1 is connected to the blind channel estimation schemes introduced in [14] and generalized in [13]. In those works, it is shown in the context of the time-invariant channel that the difference between a covariance matrix built from the observation before and after despreading is nearly proportional to $\mathbf{h}\mathbf{h}^H$, where $\mathbf{h}$ represents the (fixed) channel vector. The result of Proposition 1 can be interpreted as a generalization of the results of [13] and [14] to random time-varying channels and to the context of the UMTS specifications defined in Section II-A.

IV. Improvement of Channel Estimation Using the Channel Covariance Matrix

A. Modified Wiener Estimation

The classical Wiener channel estimator of $\mathbf{h}_m$ is defined as the orthogonal projection of $\mathbf{h}_m$ over the space generated by the components of the observed random vector $\hat{\mathbf{h}}_m$. It is thus given by

$$\begin{aligned}\mathbf{h}_m/\hat{\mathbf{h}}_m &= E\left(\mathbf{h}_m\hat{\mathbf{h}}_m^H\right)\left(E\left(\hat{\mathbf{h}}_m\hat{\mathbf{h}}_m^H\right)\right)^{-1}\hat{\mathbf{h}}_m\\ &= E\left(\mathbf{h}_m\hat{\mathbf{h}}_m^H\right)\mathbf{\Delta}_m^{-1}\hat{\mathbf{h}}_m\end{aligned}$$

where $/$ stands for the usual orthogonal projection operator in the space of finite second-order moments random variables. However, this channel estimator cannot be implemented in practice because it is impossible to estimate consistently the matrix $\mathbf{\Delta}_m$. We therefore propose to use a modified Wiener estimate defined by $\mathbf{A}_{\text{opt}}\hat{\mathbf{h}}_m$, where the matrix $\mathbf{A}_{\text{opt}}$ minimizes the cost function

$$\begin{aligned}e &= \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} e_m\\ &= \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} E(\|\mathbf{h}_m - \mathbf{A}\hat{\mathbf{h}}_m\|^2).\end{aligned}$$

The optimal matrix $\mathbf{A}_{\rm opt}$ is, of course, given by

$$\mathbf{A}_{\rm opt} = \left(\lim_{M\to\infty} \frac{1}{M} \sum_{m=0}^{M-1} E[\mathbf{h}_m \hat{\mathbf{h}}_m^*] \right) \mathbf{\Delta}_\infty^{-1}.$$

but using similar calculations as the ones developed in A and B (the details are left to the reader), it can be shown that

$$\lim_{M\to\infty} \frac{1}{M} \sum_{m=0}^{M-1} E[\mathbf{h}_m \hat{\mathbf{h}}_m^*] = \mathbf{\Gamma}.$$

Therefore, $\mathbf{A}_{\rm opt}$ coincides with $\mathbf{A}_{\rm opt} = \mathbf{\Gamma}\mathbf{\Delta}_\infty^{-1}$ and can be consistently estimated by matrix $\hat{\mathbf{\Gamma}}\hat{\mathbf{\Delta}}_\infty^{-1}$. Our modified Wiener channel estimate is, thus, the vector $\bar{\mathbf{h}}_m$ given by

$$\bar{\mathbf{h}}_m = \hat{\mathbf{\Gamma}}\hat{\mathbf{\Delta}}_\infty^{-1}\hat{\mathbf{h}}_m. \tag{17}$$

.

B. Rank Reduction of Channel Subspace

In practice, the performance of the estimate $\bar{\mathbf{h}}_m = \hat{\mathbf{\Gamma}}\hat{\mathbf{\Delta}}_\infty^{-1}\hat{\mathbf{h}}_m$ may be very poor in comparison with the performance of the true modified Wiener estimate $\mathbf{\Gamma}\mathbf{\Delta}_\infty^{-1}\hat{\mathbf{h}}_m$. This is, in particular, the case when the number of slots M used to estimate matrices $\mathbf{\Gamma}$ and $\mathbf{\Delta}_\infty$ is not large enough compared with the dimension of the matrices to be estimated; in this case, the estimates $\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{\Delta}}_\infty$ are not accurate enough. See, for example, [9] and [15] for a precise analysis when the matrix $\hat{\mathbf{\Delta}}_\infty$ is Wishart distributed.

Fortunately, the performance of the estimate $\bar{\mathbf{h}}_m$ can be improved significantly if the matrix $\mathbf{\Gamma}$ is rank deficient (or close to be rank deficient). This turns out to be the case in the context of the so-called multipath Clarke model [6] in the context of which the continuous-time impulse response $h_{a,m}(t)$ of the channel on slot m is represented by

$$h_{a,m}(t) = \sum_{k=1}^{N_t} \lambda_m(k) f_a(t-\tau_k) \tag{18}$$

where the complex coefficients $(\lambda_m(k))_{k=1,\dots,N_t}$ represent the complex amplitudes of N_t significant paths[1] on slot m, where the $(\tau_k)_{k=1,\dots,N_t}$ are the corresponding time delays, and where $f_a(t)$ represent the impulse response of the shaping filter. Usually, the time delays $(\tau_k)_{k=1,\dots,N_t}$ are assumed constant on several slots, whereas the complex amplitudes $(\lambda_m(k))_{k=1,\dots,N_t}$ are modelized by independent Gaussian random variables, the probability distribution of which is independent of m. Using (18), we get immediately that vector $\mathbf{h}_m$ can be written as

$$\mathbf{h}_m = \sum_{k=1}^{N_t} \lambda_m(k) \mathbf{f}(\tau_k)$$

where $\mathbf{f}(\tau_k)$ is a vector built from the samples of $f_a(t-\tau_k)$. If the number of paths is less than $2(L+1)$, the covariance matrix $\mathbf{\Gamma}$ of $\mathbf{h}_m$ is clearly degenerate. Note that if (18) is valid, the condition $\mathbf{\Gamma}$ degenerate is likely to hold in the context of the UMTS because the number of significant paths in an outdoor propagation channel at 2 MHz never exceeds 6 in practice [20] and because L is generally chosen greater than 10.

[1]In the Clarke model, each path is actually a superposition of elementary paths with very close delays. Moreover, the complex amplitude of the paths also depend on t, but it is usual to assume that they remain constant on the slot duration.

We thus assume that $\mathbf{\Gamma}$ is rank deficient and denote by r its rank. In this case, the channel $\mathbf{h}_m$ can be written as

$$\mathbf{h}_m = \mathbf{U}\mathbf{g}_m$$

where $\mathbf{U}$ represents the matrix build from the r eigenvectors associated with the nonzero eigenvalues of $\mathbf{\Gamma}$, and where $\mathbf{g}_m$ is an r-dimensional vector.

Let us first assume that $\mathbf{U}$ is known. The estimation of vector $\mathbf{h}_m$ reduces to the estimation of the r components of $\mathbf{g}_m$, which is a much easier problem if r is significantly smaller than the number $2(L+1)$ of components of $\mathbf{h}_m$ (recall that this condition is likely to hold in the context of UMTS). We propose to estimate vector $\mathbf{g}_m$ by means of a modified Wiener estimation scheme. For this, we remark that model (8) can be written as

$$\mathbf{x}_m = \sum_{q=0}^{Q} \mu_q (\mathcal{D}_{m,q} \otimes \mathbf{I}_2)\mathbf{U}\mathbf{g}_m + \mathbf{w}_m.$$

The conventional estimate $\hat{\mathbf{g}}_m$ of $\mathbf{g}_m$ is given by $\hat{\mathbf{g}}_m = \mathbf{U}^H\hat{\mathbf{h}}_m$. We define the following notations: $\mathbf{\Omega} = E(\mathbf{g}_m\mathbf{g}_m^H), \mathbf{\Lambda}_m = E(\hat{\mathbf{g}}_m\hat{\mathbf{g}}_m^H)$ and $\mathbf{\Lambda}_\infty = \lim_{M\to\infty}(1/M)\sum_{m=0}^{M-1}\mathbf{\Lambda}_m$. Those matrices can be estimated by $\hat{\mathbf{\Lambda}}_\infty = \mathbf{U}^H\hat{\mathbf{\Delta}}_\infty\mathbf{U}$ and $\hat{\mathbf{\Omega}} = \mathbf{U}^H\hat{\mathbf{\Gamma}}\mathbf{U}$ so that $\tilde{\mathbf{g}}_m$ defined by

$$\tilde{\mathbf{g}}_m = \hat{\mathbf{\Omega}}\hat{\mathbf{\Lambda}}_\infty^{-1}\hat{\mathbf{g}}_m$$

is an estimator of $\mathbf{g}_m$. The corresponding estimate of $\mathbf{h}_m$ is, thus, vector $\mathbf{U}\tilde{\mathbf{g}}_m$.

However, matrix $\mathbf{U}$ is, of course, unknown and replaced in practice in the above procedure by the matrix $\hat{\mathbf{U}}$ of the eigenvectors associated with the r greatest eigenvalues of matrix $\hat{\mathbf{\Gamma}}$. We denote by $\tilde{\mathbf{h}}_m$ the final rank-reduced estimate obtained by replacing matrix $\mathbf{U}$ with matrix $\hat{\mathbf{U}}$ in the above procedure. It is worth noticing that the size of matrices $\mathbf{\Omega}$ and $\mathbf{\Lambda}_\infty$ are smaller than the size of $\mathbf{\Gamma}$ and $\mathbf{\Delta}_\infty$. Therefore, the estimates $\hat{\mathbf{\Omega}}$ and $\hat{\mathbf{\Lambda}}_\infty$ are more accurate than $\hat{\mathbf{\Gamma}}$ and $\hat{\mathbf{\Delta}}_\infty$. Although $\tilde{\mathbf{h}}_m$ depends on $\hat{\mathbf{\Gamma}}$ via the eigenvectors matrix $\hat{\mathbf{U}}$, this explains why the performance of $\tilde{\mathbf{h}}_m$ is better than the performance of $\bar{\mathbf{h}}_m$ (see Section V).

We finally note that if (18) holds, the subspace associated with the r greatest eigenvalues of $\mathbf{\Gamma}$ coincides with the space generated by vectors $\mathbf{f}(\tau_k)$ for $k = 1,\dots,N_t$ (which implies that $r = N_t$). As the impulse response $f_a(t)$ of the shaping filter can be assumed to be known, it is possible to estimate $\mathbf{U}$ by estimating the time delays $(\tau_k)_{k=1,N_t}$. However, the relevance of this approach depends on (18), which is very often not exactly verified. Note, in particular, that our rank-reduction procedure uses only the assumption that $\mathbf{\Gamma}$ is not a full-rank matrix: a condition that is much more general than (18).

V. Simulations

In order to simulate the propagation channel, we have used a realistic simulator [7] developed by the research center of France Telecom. We have chosen a three-path channel with time-varying complex amplitude corresponding to a mobile speed of 50 km/h.

The spreading factor of the user of interest (i.e., the user 0) is 256. Each slot thus contains six useful QPSK symbols and four

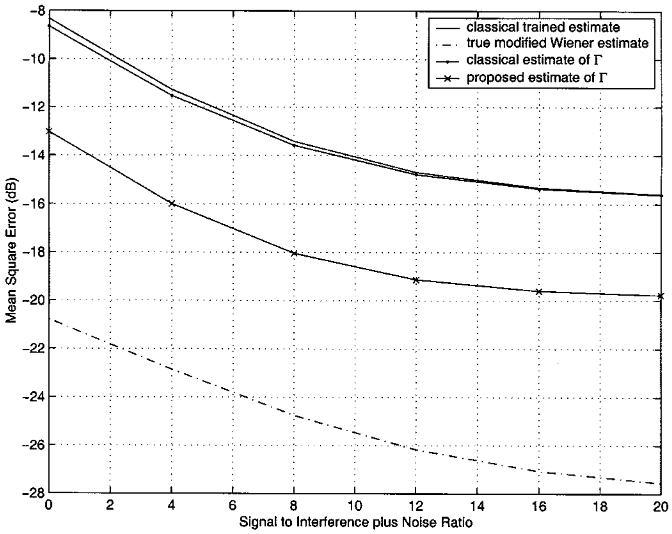

Fig. 1. Mean square error of modified Wiener estimates.

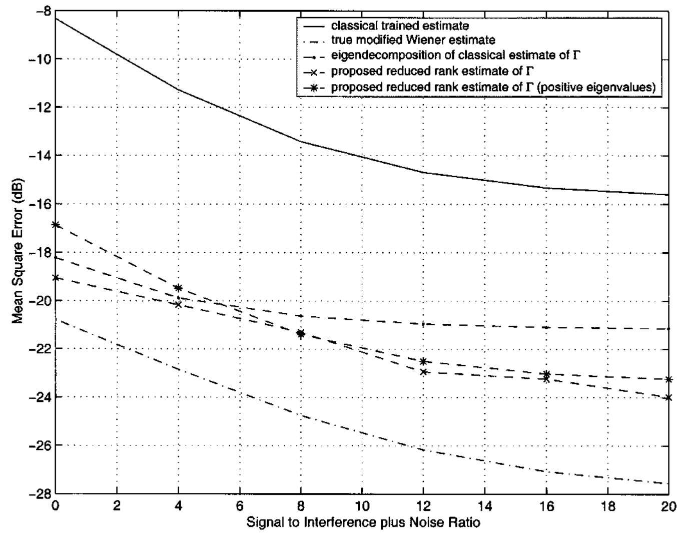

Fig. 2. Mean square error of reduced-rank modified Wiener estimates.

QPSK pilot symbols (see [1]), which are assumed to be sent with the same power.

The other users (users 1 to Q) of the cell may have different spreading factors $(N_q)_{q=1,Q}$ (recall that our results extend to this case but that we have just considered the case of identical spreading factors in order to clarify the presentation). In this case, the load of the cell is measured by the quantity ρ defined by $\rho = \sum_{q=0}^{Q}(1/N_q)$. In all the following experiments, we take $\rho \simeq (1/5)$ and assume that the power control is perfect, i.e., that the received energies per symbol of all the users coincide $E_s = N_q T_c \mu_q^2$ for $q = 0, \ldots, Q$. A second basestation and an additive white Gaussian noise are also simulated in order to achieve a realistic interference environment. The Gaussian background noise power represents 10% of the noise plus interference power.

In Fig. 1, we compare the mean square error of the conventional pilot based estimate $\hat{\mathbf{h}}_m$ with the true modified Wiener channel estimate $\mathbf{\Delta}_\infty^{-1}\mathbf{\Gamma}\hat{\mathbf{h}}_m$, the modified Wiener channel estimate based on the estimate (6) of Γ, and the modified Wiener estimate based on the proposed estimation procedure of Γ (16). Here, the number of slots M used to estimate the various matrices is $M = 240$, which, in the context of the UMTS, corresponds to a duration of 160 ms. The assumed channel duration is equal to 20 chips, i.e., $L = 20$.

We observe that the classical estimate (6) of $\mathbf{\Gamma}$ produces an unsignificant improvement, whereas our proposed estimator provides a gain between 4 and 5 dB. Nevertheless, its performances is poor in comparison with the one of the true modified Wiener estimate. Fortunately, rank-reduction procedures allow significant improvement of the mean square errors of the estimates. This claim is illustrated in Fig. 2, where we compare the performance of the proposed reduced-rank estimate of $\mathbf{\Gamma}$ with the reduced-rank estimate based on the eigendecomposition of (6). In both cases, the rank of the estimate of $\mathbf{\Gamma}$ is evaluated by the procedure proposed in [12], i.e.,

$$\hat{r} = \mathrm{argmin}_i \frac{|\hat{\lambda}_i|}{|\hat{\lambda}_{i+1}|} \tag{19}$$

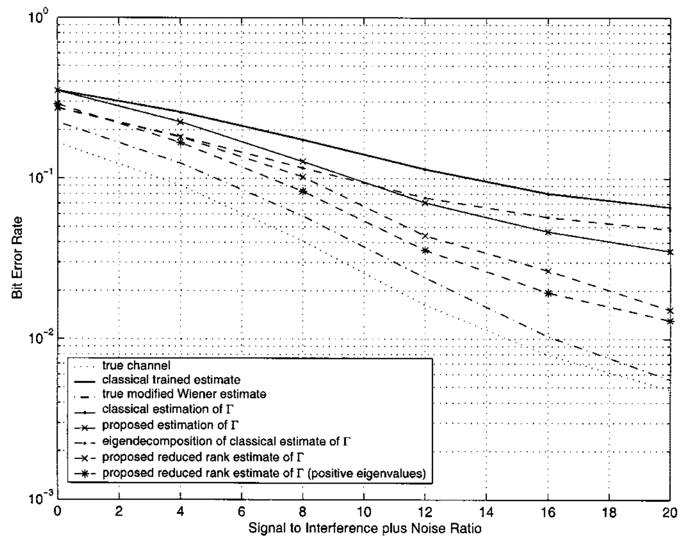

Fig. 3. Bit error rate of modified Wiener estimates.

where $\hat{\lambda}_1 \geq \hat{\lambda}_2 \geq \cdots \geq \hat{\lambda}_{2L+2}$ are the eigenvalues of the estimate of $\mathbf{\Gamma}$ arranged in decreasing order. We also plot the performance corresponding to the rank determination procedure consisting of estimating r by $\bar{r}$, which is defined as the number of positive eigenvalues of (16).

Rank reduction significantly reduces the estimation noise power (from 8 to 10 dB). For both rank-estimation methods, our proposed reduced-rank estimate of $\mathbf{\Gamma}$ provides good performance, and when the signal-to-interference plus noise ratio is greater than 8 dB, they outperform the estimate based on the eigendecomposition of the classical estimate (6) of $\mathbf{\Gamma}$.

We now compare the performances of the various channel estimates in terms of bit error rate. In Fig. 3, we compare the bit error rate associated with a conventional RAKE receiver based on the various channel estimates. In other words, the decision on symbol $b_{m,0}(l)$ is based on the argument of

$$\hat{b}_{m,0}(l) = \frac{1}{N}\sum_{k=0}^{L}\sum_{n=0}^{N-1}\hat{\mathbf{f}}_m^H \mathbf{x}_m(lN+n+k)c_0(n)s_m(lN+n)^*$$

where $\mathbf{f}_m$ represents one of the possible channel estimates. The performance of the Rake receiver associated with the true channel is also represented.

We observe that the performance of the true modified Wiener channel estimate is 2 dB less than the one of the true channel. Without rank reduction, the proposed estimate significantly outperforms the approach of (6), which behaves like the conventional trained estimate because (6) is not a consistent estimate of $\mathbf{\Gamma}$. The rank-reduction schemes of the proposed estimate of $\mathbf{\Gamma}$ allows improvement of the performances. Note that when the rank is evaluated by the number of positive eigenvalues of the estimate of $\mathbf{\Gamma}$, the performances are slightly better than if (19) is used. In any case, the two schemes significantly outperform the estimate based on the eigendecomposition of (6) as soon as the signal-to-interference plus noise ratio is greater than 8 dB.

VI. Conclusion

In this paper, we have addressed the problem of estimating consistently the covariance matrix $\mathbf{\Gamma}$ of the discrete-time version of a Rayleigh fading channel in the context of the WCDMA mode of the third-generation UMTS system. Our estimate is based on the observation that $\mathbf{\Gamma}$ can be obtained by subtracting the temporal mean of the covariance matrix of the observed signal to the temporal mean of the covariance matrix of the pilot symbol-based conventional estimate. We have also studied the performance of two Wiener-like channel estimators based on our new estimate and have compared their performances with those of a classical estimate of $\mathbf{\Gamma}$ used in the context of mono-user systems. The simulation results have shown that the new estimate outperforms quite significantly the classical one. We finally remark that our approach can be immediately generalized to the case of Ricean fading multipath channels. In this context, $\mathbf{h}_m$ is a noncentered Gaussian random vector. Its covariance matrix can be estimated as in the Rayleigh channel case, whereas the estimation of its mean is obvious.

Appendix A
Proof of Proposition 1

A. Proof of (10)

By definition, we have, for $i,j = 0,\dots,L$

$$\mathbf{R}_{x,m}(k,l) = E(\mathbf{x}_m(n+k)\mathbf{x}_m(n+l)^*).$$

Replacing $\mathbf{x}_m(n+k)$ and $\mathbf{x}_m(n+l)$ by their expressions (3), we get

$$\begin{aligned}\mathbf{R}_{x,m}(k,l) = \sum_{p,q=0}^{Q} \mu_p\mu_q E\Bigg(\sum_{i,j=0}^{L} &\mathbf{h}_m(i)\mathbf{h}_m(j)^* \\ &\times d_{m,p}(n+k-i)d_{m,q}(n+l-j)^*\Bigg) \\ &+ E(\mathbf{w}_m(n+k)\mathbf{w}_m(n+l)^*). \end{aligned} \tag{20}$$

We can notice, on one hand, that $E(\mathbf{h}_m(i)\mathbf{h}_m(j)^*) = \mathbf{\Gamma}(i,j)$ does not depend on m. On the other hand, using decomposition (1) of sequence $d_{m,q}$, we get that

$$\begin{aligned}&E(d_{m,p}(n+k-i)d_{m,q}(n+l-j)^*) \\ &\quad = c_p(n+k-i)c_q(n+l-j) \\ &\qquad \times s_m(n+k-i)s_m(n+l-j)^* \\ &\qquad \times E(v_{m,p}(n+k-i)v_{m,q}(n+l-j)^*). \end{aligned} \tag{21}$$

As the sequences transmitted by user p and q are independent for $p \neq q$, we have ,

$$\begin{aligned}&E(v_{m,p}(n+k-i)v_{m,q}(n+l-j)^*) \\ &\quad = E(v_{m,q}(n+k-i)v_{m,q}(n+l-j)^*)\delta(p-q). \end{aligned} \tag{22}$$

The term $E(v_{m,q}(n-k-i)v_{m,q}(n-l-j)^*)$ is equal to 1 if $v_{m,q}(n-k-i)$ and $v_{m,q}(n-l-j)^*$ correspond to the same symbol and to 0 otherwise. Note that this quantity does not depend on m so that the first term of the right-hand side of (20) only depends on m via the product $s_m(n-k-i)s_m(n-l-j)^*$. Using **A2**, we get that

$$\begin{aligned}&\lim_{M\to\infty} \frac{1}{M}\sum_{m=0}^{M-1} s_m(n+k-i)s_m(n+l-j)^* \\ &\quad = E_s(s(n+k-i)s(n+l-j)^*) = \delta(i-j+l-k).\end{aligned}$$

We then deduce immediately that

$$\begin{aligned}\mathbf{R}_{x,\infty}(k,l) &= \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}\mathbf{R}_{x,m}(k,l) \\ &= \sum_{q=0}^{Q}\mu_q^2 \sum_{i,j=0}^{L}\mathbf{\Gamma}(i,j)c_q(n+k-i) \\ &\quad \times c_q(n+l-j)\delta(i-j+l-k) \\ &\quad \times E(v_{m,q}(n+k-i)v_{m,q}(n+l-j)^*) \\ &\quad + \mathbf{R}_{w,\infty}(k,l) \\ &= \sum_{q=0}^{Q}\mu_q^2 \sum_{(i,j),i-j=k-l}\mathbf{\Gamma}(i,j) + \mathbf{R}_{w,\infty}(k,l). \end{aligned} \tag{23}$$

Using (12), (23) becomes

$$\mathbf{R}_{x,\infty}(k,l) = \left(\sum_{q=0}^{Q}\mu_q^2\right)\mathcal{T}(\mathbf{\Gamma})(k,l) + \mathbf{R}_{w,\infty}(k,l).$$

B. Proof of (11)

The entry $\mathbf{\Delta}_m(k,l)$ of matrix $\mathbf{\Delta}_m$ is defined by

$$\mathbf{\Delta}_m(k,l) = E(\hat{\mathbf{h}}_m(k)\hat{\mathbf{h}}_m(l)^*).$$

Using (7), we get that

$$\begin{aligned}\mathbf{\Delta}_m(k,l) = \frac{1}{N^2P^2}\sum_{n,n'=0}^{NP-1} &E(\mathbf{x}_m(n+k) \\ &\times \mathbf{x}_m(n'+l)^* d_{m,0}(n)^* d_{m,0}(n')). \end{aligned} \tag{24}$$

Therefore

$$\begin{aligned}\mathbf{\Delta}_\infty(k,l) &= \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}\mathbf{\Delta}_m(k,l)\\ &= \frac{1}{N^2P^2}\sum_{n,n'=0}^{NP-1}\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}E(\mathbf{x}_m(n+k)\\ &\quad \times \mathbf{x}_m(n'+l)^* d_{m,0}(n)^* d_{m,0}(n')).\end{aligned}$$

Recall that for each m, sequence $(d_{m,0}(n))_{n=0,NP-1}$ represents the NP chips corresponding to the P pilot symbols of slot m. Therefore, $(d_{m,0}(n))_{n=0,NP-1}$ is a known deterministic sequence, and we have

$$\begin{aligned}&E(\mathbf{x}_m(n+k)\mathbf{x}_m(n'+l)^* d_{m,0}(n)^* d_{m,0}(n'))\\ &\quad = E(\mathbf{x}_m(n+k)\mathbf{x}_m(n'+l)^*)d_{m,0}(n)^* d_{m,0}(n').\end{aligned}$$

and

$$\begin{aligned}&E(\mathbf{x}_m(n+k)\mathbf{x}_m(n'+l)^*)d_{m,0}(n)^* d_{m,0}(n')\\ &\quad = \sum_{p,q=0}^{Q}\mu_p\mu_q\sum_{i,j=0}^{L}\mathbf{\Gamma}(i,j)\\ &\qquad \times E(d_{m,p}(n+k-i)d_{m,q}(n'+l-j)^*)\\ &\qquad \times d_{m,0}(n)^* d_{m,0}(n') + E(\mathbf{w}_m(n+k)\\ &\qquad \times \mathbf{w}_m(n'+l)^*)d_{m,0}(n)^* d_{m,0}(n'). \qquad (25)\end{aligned}$$

The sum over the integers p and q in the first term of the right-hand side of (25) can be decomposed in three terms, each contributing to the value of $\lim_{M\to\infty}(1/M)E(\mathbf{x}_m(n+k)\mathbf{x}_m(n'+l)^*)d_{m,0}(n)^* d_{m,0}(n')$ and, hence, to the value of $\mathbf{\Delta}_\infty(k,l)$.

We first evaluate the contribution of the term corresponding to $p=q=0$ in (25). As $(d_{m,0}(n))_{n=0,NP-1}$ is deterministic, this contribution is given by

$$\begin{aligned}&\frac{1}{N^2P^2}\sum_{n,n'=0}^{NP-1}\sum_{i,j=0}^{L}\mathbf{\Gamma}(i,j)\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}d_{m,0}(n+k-i)\\ &\qquad \times d_{m,0}(n'+l-j)^* d_{m,0}(n)^* d_{m,0}(n').\end{aligned}$$

Let us calculate $\lim_{M\to\infty}(1/M)\sum_{m=0}^{M-1}d_{m,0}(n+k-i)d_{m,0}(n'+l-j)^* d_{m,0}(n)^* d_{m,0}(n')$. Using the decomposition (1), we get that

$$\begin{aligned}&\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}d_{m,0}(n+k-i)d_{m,0}(n'+l-j)^*\\ &\quad \times d_{m,0}(n)^* d_{m,0}(n')\\ &\qquad = c_0(n+k-i)c_0(n'+l-j)c_0(n)c_0(n')\\ &\qquad\quad \times \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}s_m(n+k-i)s_m(n'+l-j)^*\\ &\qquad\quad \times s_m(n)^* s_m(n')\\ &\qquad\quad \times v_{m,0}(n+k-i)v_{m,0}(n'+l-j)^* v_{m,0}(n)^* v_{m,0}(n').\end{aligned}$$

By Assumption **A2**, the sequence $s_m(n)$ is assumed to coincide with the realization of an i.i.d. sequence of QPSK symbols. As $|v_{m,0}(n+k-i)v_{m,0}(n'+l-j)^* v_{m,0}(n)^* v_{m,0}(n')| = 1$, a straightforward generalization of the strong law of large numbers implies that

$$\begin{aligned}&\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}s_m(n+k-i)s_m(n'+l-j)^* s_m(n)^* s_m(n')\\ &\quad \times v_{m,0}(n+k-i)v_{m,0}(n'+l-j)^* v_{m,0}(n)^* v_{m,0}(n')\\ &\qquad = \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}E_s(s(n+k-i)\\ &\qquad\quad \times s(n'+l-j)^* s(n)^* s(n'))v_{m,0}(n+k-i)\\ &\qquad\quad \times v_{m,0}(n'+l-j)^* v_{m,0}(n)^* v_{m,0}(n').\end{aligned}$$

Since $s(n)$ is an i.i.d. QPSK sequence, $E_s(s(n+k-i)s(n'+l-j)^* s(n)^* s(n'))$ is given by

$$\begin{aligned}&E_s(s(n+k-i)s(n'+l-j)^* s(n)^* s(n'))\\ &\quad = \delta(k-i)\delta(l-j) + \delta(n'-n)\delta(k-i+j-l)\\ &\qquad - \delta(k-i)\delta(j-l)\delta(n'-n).\end{aligned}$$

From this, we get immediately that

$$\begin{aligned}&\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}s_m(n+k-i)s_m(n'+l-j)^* s_m(n)^* s_m(n')\\ &\quad \times v_{m,0}(n+k-i)v_{m,0}(n'+l-j)^* v_{m,0}(n)^* v_{m,0}(n')\\ &\qquad = \delta(k-i)\delta(l-j) + \delta(n'-n)\delta(k-i+j-l)\\ &\qquad\quad - \delta(k-i)\delta(j-l)\delta(n'-n) \qquad (26)\end{aligned}$$

and that

$$\begin{aligned}&\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}d_{m,0}(n+k-i)d_{m,0}(n'+l-j)^*\\ &\quad d_{m,0}(n)^* d_{m,0}(n')\\ &\qquad = \delta(k-i)\delta(l-j) + \delta(n'-n)\delta(k-i+j-l)\\ &\qquad\quad - \delta(k-i)\delta(j-l)\delta(n'-n). \qquad (27)\end{aligned}$$

Putting all the pieces together, we obtain that the contribution of the term corresponding to $p=q=0$ in (25) to $\mathbf{\Delta}_\infty(k,l)$ is equal to

$$\left(1-\frac{1}{NP}\right)\mathbf{\Gamma}(k,l) + \frac{1}{NP}\mathcal{T}(k,l).$$

The second term contributing to the summation in (25) is the sum over the integers $(0,q)$ for $q \geq 1$ and the integers $(q,0)$ for $q \geq 1$. For each $q \geq 1$ and each m, the sequence $(d_{m,q}(n))_{n=0,\ldots,NP-1}$ is unknown because the pilot symbol sequences assigned to the user q for $q \geq 1$ are unknown. It is therefore relevant to represent the sequence $(d_{m,q}(n))_{n=0,\ldots,NP-1}$ as a random sequence of *centered random variables* (see Assumption **A3**). Using the relation $E(d_{m,q}(n)) = 0$ for $q \geq 1$, it is easy to show that the contribution of the sum over the integers $(0,q)$ for $q \geq 1$ and the integers $(q,0)$ for $q \geq 1$ is identically zero.

The contribution of the sum over (p,q) with $p \geq 1$ and $q \geq 1$ remains to be evaluated. By Assumption **A3**, the sequences $d_{m,p}$ and $d_{m,q}$ for $p \neq q$ are statistically independent. Therefore, the contribution of the sum over indices (p,q) for $p \geq 1$ and $q \geq 1$ reduces to the sum over the integers $p = q \geq 1$.

Let us first evaluate $\lim_{M\to\infty}(1/M)\sum_{m=0}^{M-1} E(d_{m,q}(n+k-i)d_{m,q}(n'+l-j)^*)d_{m,0}(n)^* d_{m,0}(n')$. Using the decomposition (1), this term is equal to

$$\begin{aligned} &c_q(n+k-i)c_q(n'+l-j)c_0(n)c_0(n') \\ &\times \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} s_m(n+k-i)s_m(n'+l-j)^* \\ &\times s_m(n)^* s_m(n')E(v_{m,q}(n+k-i) \\ &\times v_{m,q}(n'+l-j)^*)v_{m,0}(n)^* v_{m,0}(n'). \end{aligned} \tag{28}$$

As $E(v_{m,q}(n+k-i)v_{m,q}(n'+l-j)^*)$ does not depend on m, it is denoted by $E(v_q(n+k-i)v_q(n'+l-j)^*)$ in the following. Therefore, (28) is equal to

$$\begin{aligned} &c_q(n+k-i)c_q(n'+l-j)c_0(n)c_0(n') \\ &\times E(v_q(n+k-i)v_q(n'+l-j)^*) \\ &\times \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} s_m(n+k-i)s_m(n'+l-j)^* \\ &\times s_m(n)^* s_m(n')v_{m,0}(n)^* v_{m,0}(n') \end{aligned}$$

by a generalization of the strong law of large numbers.

Using again the observation that $|v_{m,q}(n)| = 1$ for each integer m,q,n this term is then equal to

$$\begin{aligned} &c_q(n+k-i)c_q(n'+l-j)c_0(n)c_0(n') \\ &\times E(v_q(n+k-i)v_q(n'+l-j)^*) \\ &\times \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} E_s(s(n+k-i) \\ &\times s(n'+l-j)^* s(n)^* s(n'))v_{m,0}(n)^* v_{m,0}(n'). \end{aligned} \tag{29}$$

Using $E_s(s(n+k-i)s(n'+l-j)^* s(n)^* s(n')) = \delta(k-i)\delta(l-j)+\delta(n'-n)\delta(k-i+j-l)-\delta(k-i)\delta(j-l)\delta(n'-n)$, (29) reduces to

$$\begin{aligned} &c_q(n+k-i)c_q(n'+l-j)c_0(n)c_0(n') \\ &\times \left(E(v_q(n)v_q(n')^*)\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} v_{m,0}(n)^* v_{m,0}(n')\right) \\ &\times (\delta(k-i)\delta(l-j)+\delta(n-n')\delta(k-i+j-l) \\ &-\delta(n'-n)\delta(k-i)\delta(j-l)). \end{aligned}$$

Therefore

$$\begin{aligned} &\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} E(d_{m,q}(n+k-i) \\ &d_{m,q}(n'+l-j)^*)d_{m,0}(n)^* d_{m,0}(n') \\ &= c_q(n)c_q(n')c_0(n)c_0(n')\left(E(v_q(n)v_q(n')^*)\right. \\ &\left.\times \lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} v_{m,0}(n)^* v_{m,0}(n')\right) \\ &\times \delta(k-i)\delta(l-j) + c_q(n+k-i)c_q(n+k-i)^* \\ &\times c_0(n)^* c_0(n)\delta(n'-n)\delta(k-i+j-l) \\ &- c_q(n)c_q(n)^* c_0(n)^* c_0(n)\delta(k-i)\delta(j-l)\delta(n'-n). \end{aligned} \tag{30}$$

We note that $E(v_q(n)v_q(n')^*) = 1$ if n and n' correspond to the same symbol (i.e., if n and n' belong both to $\{kN,\ldots,kN+N-1\}$ for some integer k) and 0 otherwise. Therefore

$$\begin{aligned} &\sum_{n,n'=0}^{NP-1}\sum_{q=1}^{Q}\sum_{i,j=0}^{L}\mathbf{\Gamma}(i,j)c_q(n)c_q(n')c_0(n)c_0(n') \\ &\times E(v_q(n)v_q(n')^*)\left(\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} v_{m,0}(n)^* v_{m,0}(n')\right) \\ &\times \delta(k-i)\delta(l-j) \\ &= \mathbf{\Gamma}(k,l)\sum_{n,n'=0}^{NP-1} c_q(n)c_q(n')c_0(n)c_0(n') \\ &\times E(v_q(n)v_q(n')^*)\left(\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1} v_{m,0}(n)^* v_{m,0}(n')\right) \end{aligned}$$

is easily seen to be equal to 0 because vectors $(c_0(0),\ldots,c_0(N-1))^T$ and $(c_q(0),\ldots,c_q(N-1))^T$ are orthogonal for $q \neq 0$. Hence, the first term of (30) does not contribute to the value of $\mathbf{\Delta}_\infty(k,l)$. Therefore, the only term of $\lim_{M\to\infty}(1/M)\sum_{m=0}^{M-1} E(d_{m,q}(n+k-i)d_{m,q}(n'+l-j)^*)d_{m,0}(n)^* d_{m,0}(n')$ contributing to $\mathbf{\Delta}_\infty(k,l)$ is $\delta(n'-n)\delta(k-i+j-l)-\delta(k-i)\delta(j-l)\delta(n'-n)$ (recall that $c_q(n) = \pm 1$ for each q and each n). From this, it follows immediately that the contribution of the sum over integers $(p,q), p \geq 1, q \geq 1$ to $\mathbf{\Delta}_\infty(k,l)$ is equal to

$$\frac{1}{NP}\left(\sum_{q=1}^{Q}\mu_q^2\right)(\mathcal{T}(\mathbf{\Gamma})(k,l)-\mathbf{\Gamma}(k,l)).$$

In order to complete the proof of (11), we still need to calculate the contribution of the additive noise w to $\mathbf{\Delta}_\infty(k,l)$, which is easily seen to be equal to $(1/NP)\mathbf{R}_{x,\infty}(k,l)$. Putting all pieces together, we get that

$$\begin{aligned} \mathbf{\Delta}_\infty(k,l) = \mathbf{\Gamma}(k,l) &+ \frac{1}{NP}\left(\sum_{q=0}^{Q}\mu_q^2\right)(\mathcal{T}(\mathbf{\Gamma})(k,l) \\ &-\mathbf{\Gamma}(k,l)) + \frac{1}{NP}\mathbf{R}_{x,\infty}(k,l). \end{aligned}$$

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REFERENCES

[1] *Physical Channels and Mapping of Transport Channel Onto Physical Channels (FDD) (Release 1999)*, Mar. 2000.

[2] N. Ben Rached and J.-L. Dornstetter, "Time-weighted transmission channel estimation, patent 9 800 734,", Apr. 10 1998.

[3] P. Billingsley, *Convergence of Probability Measures*. New York: Wiley, 1968.

[4] V. Buchoux, O. Cappé, É. Moulines, and A. Gorokhov, "On the performance of semi-blind subspace-based channel estimation," *IEEE Trans. Signal Processing*, vol. 48, pp. 1750–1759, June 2000.

[5] G. Caire and U. Mitra, "Training sequence design for adaptive equalization of multi-user systems," in *IEEE Conf. Rec. Thirty-Second Asilomar Conf. Signals, Syst., Comput.*, vol. 2, 1998, pp. 1479–1483.

[6] R. H. Clarke, "A statistical theory of mobile radio reception," *Bell Syst. Tech. J.*, vol. 47, pp. 987–1000, 1968.

[7] P. Laspougeas, P. Pajusco, and J.-C. Bic, "Radio propagation in urban small cells environment at 2 GHz: Experimental spatio-temporal characterization and spatial wideband channel model," *Proc. IEEE Veh. Technol. Conf.*, Sept. 2000.
[8] A. W. Fuxjaeger and R. A. Itlis, "Adaptive parameter estimation using parallel Kalman filtering for spread spectrum system," *IEEE Trans. Commun.*, vol. 42, no. 6, pp. 2227–2230, June 1994.
[9] N. R. Goodman, "Statistical analysis based on a certain multivariate Gaussian distribution," *Ann. Math. Stat.*, vol. 34, pp. 152–177, Mar. 1963.
[10] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.
[11] W. Y. C. Lee, *Mobile Communications Engineering*. New York: McGraw-Hill, 1982.
[12] A. P. Liavas, P. A. Regalia, and J.-P. Delmas, "Robustness of least-squares and subspace methods for blind channel identification/equalization with respect to effective channel undermodeling/overmodeling," *IEEE Trans. Signal Processing*, vol. 47, pp. 1636–1645, June 1999.
[13] H. Liu and M. D. Zoltowski, "Blind equalization in antenna array CDMA systems," *IEEE Trans. Signal Processing*, vol. 45, pp. 161–172, Jan. 1997.
[14] A. Naguib, A. Paulraj, and T. Kailath, "Capacity improvement with base station antenna array in cellular CDMA," *IEEE Trans. Veh. Technol.*, vol. 43, pp. 691–698, Aug. 1994.
[15] I. S. Reed, J. D. Mallet, and L. E. Brennan, "Rapid convergence rate in adaptive arrays," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-10, pp. 853–863, Nov. 1974.
[16] D. T. M. Slock, J. Ayadi, and E. De Carvalho, "Blind and semi-blind maximum likelihood methods for FIR multichannel identification," in *Proc. ICASSP*, vol. 6, 1998, p. 3185.
[17] M. K. Tsatsanis, G. B. Giannakis, and G. Zhou, "Estimation and equalization of fading channels with random coefficients," *Signal Process.*, vol. 53, pp. 211–229, 1996.
[18] M. K. Tsatsanis and Z. Xu, "Pilot symbol assisted modulation in frequency selective fading wireless channels," *IEEE Trans. Signal Processing*, vol. 48, pp. 2353–2365, Aug. 2000.
[19] H. A. Cirpan and M. K. Tsatsanis, "Stochastic maximum likelihood methods for semi-blind channel equalization," *IEEE Signal Processing Lett.*, pp. 21–24, Jan. 1998.
[20] *Selection Procedures for the Choice of Radio Transmission Technologies of the UMTS (UMTS 30.03 Version 3.2.0)*, Apr. 1998. Tech. Spec. TR 101 112.
[21] A. Van der Veen, S. Talwar, and A. Paulraj, "Joint angle and delay estimation using shift invariance properties," *IEEE Trans. Signal Processing*, vol. 45, pp. 173–190, Jan. 1997.
[22] X. Zhuang and A. L. Swindlehurst, "A space-time semi-blind equalizer based on constant modulus and decision-direction," in *Proc. Conf. Rec. Thirty-Third Asilomar Conf. Signals, Syst., Comput.*, vol. 2, 1999, pp. 1017–1021.

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