On the fluctuations of the SINR at the output of the Wiener filter for non centered channels: the non Gaussian case

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Introduction

$\hookrightarrow$ **Ricean Multiple-Input Multiple-Output (MIMO) channels** : We are interested by non-Gaussian channels which admits a deterministic line-of-sight (LOS) component.

$\hookrightarrow$ **LWR (Linear Wiener Receiver)** : In the context of multidimensional signals, the LWR is the linear receiver that achieves the lowest level of interference.

$\hookrightarrow$ **SINR (Signal-to-Interference plus Noise Ratio)** : To evaluate the performance of the Wiener filter, we focuse on the study of the associated SINR at its output.

Problem statement

- **Hypotheses and notations**

$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$

$\mathbf{n}$: AWNG, $\mathbb{E}(\mathbf{n}\mathbf{n}^* = \rho \mathbf{I}_N)$

$\mathbf{r}$: $(N \times 1)$ received vector

$\mathbf{s}$: $((P+1) \times 1)$ transmitted symbols

$\mathbf{H}(N \times (P+1))$ channel matrix

Channel Matrix Model:

$$\mathbf{H} = \frac{1}{\sqrt{P}}\left(\sqrt{\frac{1}{1+K}}\mathbf{X} + \sqrt{\frac{K}{K+1}}\tilde{A}\right) \triangleq [\mathbf{y}+\mathbf{b} \quad \mathbf{Y}+\mathbf{B}],$$

where,
– K is the Rice factor,
– $\mathbf{X}$ random matrix with i.i.d. entries with zero mean and unit variance,and
– $\tilde{A}$ is the deterministic matrix which stands for the line-of-sight component.

- **LWR estimator and the SINR**

The estimate of s_0 at the LWR:

$$\hat{s}_0 = (\mathbf{y}+\mathbf{b})^*\left((\mathbf{Y}+\mathbf{B})(\mathbf{Y}+\mathbf{B})^* + \rho \mathbf{I}_p\right)^{-1}\mathbf{r},$$

Then, the SINR is given by:

$$\beta_P = (\mathbf{y}+\mathbf{b})^* \mathbf{Q}\, (\mathbf{y}+\mathbf{b}),$$

where, $\mathbf{Q} = \left((\mathbf{Y}+\mathbf{B})(\mathbf{Y}+\mathbf{B})^* + \rho \mathbf{I}_N\right)^{-1}$.

$\hookrightarrow$ **Goal** : Understanding the asymptotic behavior (first and second order) of the SINR, under the asymptotic regime (when the dimensions of the channel matrix tend to infinity with the same rate).

Theoretical Results

Important results: Deterministic system For any $\rho > 0$, the deterministic system:

$$\begin{cases} \delta_p(\rho) = \frac{1}{P}\mathrm{Tr}\,\mathbf{T}_p(\rho) \\ \tilde{\delta}_p(\rho) = \frac{1}{P}\mathrm{Tr}\,\tilde{\mathbf{T}}_p(\rho), \end{cases}$$

where $\mathbf{T}_p$ and $\tilde{\mathbf{T}}_p$ are the matrices

$$\begin{cases} \mathbf{T}_p(\rho) = \left(\rho(K+1)(1+\tilde{\delta}_p(\rho))\mathbf{I}_N + \frac{K\mathbf{A}\mathbf{A}^*}{P(1+\delta_p(\rho))}\right)^{-1} \\ \tilde{\mathbf{T}}_p(\rho) = \left(\rho(K+1)(1+\delta_p(\rho))\mathbf{I}_p + \frac{K\mathbf{A}^*\mathbf{A}}{P(1+\tilde{\delta}_p(\rho))}\right)^{-1} \end{cases}$$

admits a unique solution $(\delta_P, \tilde{\delta}_P)$ in $(0,\infty)^2$.

First order asymptotic behavior Under the asymptotic regime, we have:

$$\beta_p(\rho) - \bar{\beta}_p(\rho) \xrightarrow[p\to\infty]{a.s} 0, \qquad \text{where, } \bar{\beta}_p = \frac{1}{P}\mathrm{Tr}\,\mathbf{T}_p + K\mathbf{a}^*\mathbf{T}_p\mathbf{a}.$$

The Bias We have:

$$\mathbb{E}\beta_{\mathbf{p}} - \bar{\beta}_{\mathbf{p}} \xrightarrow[\mathbf{p}\to\infty]{} \mathbf{0}$$

Second order result Let $\gamma = \frac{1}{P}\mathrm{Tr}(\mathbf{T}^2)$, $\tilde{\gamma} = \frac{1}{P}\mathrm{Tr}\tilde{\mathbf{T}}$, $\mathbf{S} = \mathrm{diag}(\mathbf{T})$ and $\tilde{\mathbf{S}} = \mathrm{diag}(\tilde{\mathbf{T}})$. Let κ be the fourth cumulant of the entries of $\mathbf{X}$ given by $\kappa = \mathbb{E}|X_{1,1}|^4 - 2$. Define Δ_p, α_p and ξ_p as:

$$\begin{aligned} \Delta_p &= \left(1 - \frac{K}{P(1+\delta)^2}\mathrm{Tr}(\mathbf{A}\mathbf{A}^*\mathbf{T}^2)\right)^2 - \rho^2(K+1)^2\gamma\tilde{\gamma}, \\ \alpha_p &= \frac{1}{\gamma(1+\delta)^4}\left[\gamma\left(\frac{K}{P}\mathrm{Tr}\mathbf{T}^2\mathbf{A}\mathbf{A}^* + K^2\mathbf{a}^*\mathbf{T}\mathbf{A}\mathbf{A}^*\mathbf{T}\mathbf{a}\right)\right. \\ &\quad \left. + \left((1+\delta)^2 - \frac{K}{P}\mathrm{Tr}\mathbf{T}^2\mathbf{A}\mathbf{A}^*\right)\left(\gamma + K\mathbf{a}^*\mathbf{T}^2\mathbf{a}\right)\right]^2, \\ \xi_p &= \rho^2(K+1)^2K^2\frac{1}{P}\mathrm{Tr}\tilde{\mathbf{S}}^2\sum_{i=1}^{N}[\mathbf{T}\mathbf{a}\mathbf{a}^*\mathbf{T}]_{i,i}^2 \\ &\quad + \frac{K^4}{(1+\delta)^4}\frac{1}{P}\mathrm{Tr}\mathbf{S}^2\sum_{i=1}^{N}|\mathbf{a}^*\mathbf{T}\mathbf{a}_i\mathbf{a}_i^*\mathbf{T}\mathbf{a}|^2 + \frac{1}{P}\mathrm{Tr}\mathbf{S}^2. \end{aligned}$$

Then, the SINR satisfies:

$$\sqrt{\frac{P}{\Omega_p^2}}\left(\beta_p - \bar{\beta}_p\right) \xrightarrow[P\to\infty]{\mathcal{D}} \mathcal{N}(0,1), \ \text{where, } \Omega_p^2 = \frac{\alpha_p}{\Delta_p} - \frac{K^2(\mathbf{a}^*\mathbf{T}\mathbf{a})^2}{\gamma} + \kappa\xi_p.$$

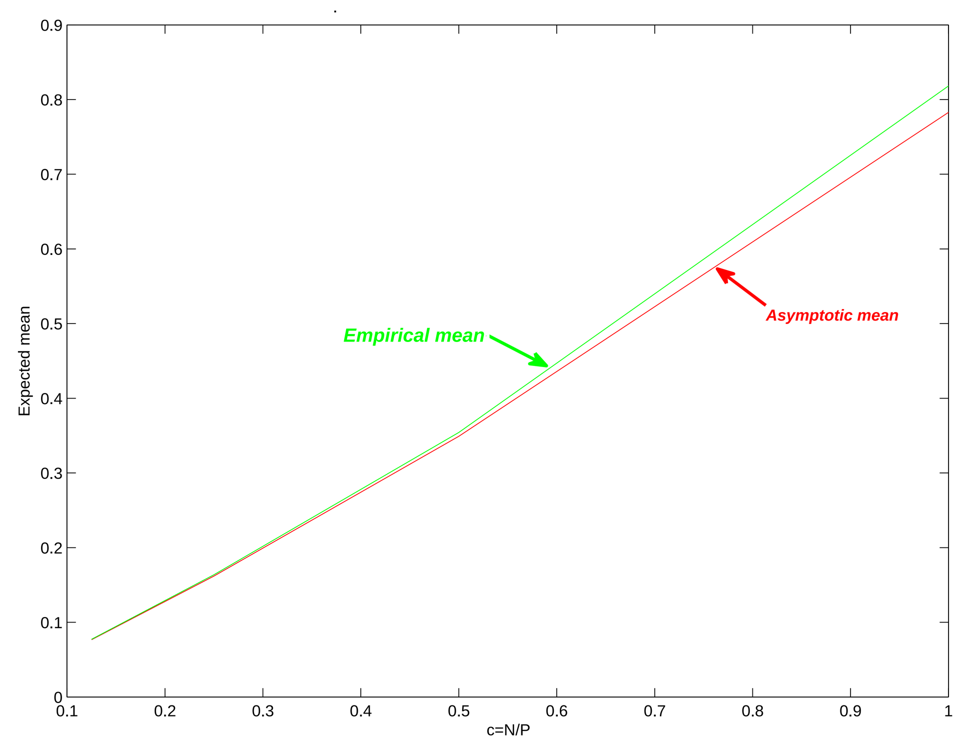
Numerical Results

The context

- $\tilde{\mathbf{A}} = [\mathbf{a}(\alpha_1), \ldots, \mathbf{a}(\alpha_{p+1})]$, where $\mathbf{a}(\alpha_i) = \left[1, e^{j\alpha}, \ldots, e^{j(N-1)\alpha}\right]^T$ is a directional vector,
- $\mathbf{X}_{i,j} = r_{i,j}exp(\theta_{i,j})$, where $\theta_{i,j}$ are i.i.d. uniform phase variables over $[0, 2\pi]$, and $r_{i,j}$ are i.i.d. real positive random variables having Nakagami-m distribution.

Empirical and asymptotic means with respect to $c = N/P$

- $\omega = 1$, $\mu = 1$,
- $\rho = 0.5$ and $K = 1$.



Empirical and asymptotic variances with respect to K

- $\mu = 0.6$, $\omega = 1$,
- $\rho = 0.5$.