On the fluctuations of the SINR at the output of the Wiener filter for non centered channels: the non Gaussian case

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Introduction

Ricean Multiple-Input Multiple-Output (MIMO) channels: We are interested by non-Gaussian channels which admits a deterministic line-of-sight (LOS) component.

LWR (Linear Wiener Receiver): In the context of multidimensional signals, the LWR is the linear receiver that achieves the lowest level of interference.

SINR (Signal-to-Interference plus Noise Ratio): To evaluate the performance of the Wiener filter, we focus on the study of the associated SINR at its output.

Problem statement

• Hypotheses and notations

\[ \mathbf{r} = \mathbf{H}_\mathbf{A} \mathbf{s} + \mathbf{w} \]

\( \mathbf{r} \) \((N \times 1)\) received vector \( \mathbf{w} \) : AWNG, \( \mathbb{E}(\mathbf{w}^* \mathbf{w}) = \rho \mathbf{I}_N \)

\( \mathbf{H} \) \((N \times (P+1))\) channel matrix

Channel Matrix Model:

\[ \mathbf{H} = \frac{1}{\sqrt{P}} \left( \sqrt{\frac{K}{K+1}} \mathbf{X} + \sqrt{\frac{1}{K+1}} \mathbf{I} \right) \Xi [\mathbf{y} + \mathbf{b} \ Y + \mathbf{B}], \]

where,

\( K \) is the Rice factor,

\( \mathbf{X} \) random matrix with i.i.d. entries with zero mean and unit variance, and

\( \Xi \) is the deterministic matrix which stands for the line-of-sight component.

• LWR estimator and the SINR

The estimate of \( s_0 \) at the LWR:

\[ \hat{s}_0 = [\mathbf{y} + \mathbf{b}]^\dagger [\mathbf{y} + \mathbf{b}] [\mathbf{Y} + \mathbf{B}]^\dagger + \rho \mathbf{I}_N^{-1} \mathbf{r}, \]

Then, the SINR is given by:

\[ \beta_T = [\mathbf{y} + \mathbf{b}]^\dagger \mathbf{Q} [\mathbf{y} + \mathbf{b}] \]

where, \( \mathbf{Q} = (\mathbf{Y} + \mathbf{B}) (\mathbf{Y} + \mathbf{B})^\dagger + \rho \mathbf{I}_N^{-1} \)

• Goal: Understanding the asymptotic behavior (first and second order) of the SINR, under the asymptotic regime (when the dimensions of the channel matrix tend to infinity with the same rate).

Theoretical Results

Important results: Deterministic system

For any \( \rho > 0 \), the deterministic system:

\[ \begin{cases} \hat{\beta}(\rho) = \frac{1}{\sqrt{P}} T_\mathbb{R}_\beta(\rho) \\ \hat{\beta}_T(\rho) = \frac{1}{\sqrt{P}} \tilde{T}_\mathbb{R}_\beta(\rho) \end{cases} \]

where \( T_\mathbb{R} \) and \( \tilde{T}_\mathbb{R} \) are the matrices

\[ \begin{align*} T_\mathbb{R}_\beta(\rho) &= \left( \rho (K+1) (1+\hat{\beta}(\rho)) \mathbf{I}_N + \frac{\mathbf{A} \mathbf{A}}{\rho Tr(\mathbf{A})} \right)^{-1} \\ \tilde{T}_\mathbb{R}_\beta(\rho) &= \left( \rho (K+1) (1+\hat{\beta}_T(\rho)) \mathbf{I}_N + \frac{\mathbf{A} \mathbf{A}}{\rho Tr(\mathbf{A})} \right)^{-1} \end{align*} \]

admits a unique solution \( (\hat{\beta}, \hat{\beta}_T) \) in \((0, \infty)^2\).

First order asymptotic behavior

Under the asymptotic regime, we have:

\[ \hat{\beta}_T(\rho) - \hat{\beta}(\rho) \xrightarrow{p \to \infty} 0 \]

where, \( \hat{\beta}_T = \frac{1}{\sqrt{P}} T_\beta + K a T_\alpha a \)

The Bias

We have:

\[ \mathbb{E}(\hat{\beta}_T - \hat{\beta}) \xrightarrow{p \to \infty} 0 \]

Second order result

Let \( \gamma = \frac{1}{\sqrt{P}} (T_\beta)^T, \xi = \frac{1}{\sqrt{P}} \mathbf{S} = \text{diag}(T) \) and \( \tilde{S} = \text{diag}(\tilde{T}) \). Let \( \kappa \) be the fourth cumulant of the entries of \( X \) given by \( \kappa = \mathbb{E}(X_{1,1}^4) - 2 \).

Define \( \Delta_p, \alpha_p \) and \( \zeta_p \) as:

\[ \begin{align*} \Delta_p &= 1 - \frac{1}{\rho} \frac{K}{K+1} \left( \frac{(\mathbf{A}^T \mathbf{A})^T}{\rho Tr(\mathbf{A})} \right)^2 - \frac{1}{\rho (K+1)^2} \\ \alpha_p &= \frac{1}{\rho} \left( \frac{K}{(K+1)^2} \right)^2 \left( \frac{1}{\rho (K+1)^2} \right)^2 \\ \zeta_p &= \frac{1}{\rho} \frac{1}{\mathbb{E} \left[ X_{11}^4 \right] - 2} \frac{N}{\rho} \sum_{i=1}^{N} [\mathbf{a} \mathbf{T}_\alpha \mathbf{a}]^2 + \frac{1}{\rho} \mathbb{E} \left[ \mathbf{S}^2 \right] \end{align*} \]

Then, the SINR satisfies:

\[ \sqrt{\frac{P}{\mathbb{E}}} (\hat{\beta}_T - \hat{\beta}) = O_p(\kappa) \]

Numerical Results

The context

\[ \mathbf{A} = [a(n_1), \ldots, a(n_m)], \quad \text{where} \quad a(n) = [1, e^{\iota \theta_2}, \ldots, e^{i(N-1)\theta_2}]^T \]

\( \tilde{A} \) is a directional vector.

\[ \mathbf{X}_{ij} = r_{ij} \exp(i \theta_{ij}), \]

\( \theta_{ij} \) are i.i.d. uniform phase variables over \([0, 2\pi]\), and

\( r_{ij} \) are i.i.d. real positive random variables having Nakagami-m distribution.

Empirical and asymptotic means with respect to \( c = N/P \)

\[ \omega = 1, \mu = 1, \]

\[ \rho = 0.5 \quad \text{and} \quad K = 1. \]

Empirical and asymptotic variances with respect to K

\[ \mu = 0.6, \omega = 1, \rho = 0.5. \]