2021 IEEE Statistical Signal Processing Workshop (SSP) | 978-1-7281-5767-2/21/$31.00 ©2021 IEEE | DOI: 10.1109/SSP49050.2021.9513815

TESTING UNCORRELATION OF MULTI-ANTENNA SIGNALS USING LINEAR SPECTRAL STATISTICS OF THE SPATIO-TEMPORAL SAMPLE AUTOCORRELATION MATRIX

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ABSTRACT

We consider the use of the spatio-temporal sample autocorrelation matrix in order to determine whether the signals received by a distributed antenna system are spatially correlated. The asymptotic behavior of linear spectral statistics built from this matrix is studied, assuming that (i) the number of antennas, (ii) the sample size and (iii) the number of tested time lags all converge to infinity. In this asymptotic regime, linear spectral statistics of the spatio-temporal sample correlation matrix are shown to be asymptotically equivalent to the corresponding functional average with respect to a Marchenko-Pastur distribution. This means that the eigenvalue distribution of the original sample autocorrelation matrix essentially behaves as a sample covariance matrix of spatio-temporal white noise with equivalent dimensions. This result turns out to be useful in order to address the problem of detecting the presence of wideband directional signals with a number of uncalibrated receivers distributed over a large area.

Index Terms— Correlation tests, multivariate time series, random matrix theory, wideband signal detection.

1. INTRODUCTION

We consider the problem of testing whether a collection of wideband signals received by a large number of spatially distributed antennas are correlated. The main motivation for this problem in array signal processing is the detection of the presence of wideband directional sources using a large number of receivers that are subject to calibration and synchronization errors. Since the received signals are generally wideband, they can be best modeled as time series. Under these challenging circumstances, and given the difficulty of establishing valid signal models for such a general propagation environment, one possible way of detecting the presence of directional signals is by establishing whether the signals that are measured at the different antennas are statistically independent. If the signals are assumed to be zero-mean and Gaussian distributed, the problem is equivalent to testing uncorrelation among the different time series.

Uncorrelation tests among multiple time series have traditionally been formulated from two different perspectives, namely frequency-domain approaches and lag-domain approaches. Frequency-domain methods establish that the different time series are uncorrelated when the spectral coherence matrix is equal to the identity over all the spectrum, e.g. [1, 2, 3, 4]. Lag-domain approaches, on the contrary, directly examine the autocorrelation coefficients between the different time series taken at distinct time lags, e.g. [5, 6, 7, 8, 9]. In this paper, we focus on this second family of approaches, which directly work on the sample autocorrelation matrix series between pairs of time series. For a similar treatment of frequency-domain approaches, the reader is referred to [10, 11].

Let $y_m[n]$ denote the signal that is captured at the mth antenna, $m = 1, \ldots, M$, where M is the total number of receive antennas. Assuming that the signals are complex circularly symmetric Gaussian stationary processes with zero mean, they will be uncorrelated if and only if

$$\mathbb{E}\left[y_m[n+\ell]y_k^*[n]\right] = r_m(\ell)\delta_{m=k} \tag{1}$$

for all $\ell \in \mathbb{Z}$, where $\left(r_m(\ell)\right)_{\ell\in\mathbb{Z}}$ is the covariance sequence of the mth received signal, which can be expressed as the inverse Fourier transform of the associated spectral density $\mathcal{S}_m(\nu)$, namely

$$r_m(\ell) = \int_0^1 \mathcal{S}_m(\nu)\, \mathrm{e}^{2\pi \mathrm{i}\nu\ell} d\nu. \tag{2}$$

In practice, the right hand side of (1) is replaced with the empirical mean estimate taken from N consecutive samples of the received signals, that is $\hat{r}_{m,m'}(n+\ell, n)$ where

$$\hat{r}_{m,m'}(n, n') = \frac{1}{N}\sum_{n=0}^{N-1} y_m[n]y_{m'}^*[n']$$

which is computed for a finite span of time lags, namely $\ell = -(L-1), \ldots, L-1$ for some design integer L. In order to formalize the construction of uncorrelation tests based on the above empirical estimates, let us consider the $ML \times ML$ spatio-temporal empirical covariance matrix $\widehat{\mathcal{R}}_L$. This matrix is structured in blocks of size $L \times L$ associated to each pair of series. More specifically, the $L \times L$ block matrix in the position (m, m') for $1 \leq m, m' \leq M$ can be denoted as $\widehat{\mathcal{R}}_L^{(m,m')}$ and corresponds to the sample cross-covariance matrix between the signals received at antennas m and m'. The (l_1, l_2)th entry of this matrix can therefore be written as

$$\left(\widehat{\mathcal{R}}_L^{(m,m')}\right)_{l_1,l_2} = \hat{r}_{m,m'}(n+l_1, n+l_2).$$

If we keep M, L fixed and let $N \to \infty$ assuming that the signals are independent, the spatio-temporal sample covariance matrix $\widehat{\mathcal{R}}_L$ converges almost surely to a block diagonal matrix with $L \times L$ Toeplitz blocks $\mathcal{R}_{m,L}$, $m = 1, \ldots, M$, with entries

$$\left(\mathcal{R}_{m,L}\right)_{l_1,l_2} = r_m(l_1 - l_2). \tag{3}$$

This means that, if N is sufficiently large, one can test whether the signals are uncorrelated by checking how far $\widehat{\mathcal{R}}_L$ is from a block

This work has been partially funded by the French government through the Bézout Labex (ANR-10-LABX-58) and the HIDITSA project (ANR-17-CE40-0003) and by the Spanish government under the Aristides project (RTI2018-099722-B-I00).

diagonal matrix. This is the main motivation for considering the sample spatio-temporal correlation matrix, defined as

$$\widehat{\mathcal{R}}_{\mathrm{corr},L} = \widehat{\mathcal{B}}_L^{-1/2}\widehat{\mathcal{R}}_L\widehat{\mathcal{B}}_L^{-1/2}$$

where $\widehat{\mathcal{B}}_L$ is the block diagonal matrix with blocks $\widehat{\mathcal{R}}_L^{(m,m)}$, $m = 1,\ldots,M$. If N is sufficiently large with respect to the other parameters (in the sense that $ML/N \ll 1$) one would expect $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ to be close to the identity matrix provided that the observed signals are uncorrelated.

A key aspect of the above discussion is how to choose the maximum tested lag L in a practical setting. On the one hand, L should be sufficiently large, because this allows to identify correlations among signals collected at different antennas associated to large time lags. For example, if a jammer source transmits white noise and is received at two different antennas with a time difference of more than L samples, it will be perceived as completely uncorrelated signals at these two antennas. On the other hand, L should be chosen sufficiently small so that $ML/N \ll 1$ in order to make the estimation error $\|\widehat{\mathcal{R}}_{\mathrm{corr},L} - \mathbf{I}_{ML}\|$ reasonably low when the signals are indeed uncorrelated. The situation is specially challenging when the number of collected signals M is large and the number of observations N is limited, because the condition $ML/N \ll 1$ requires the selection of a small value for L, thus drastically limiting the efficiency of the uncorrelation tests based on $\|\widehat{\mathcal{R}}_{\mathrm{corr},L} - \mathbf{I}_{ML}\|$. Our objective here is to study the spectral behavior of $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ in asymptotic regimes where M, N, L converge towards $+\infty$ in such a way that $c_N = \frac{ML}{N}$ converges towards a non zero constant $c_* \in (0,+\infty)$. This asymptotic regime is much more relevant than conventional asymptotics ($N \to \infty$ for fixed ML) because it corresponds to a situation where size of the spatio-temporal sample autocorrelation matrix $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ is comparable in magnitude to the number of samples used in the underlying sample covariance estimators.

There exist several studies in the literature that have specifically focused on this large dimensional asymptotic regime (see, e.g. [12, 13, 14]), although most of them have considered the case where L is fixed. In fact, a number of previous works have investigated the behavior of the autocovariance matrix for a given fixed time lag (see [15, 16, 17, 18, 19, 20, 21]). The case of large L has been investigated in [22] and [23], which also addressed the asymptotic regime considered in the present paper. More specifically, [22] assumed that the M mutually independent time series are i.i.d. Gaussian white noise and established that the empirical eigenvalue distribution of the sample spatio-temporal covariance matrix $\widehat{\mathcal{R}}_L$ converges to the Marchenko-Pastur distribution. In [23], a more general situation in which the different time series were independent but were allowed to present non-trivial statistical dependencies in the time domain. Under the present asymptotic conditions, it was established that the empirical eigenvalue distribution has also a deterministic behavior.

In this paper, we propose to study the behavior of spectral statistics built from the eigenvalues of $\widehat{\mathcal{R}}_{\mathrm{corr},L}$, which will be denoted by $(\hat{\lambda}_{k,N})_{k=1,\ldots,ML}$. More specifically, we will consider linear spectral statistics (LSS) built from this matrix, which take the general form

$$\widehat{\phi}_N = \frac{1}{ML}\mathrm{Tr}\left[\phi\left(\widehat{\mathcal{R}}_{\mathrm{corr},L}\right)\right] = \frac{1}{ML}\sum_{k=1}^{ML}\phi\left(\hat{\lambda}_{k,N}\right) \tag{4}$$

where ϕ is assumed to be a sufficiently smooth function. Our interest is mainly on the behavior of this type of statistics for the characterization of uncorrelation tests under the null hypothesis, which corresponds to the situation where the different signals are independent but present non-trivial statistical dependence in the time domain.

2. ASYMPTOTIC DETERMINISTIC EQUIVALENT OF $\widehat{\phi}_N$

Our objective is to characterize the behavior of linear spectral statistics as given in (4) when the three parameters M, L, N grow without bound. We will strongly rely on the following asymptotic and statistical assumptions.

Asymptotic assumptions. We will let $M \to +\infty, N \to +\infty$ in such a way that $c_N = \frac{ML}{N} \to c_\star$, where $0 < c_\star < +\infty$, and that $L = L(N) = \mathcal{O}(N^\beta)$ for some constant $\beta \in (0,1)$.

Special emphasis will be given to the case where β is small. Indeed, the scenario with small β can be associated with configurations operating with a large number of receiving antennas M. In this case, the conventional regime ($ML \ll N$) could only be achieved with a very small L, which would seriously limit the correlation identification capabilities among the different received signals.

Statistical assumptions. We will assume that the signals received by the M antennas, that is $y_m[n]$, $m \geq 1$, are mutually independent, stationary, zero mean and circularly symmetric Gaussian distributed time series with autocovariance sequences $(r_m(k))_{k\in\mathbb{N}}$ in (1) and associated spectral densities $\mathcal{S}_m(\nu)$ as in (2). We will assume that the spectral densities are uniformly bounded above and below, that is

$$0 < \inf_{m\geq 1}\min_{\nu\in[0,1]}\mathcal{S}_m(\nu) \leq \sup_{m\geq 1}\max_{\nu\in[0,1]}\mathcal{S}_m(\nu) < +\infty.$$

Furthermore, the autocovariance sequence r_m will be assumed to decay sufficiently fast in the lag domain, so that

$$\sup_{m\geq 1}\sum_{n=-\infty}^{\infty}(1+|n|)^{\gamma_0}\,|r_m(n)| < \infty \tag{5}$$

for some $\gamma_0 > 0$. Finally, if we denote by $\mathbf{r}_M$ the M-dimensional sequence of covariances, namely $\mathbf{r}_M(k) = [r_1(k),\ldots,r_M(k)]^T$, we will also need to assume that

$$\sup_{M\geq 1}\frac{1}{\sqrt{M}}\sum_{k\in\mathbb{Z}}\|\mathbf{r}_M(k)\| < +\infty.$$

We will see next that, with exponentially high probability, the empirical eigenvalue distribution of the $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ has the same asymptotic behavior as a deterministic scalar measure $\mu_N(\lambda)$ that essentially depends on the autocovariance sequences of the signals received at the M different antennas. This measure will be useful to characterize the asymptotics of the linear spectral statistics $\widehat{\phi}_N$ whenever the autocovariance sequences (or the associated spectral densities) of the different time series are known beforehand. Later, we will see that this deterministic measure $\mu_N(\lambda)$ can be further approximated by the Marchenko-Pastur distribution of parameter c_N, a result that can be used to characterize the asymptotic behavior of $\hat{\phi}_N$ in the more common situation where the covariance structure of the different signals is unknown.

In order to introduce the measure $\mu_N(\lambda)$, we need to consider some matrix Toeplitzation operators that were originally used in [23], which inherently depend on the covariance sequences $(r_m)_{m\geq 1}$ of the received signals. In order to introduce these operators, for $\nu \in [0,1]$ and $R \in \mathbb{N}$, we define the column vector

$$\mathbf{d}_R(\nu) = \left(1, \mathrm{e}^{2i\pi\nu}, \ldots, \mathrm{e}^{2i\pi(R-1)\nu}\right)^T \tag{6}$$

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and let $\mathbf{a}_R(\nu)$ denote the corresponding normalized vector, i.e. $\mathbf{a}_R(\nu) = R^{-1/2}\mathbf{d}_R(\nu)$. With these two definitions, we are now able to introduce the Toeplitzation operators used to define the above deterministic measure $\mu_N(\lambda)$. For a given squared matrix $\mathbf{M}$ with dimensions $R \times R$, we define $\Psi_K^{(m)}(\mathbf{M})$, $m = 1, \ldots, M$, as the $K \times K$ Toeplitz matrix given by

$$\Psi_K^{(m)}(\mathbf{M}) = \int_0^1 \mathcal{S}_m(\nu)\, \mathbf{a}_R^H(\nu)\, \mathbf{M} \mathbf{a}_R(\nu)\, \mathbf{d}_K(\nu)\, \mathbf{d}_K^H(\nu)\, d\nu.$$

The above operator is the key building block that defines Ψ and $\overline{\Psi}$, which are the ones that determine the master equations that define $\mu_N(\lambda)$. Indeed, consider an $N \times N$ matrix $\mathbf{M}$. We define $\Psi(\mathbf{M})$ as an $ML \times ML$ block diagonal matrix with mth diagonal block given by $\Psi_L^{(m)}(\mathbf{M})$, namely

$$\Psi(\mathbf{M}) = \mathrm{Bdiag}\left(\Psi_L^{(1)}(\mathbf{M}), \ldots, \Psi_L^{(M)}(\mathbf{M})\right). \tag{7}$$

Finally, consider an $ML \times ML$ matrix $\mathbf{M}$, and let $\mathbf{M}_{m,m}$ denote its mth $L \times L$ diagonal block. We define $\overline{\Psi}(\mathbf{M})$ as the $N \times N$ matrix given by

$$\overline{\Psi}(\mathbf{M}) = \frac{1}{M}\sum_{m=1}^{M} \Psi_N^{(m)}(\mathbf{M}_{m,m}). \tag{8}$$

Having introduced the above operators, we are now ready to present the master equations that define the deterministic measure $\mu_N(\lambda)$. Consider a $z \in \mathbb{C}^+$ (the upper complex semiplane) and the following pair of equations in $\mathbf{T}_N(z), \widetilde{\mathbf{T}}_N(z)$:

$$\mathbf{T}_N(z) = -\frac{1}{z}\left(\mathbf{I}_{ML} + \mathcal{B}_L^{-1/2}\Psi\left(\widetilde{\mathbf{T}}_N^T(z)\right)\mathcal{B}_L^{-1/2}\right)^{-1} \tag{9}$$

$$\widetilde{\mathbf{T}}_N(z) = -\frac{1}{z}\left(\mathbf{I}_N + c_N\overline{\Psi}^T\left(\mathcal{B}_L^{-1/2}\mathbf{T}_N(z)\mathcal{B}_L^{-1/2}\right)\right)^{-1}. \tag{10}$$

where $\mathcal{B}_L$ is an $ML \times ML$ block diagonal matrix with $L \times L$ blocks $\mathcal{R}_{m,L}$, $m = 1, \ldots, M$ as defined in (3). It can be shown that there exists a unique pair of solutions $\mathbf{T}_N(z), \widetilde{\mathbf{T}}_N(z)$ to the above equations in the set of matrix-valued Stieltjes transforms of positive matrix measures carried by $\mathbb{R}^+$ of sizes $ML \times ML$ and $N \times N$ respectively. Then, $\mu_N(\lambda)$ is defined as the deterministic measure with Stieltjes transform $t_N(z) = (ML)^{-1}\,\mathrm{Tr}(\mathbf{T}_N(z))$. We recall that the measure $\mu_N(\lambda)$ can be retrieved from its Stieltjes transform $t_N(z)$ via the Stieltjes inverse formula, that is

$$\int_a^b d\mu_N(\lambda) = \lim_{y \to 0^+}\frac{1}{\pi}\int_a^b \mathrm{Im}\left[t_N(x + \mathrm{i}y)\right]dx$$

for any two continuity points a, $b \in \mathbb{R}$. Having introduced this deterministic measure, we are now in the position to present the first result of this paper, which is proven in [24].

Theorem 1. *Consider the above statistical and asymptotic assumptions. Let $\beta < 4/5$ and assume that ϕ is a smooth function with compact support on the positive real axis. Then, for every small $\epsilon > 0$ there exists an $r > 0$ independent of N such that*

$$\mathbb{P}\left(\left|\hat{\phi}_N - \int_{\mathbb{R}^+}\phi(\lambda)d\mu_N(\lambda)\right| > \frac{N^\epsilon}{N^{\min(\beta\gamma_0, 1-\beta)}}\right) < \mathrm{e}^{-N^r} \tag{11}$$

for all N sufficiently large.

The above theorem establishes that, with exponentially high probability, linear spectral statistics of the spatio-temporal sample autocorrelation matrix are asymptotically close to a deterministic quantity that can be completely characterized from the deterministic scalar measure $\mu_N(\lambda)$, with an error term that decays as $\mathcal{O}(N^{-\min(\beta\gamma_0, 1-\beta)})$. In order to determine the asymptotic deterministic equivalent of the statistic $\hat{\phi}_N$, we need the deterministic measure $\mu_N(\lambda)$, which inherently depends on the spectral densities of the signals received at the different antennas. In most applications, these spectral densities are completely unknown, so that $\mu_N(\lambda)$ cannot possibly be determined. In these cases, we may consider an additional approximation step, which shows that $\mu_N(\lambda)$ is asymptotically close to a Marchenko-Pastur distribution of parameter c_N, up to an additional deterministic error term.

Before we present this second approximation, it is worth pointing out that the assumption that the function ϕ has compact support can be relaxed under some special conditions. For example, the above result is equally valid for the function $\phi(\lambda) = (\lambda - 1)^2$, which gives rise to the famous Frobenius norm test (sum of the squared modulus of the off-diagonal entries of $\widehat{\mathcal{R}}_{\mathrm{corr},L}$), which corresponds to $\hat{\phi}_N = ||\widehat{\mathcal{R}}_{\mathrm{corr},L} - \mathbf{I}_{ML}||_F^2$. We refer the reader to [24, Remark 6.1] for further details on when Theorem 1 can still be applied to functions without compact support.

3. APPROXIMATION BY A MARCHENKO-PASTUR LAW

Let us now turn to the more common situation where the spectral densities of the received signals are unknown, so that one has to rely on a less accurate asymptotic approximation of the linear spectral statistic. The following result is established in [24] by using results on orthogonal polynomials associated to the measures $\mathcal{S}_m(\nu)d\nu$ [25, 26]. We will denote by μ_{mp,c_N} the Marchenko-Pastur distribution with parameter $c_N = \frac{ML}{N}$. We recall that μ_{mp,c_N} is the limit of the empirical eigenvalue distribution of a large random matrix $N^{-1}\mathbf{X}\mathbf{X}^H$ where $\mathbf{X}$ is an $ML \times N$ random matrix with i.i.d. entries having zero mean and unit variance.

Theorem 2. *Consider the statistical and asymptotic assumptions at the beginning of Section 2. Then, for every $\gamma < \gamma_0$, $\gamma \neq 1$ and every compactly supported smooth function ϕ on the positive real axis, we have*

$$\left|\int_{\mathbb{R}^+}\phi(\lambda)d\mu_N(\lambda) - \int_{\mathbb{R}^+}\phi(\lambda)d\mu_{mp,c_N}(\lambda)\right| < \kappa\frac{1}{N^{2\beta\min(\gamma,1)}} \tag{12}$$

for some universal constant $\kappa > 0$.

The above theorem basically establishes a further level of approximation of the original linear spectral statistic $\hat{\phi}_N$, which can be asymptotically described by the Marchenko-Pastur law (thus not requiring the knowledge of the individual spectral densities $\mathcal{S}_m(\nu)$). The price to pay is an additional error term, this time decaying to zero with speed $\mathcal{O}(N^{-2\beta\min(\gamma,1)})$. Unfortunately, the error term in (12) becomes the dominant one as soon as $\beta < 1/3$ if $\gamma_0 > 1$. Note that the situation where β is small (or, equivalently, $L \ll M$) is the most relevant asymptotic scenario. Otherwise, the ratio M/N converges quickly towards 0, which in practice represents situations in which $M \ll N$. Therefore, it may be possible to choose a reasonably large value of L such that $\frac{ML}{N} \ll 1$ and therefore the simpler asymptotic regime where $\frac{ML}{N} \to 0$ may be relevant enough.

In conclusion, we observe that when $\beta < 1/3$ and $\gamma_0 > 1$, the dominant error incurred by approximating the linear spectra statistic $\hat{\phi}_N$ as an integral with respect to the Marchenko-Pastur law is in fact an unknown deterministic term as established in (12).

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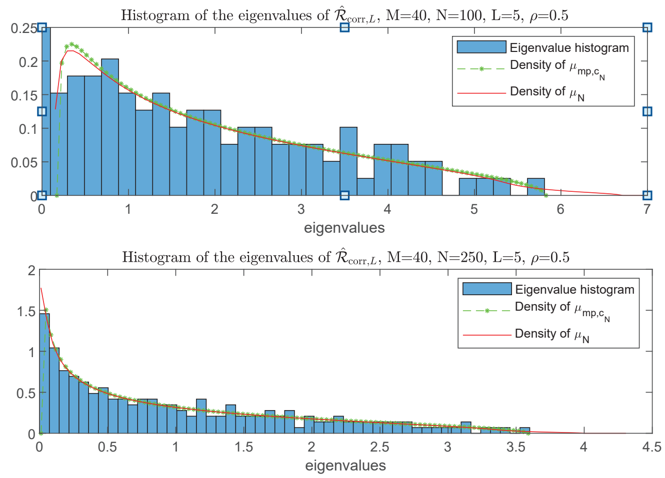

Fig. 1. Histogram of the eigenvalues of $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ in comparison with the distributions μ_N and μ_{mp,c_N} for different values of M, N, L with $\rho = 0.5$. The upper plot corresponds to a situation where $c_N > 1$ whereas the lower plot represents the case $c_N < 1$.

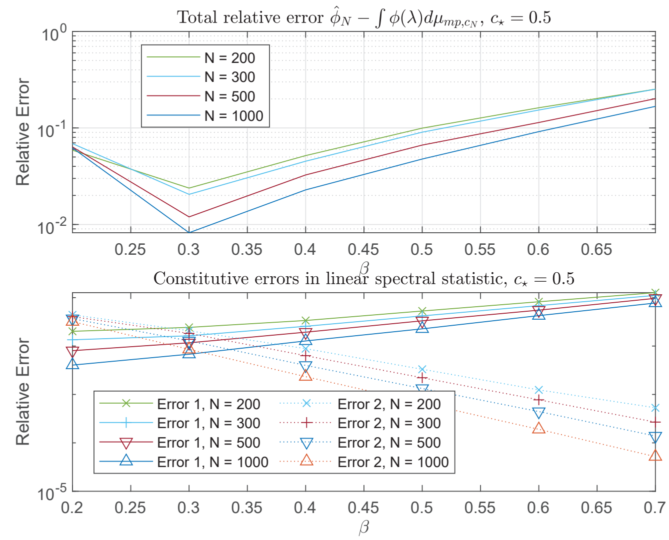

Fig. 2. Evolution of the error of $\hat{\phi}_N$ with respect to the Marchenko-Pastur limit as a function of β.

4. NUMERICAL VALIDATION

We consider here a simple example in which the M independent time series are all autoregressive processes of order one AR(1) with parameter ρ and unit power. By this, we mean that we generate each time series independently by the recursion $y_m[n+1] = \rho y_m[n] + e_m[n]$ where $e_m[n] \sim \mathcal{N}_{\mathbb{C}}(0, 1 - |\rho|^2)$. We first compare the empirical eigenvalue distribution of the sample cross correlation matrix $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ with the scalar deterministic measure μ_N and the Marchenko-Pastur distribution with parameter c_N, that is μ_{mp,c_N}. Figure 1 represents the histogram of the eigenvalues of $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ together with the densities of μ_N and μ_{mp,c_N} for different values of M, N, L. In general terms, we observe that the Marchenko-Pastur law is a very good approximation of the actual empirical eigenvalue distribution, even for relatively low values of M, L. In fact, it is difficult to spot the difference between the two densities of μ_N and μ_{mp,c_N}, which appear to be more apparent near the endpoints of their corresponding supports.

Next, consider a correlation detection test statistic consisting of the sum of the squared value of all the off-diagonal entries of $\widehat{\mathcal{R}}_{\mathrm{corr},L}$. As mentioned above, this corresponds to a linear spectral statistic of $\widehat{\mathcal{R}}_{\mathrm{corr},L}$ built with the function $\phi(\lambda) = (\lambda - 1)^2$. For this particular choice of the function $\phi(\lambda)$, we can establish that

$$\int \phi(\lambda) d\mu_N(\lambda) = c_N + c_N \frac{1}{ML} \mathrm{Tr}\left(\mathcal{B}_L^{-1} \boldsymbol{\Psi}(\mathbf{E}_N)\right) \tag{13}$$

where we have introduced the $N \times N$ matrix $\mathbf{E}_N$, defined as

$$\mathbf{E}_N = \int_0^1 \frac{1}{M} \sum_{m=1}^{M} \left(\mathcal{S}_m(\nu) \mathbf{a}_L^H(\nu) \mathcal{R}_{m,L}^{-1} \mathbf{a}_L(\nu) - 1 \right) \mathbf{d}_N(\nu) \mathbf{d}_N^H(\nu) d\nu$$

with $\mathcal{R}_{m,L}$ defined in (3). On the other hand, we can also establish that $\int \phi(\lambda) d\mu_{mp,c_N}(\lambda) = c_N$. In the upper plot of Figure 2 we evaluate the error between $\hat{\phi}_N$ and the corresponding integral of $\phi(\lambda)$ with respect to the Marchenko–Pastur distribution. For each experiment, we fixed the three parameters $c_\star$, N and β and considered a set of $M = [(c_\star N)^{1-\beta}]$ independent signals and $L = [(c_\star N)^{\beta}]$ time lags, where $[x]$ here denotes the integer that is closest to x. The errors are represented as the square root of the empirical mean of the corresponding normalized difference, averaged over the 10^4 realizations of the AR(1) processes. In the lower plot, we represent the two constituent errors that are characterized in Theorems 1 and 2 respectively. "Error 1" (solid lines) represents the square root of the empirical mean of $|\hat{\phi}_N - \int \phi(\lambda) d\mu_N|^2$, and "Error 2" (dotted lines) represents $|\int \phi(\lambda) d\mu_N - \int \phi(\lambda) d\mu_{mp,N}|$.

These results tend to confirm the fact that the error between the considered statistic and its asymptotic deterministic approximation tends to be dominated by two different phenomena depending on whether $M \ll L$ (large β) or $M \gg L$ (small β). In the fist case, the main contribution to the error corresponds to the term $\hat{\phi}_N - \int \phi(\lambda) d\mu_N$ (Error 1). We recall that, since the correlation sequence considered here decays exponentially to zero, this error term is dominated by $N^{-(1-\beta)}$, which in particular increases with β. Conversely, when $M \gg L$ (small β), the error is dominated by the difference between the two measures μ_N and $\mu_{mp,N}$. We have seen that this error term is dominated by a term of order $N^{-2\beta}$, which in particular decreases with β. Observe also that the optimum choice of β in terms of approximation error appears to be close to $1/3$, which corresponds to the case where the two error rates coincide.

5. CONCLUSIONS

We have examined the problem of testing uncorrelation via linear spectral statistics of the spatio-temporal sample autocorrelation matrix under the hypotheses that the signals received at the different antennas are mutually uncorrelated. In the large dimensional regime, these statistics can be well approximated by deterministic equivalents depending on the constituent spectral densities. A closed form expression has been derived for the Stieltjes transform of the deterministic measure associated to this asymptotic equivalent. Finally, it has been shown that (up to an additional error term) one can also express these deterministic equivalents as the corresponding integrals with respect to a Marchenko-Pastur distribution, which do not require the knowledge of the covariance structure of the signals.

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