Outage Performance of Cooperative Small-Cell Systems Under Rician Fading Channels

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***Abstract*—We consider a general class of Rician fading multiple-input multiple-output (MIMO) channels, modeled by a random, non-centered channel matrix with a variance profile, i.e., the independent elements of the matrix are allowed to have each a different mean and variance. This channel model is motivated by the recent interest in cooperative small-cell systems where several densely deployed base stations (BSs) cooperatively serve multiple user terminals (UTs). We study the fluctuations of the mutual information of this channel under the form of a central limit theorem (CLT) and provide an explicit expression of the asymptotic variance. The result can be used to compute an approximation of the outage probability of such channels. Although the derived expressions are only tight in the large system limit, we show by simulations that they provide very accurate approximations for realistic system dimensions.**

I. Introduction

With an exponentially growing demand for mobile data services, operators are facing the challenge of how to increase the capacity of their networks: No breakthroughs in coding or modulation schemes are to be expected and additional spectrum resources are scarce [1]. This development has stimulated research on interference cancellation techniques [2], multicell processing [3], and cognitive radio [4] to improve the spectral efficiency of today's wireless networks. However, none of these techniques is likely to carry the expected increase in mobile data traffic alone and a further network densification seems necessary. Since simply deploying more macro cell base stations (BSs) causes prohibitive capital and operational expenditures, a new concept of massive network densification, known as "Small-Cell Networks (SCNs)", is of current investigation [5]. In short, SCNs are based on the idea of a very dense deployment of self-organizing, low-cost, low-power BSs which could potentially provide unprecedented network capacities in an economically viable way.

Although a promising concept, SCNs pose also many new challenges to the system design. Among numerous other reasons (see, e.g., [5]), this is because smaller cell sizes cause significant changes to the wireless link. With necessarily lower antenna heights, the wave propagation becomes less predictable and the channels between the BSs and the user terminals (UTs) are likely to contain strong line-of-sight (LOS) components. Moreover, some form of cooperation between the BSs becomes mandatory in highly mobile environments since hard handovers between the small-cell BSs would occur otherwise far too frequently.

We consider in this paper a general channel model, well-suited for the study of cooperative small-cell systems. More precisely, we assume a Rician fading channel, where each complex channel gain between a transmitter and a receiver is allowed to have a different mean and variance. The latter assumption is relevant to cooperative small-cell systems since a UT might be simultaneously served by multiple BSs to each of which it has a channel with a different path loss. The aim of this paper is to study the fluctuations of the mutual information of this channel around its mean, or more precisely around a deterministic approximation of its mean [6], when the channel dimensions grow large. The result is established under the form of a central limit theorem (CLT) and can be used, e.g., to compute a close approximation of the outage probability.

The fluctuations of the mutual information around its deterministic approximation have been studied in several works. For channel matrices with Gaussian entries, the replica method, an approach borrowed from mechanical statistics, has been often pushed forward. Although not fully mathematically rigorous, this method has provided exact results for Rayleigh [7] and Rician fading [8] channels with a separable variance profile. The use of advanced tools of random matrix theory allowed the treatment of more sophisticated channel models and to provide sound mathematical proofs. Rayleigh fading channels with a separable variance profile and a general variance profile with arbitrary colored noise were considered in [9] and [10], respectively. Also more general channel matrix models composed of independent and identically distributed (i.i.d.) entries, not necessarily Gaussian, and a deterministic Rician component [11] and channel matrices with centered entries and a general variance profile [12] have been treated recently. The case of a channel matrix with a general variance profile *and* a Rician component has not been covered so far.

It shall be noticed that although Theorem 1 is a by-product of [6], the fluctuation results presented in this contribution (Theorem 3) are neither by-products of [12] (where no Rician component appears) nor of [10] (see also Remark 1 below). It is indeed customary, and somehow unfortunate, in large random matrix theory that new computations must be performed from the very beginning whenever a substantial change of the model (with respect to previously studied models) is considered. This is the case in this work. Thus, apart from being of practical interest for the performance analysis of cooperative small-cell systems, our result is also a novel contribution to the field of random matrix theory in two directions: a) we

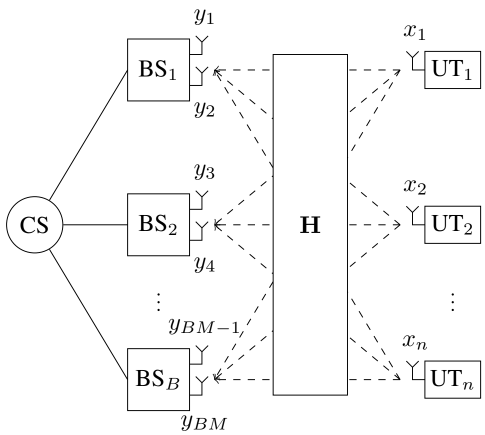

Fig. 1. Schematic system model for the case $M = 2$.

give a comprehensive outline of the proof of Theorem 3, b) we provide a straightforward way of computing the variance based on the fact that it can be written as the logarithm of a well-defined Jacobian matrix. We would also like to stress that this formula for the variance, which was already used in [10] in a different context, is: (i) straightforward, (ii) can certainly be extended to other complex models and circumvents a complete mathematical development[1] which is usually extremely involved, cf. [12], [9], [13], (iii) and applies beyond the range of the replica method.

II. System model

We consider a multi-cell, Rician fading, uplink channel from n single-antenna UTs to B BSs, each equipped with M antennas. The BSs are connected to a central station (CS) via orthogonal, delay- and error-free backhaul links of infinite capacity. We assume that the CS jointly processes the received signals from all BSs. This model corresponds to a cooperative small-cell system where several BSs are connected together to a cluster or "virtual cell" [5] which appears as a single, distributed BS to the UTs. Apart from providing the well known gains of multi-cell processing (network MIMO) [3], virtual cells are mainly used in this context to reduce the number of handovers between the small cells. A schematic diagram of the system model is shown in Fig. 1. The stacked receive vector of all BSs $\mathbf{y} = [y_1, \ldots, y_{BM}]^\mathsf{T} \in \mathbb{C}^{BM}$ at a given time reads

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

where $\mathbf{x} = [x_1, \ldots, x_n]^\mathsf{T} \in \mathbb{C}^n$ is the vector of the transmitted signals of all UTs, $\mathbf{H} = [\mathbf{H}_1^\mathsf{T} \cdots \mathbf{H}_B^\mathsf{T}]^\mathsf{T} \in \mathbb{C}^{BM\times n}$ is the aggregated channel matrix from all UTs to all BSs and $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \rho \mathbf{I}_{BM})$ is a vector of additive noise. The UTs are subject to the transmit power constraint $\mathbb{E}\left[|x_j|^2\right] \leq 1, \forall j$. We model the channel from the UTs to the b^{th} BS by the channel matrix $\mathbf{H}_b \in \mathbb{C}^{M\times n}$ whose elements are given as

$$[\mathbf{H}_b]_{m,k} = \underbrace{\sqrt{\frac{(1-\kappa_{bk})\,\ell_{bk}}{n}}\, w_{mk}^b}_{\text{Rayleigh component}} + \underbrace{\sqrt{\frac{\kappa_{bk}\,\ell_{bk}}{n}}\, e^{j\phi_{mk}^b}}_{\text{LOS component}}$$

where the w_{mk}^b are i.i.d. standard complex Gaussian random variables, ℓ_{bk} is the inverse path-loss and $\kappa_{bk} \in [0,1]$ the Rician parameter of the channel between UT k and BS b, and $\phi_{mk}^b \in [0, 2\pi)$ is the phase of the specular component of the channel between UT k and the mth antenna of BS b. In general, the larger κ_{bk}, the more deterministic is the channel; for $\kappa_{bk} = 0$, the channel is purely Rayleigh faded. Define the matrices $\mathbf{\Sigma} = [\mathbf{\Sigma}_1^\mathsf{T} \cdots \mathbf{\Sigma}_B^\mathsf{T}]^\mathsf{T} \in \mathbb{R}_+^{BM\times n}$ and $\mathbf{A} = [\mathbf{A}_1^\mathsf{T} \cdots \mathbf{A}_B^\mathsf{T}]^\mathsf{T} \in \mathbb{C}^{BM\times n}$, where the matrices $\mathbf{\Sigma}_b \in \mathbb{R}_+^{M\times n}$ and $\mathbf{A}_b \in \mathbb{C}^{M\times n}$ are given as

$$[\mathbf{\Sigma}_b]_{m,k} = (1-\kappa_{bk})\ell_{bk}\ , \quad [\mathbf{A}_b]_{m,k} = \sqrt{\frac{\kappa_{bk}\ell_{bk}}{n}} e^{j\phi_{mk}^b}\ .$$

Under these assumptions, the elements $[\mathbf{H}]_{i,j}$ of the aggregated channel matrix $\mathbf{H}$ are independent, circular symmetric complex Gaussian random variables with mean a_{ij} and variance σ_{ij}^2/n, where a_{ij} and σ_{ij}^2 are the (i,j) elements of the matrices $\mathbf{A}$ and $\mathbf{\Sigma}$, respectively.

Assuming that all UTs apply complex Gaussian codebooks and that the channel $\mathbf{H}$ is fully known at the CS, the normalized ergodic mutual information of the channel is given by $I(\rho) = \mathbb{E}[\mathcal{I}(\rho)]$, where

$$\mathcal{I}(\rho) = \frac{1}{N}\log\det\left(\mathbf{I}_N + \frac{1}{\rho}\mathbf{H}\mathbf{H}^\mathsf{H}\right)$$

and we defined $N = BM$. In the next sections, we provide a deterministic approximation $V(\rho)$ of $I(\rho)$ and study the fluctuations of the random variable $N(\mathcal{I}(\rho) - V(\rho))$ under the assumption that N and n grow large at the same speed.

III. Deterministic Approximation of $I(\rho)$

For fixed size dimensions, finding a closed-form expression for the ergodic mutual information $I(\rho)$ is only possible in certain academic cases, for example when the channel matrix $\mathbf{H}$ is standard complex Gaussian [14]. For more realistic scenarios, the study of the mutual information has often been performed in the asymptotic regime, i.e., when $N, n \to \infty$ in such a way that

$$0 < \liminf \frac{N}{n} \leq \limsup \frac{N}{n} < \infty\ .$$

However, even in the asymptotic setting, the mutual information still does not have a closed-form expression, in general. Deterministic quantities which depend solely on the statistical properties of the channel and which are given as the solution of several implicit equations are therefore introduced. As we will see next, such quantities play a key role for the approximation of the mutual information and its fluctuations.

For $j \in \{1, \ldots, n\}$ and $i \in \{1, \ldots, N\}$, define the matrices:

$$\mathbf{D}_j = \operatorname{diag}\left(\sigma_{1j}^2, \ldots, \sigma_{Nj}^2\right)\ , \quad \widetilde{\mathbf{D}}_i = \operatorname{diag}\left(\sigma_{i1}^2, \ldots, \sigma_{in}^2\right)\ .$$

[1] Although its rigorous justification remains based on a posteriori mathematical developments.

Denote by $\mathbb{C}_+ = \{z \in \mathbb{C} : \mathrm{Im}(z) > 0\}$, and by $\mathcal{S}$ the class of functions f analytic over $\mathbb{C}_+$, such that $f : \mathbb{C}_+ \to \mathbb{C}_+$ and $\lim_{y\to\infty} -\mathbf{i}yf(\mathbf{i}y) = 1$, where $\mathbf{i} = \sqrt{-1}$.[2] The following theorem provides a deterministic equivalent of the ergodic mutual information $\mathcal{I}(\rho)$:

Theorem 1 (Deterministic Equivalent): [6, Theorems 2.4, 4.1] Under some mild technical assumptions:

(i) The following set of $N + n$ deterministic equations,

$$\psi_i(z) = \frac{1}{\rho\left(1 + \frac{1}{n}\mathrm{tr}\widetilde{\mathbf{D}}_i\widetilde{\mathbf{T}}(z)\right)}, \quad 1 \le i \le N$$

$$\widetilde{\psi}_j(z) = \frac{1}{\rho\left(1 + \frac{1}{n}\mathrm{tr}\mathbf{D}_j\mathbf{T}(z)\right)}, \quad 1 \le j \le n$$

where

$$\begin{aligned}
\mathbf{\Psi}(z) &= \mathrm{diag}\left(\psi_i(z),\, 1 \le i \le N\right) \\
\widetilde{\mathbf{\Psi}}(z) &= \mathrm{diag}\left(\widetilde{\psi}_j(z),\, 1 \le j \le n\right) \\
\mathbf{T}(z) &= \left(\mathbf{\Psi}(z)^{-1} + \rho\mathbf{A}\widetilde{\mathbf{\Psi}}(z)\mathbf{A}^{\mathsf{H}}\right)^{-1} \\
\widetilde{\mathbf{T}}(z) &= \left(\widetilde{\mathbf{\Psi}}(z)^{-1} + \rho\mathbf{A}^{\mathsf{H}}\mathbf{\Psi}(z)\mathbf{A}\right)^{-1}
\end{aligned}$$

admits a unique solution $(\psi_1(z), \dots, \psi_N(z), \widetilde{\psi_1}(z), \dots, \widetilde{\psi_n}(z)) \in \mathcal{S}^{N+n}$ for $z \in \mathbb{C} \setminus \mathbb{R}^+$.

(ii) Let $\rho > 0$ and consider the quantity:

$$\begin{aligned}
V(\rho) =& \frac{1}{N}\log\det\left(\frac{\mathbf{\Psi}(-\rho)^{-1}}{\rho} + \mathbf{A}\widetilde{\mathbf{\Psi}}(-\rho)\mathbf{A}^{\mathsf{H}}\right) \\
&+ \frac{1}{N}\log\det\left(\frac{\widetilde{\mathbf{\Psi}}(-\rho)^{-1}}{\rho}\right) \\
&- \frac{\rho}{Nn}\sum_{\substack{i=1,\dots,N \\ j=1,\dots,n}} \sigma_{ij}^2 \mathbf{T}_{ii}(-\rho)\widetilde{\mathbf{T}}_{jj}(-\rho) \ .
\end{aligned}$$

Then, the following holds true:

$$I(\rho) - V(\rho) \xrightarrow[N,n\to\infty]{} 0 \ .$$

In the following, $z = -\rho$. If we define $\delta_j = \frac{1}{n}\mathrm{tr}\mathbf{D}_j\mathbf{T}$, $\widetilde{\delta}_i = \frac{1}{n}\mathrm{tr}\widetilde{\mathbf{D}}_i\widetilde{\mathbf{T}}$, and let $\mathbf{\Delta} = \mathrm{diag}\left(\delta_j, 1 \le j \le n\right)$ and $\widetilde{\mathbf{\Delta}} = \mathrm{diag}\left(\widetilde{\delta}_i, 1 \le i \le N\right)$, then, the system of $N + n$ equations in Theorem 1 (i) can be written in an equivalent way as:

$$\begin{aligned}
\delta_j &= \Gamma_j(\mathbf{\Delta}, \widetilde{\mathbf{\Delta}}) \ , \quad 1 \le j \le n \\
\widetilde{\delta}_i &= \widetilde{\Gamma}_i(\mathbf{\Delta}, \widetilde{\mathbf{\Delta}}) \ , \quad 1 \le i \le N
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_j(\mathbf{\Delta}, \widetilde{\mathbf{\Delta}}) &\triangleq \frac{1}{n}\mathrm{tr}\mathbf{D}_j\left[\rho\left(\mathbf{I}_N + \widetilde{\mathbf{\Delta}}\right) + \mathbf{A}\left(\mathbf{I}_n + \mathbf{\Delta}\right)^{-1}\mathbf{A}^{\mathsf{H}}\right]^{-1} \\
\widetilde{\Gamma}_i(\mathbf{\Delta}, \widetilde{\mathbf{\Delta}}) &\triangleq \frac{1}{n}\mathrm{tr}\widetilde{\mathbf{D}}_i\left[\rho\left(\mathbf{I}_n + \mathbf{\Delta}\right) + \mathbf{A}^{\mathsf{H}}\left(\mathbf{I}_N + \widetilde{\mathbf{\Delta}}\right)^{-1}\mathbf{A}\right]^{-1}
\end{aligned}$$

As we will see in the next section, the functions Γ_j and $\widetilde{\Gamma}_i$ will help in providing a concise expression for the asymptotic variance of the random variable $N(\mathcal{I}(\rho) - V(\rho))$. Prior to that, let us consider the case of a separable variance profile for which Theorem 1 (i) simplifies to the following result:

Theorem 2: Assume $\sigma_{ij}^2 = d_i\widetilde{d}_j$. Let $\mathbf{D} = \mathrm{diag}(d_i, 1 \le i \le N)$ and $\widetilde{\mathbf{D}} = \mathrm{diag}(\widetilde{d}_j, 1 \le j \le n)$. Then, the following system of implicit equations

$$\delta = \Gamma_1(\delta, \widetilde{\delta}) \ , \qquad \widetilde{\delta} = \Gamma_2(\delta, \widetilde{\delta})$$

where

$$\begin{aligned}
\Gamma_1(\delta, \widetilde{\delta}) &\triangleq \frac{1}{n}\mathrm{tr}\mathbf{D}\left[\rho\left(\mathbf{I}_N + \widetilde{\delta}\mathbf{D}\right) + \mathbf{A}(\mathbf{I}_n + \delta\widetilde{\mathbf{D}})^{-1}\mathbf{A}^{\mathsf{H}}\right]^{-1} \\
\Gamma_2(\delta, \widetilde{\delta}) &\triangleq \frac{1}{n}\mathrm{tr}\widetilde{\mathbf{D}}\left[\rho\left(\mathbf{I}_n + \delta\widetilde{\mathbf{D}}\right) + \mathbf{A}^{\mathsf{H}}(\mathbf{I}_N + \widetilde{\delta}\mathbf{D})^{-1}\mathbf{A}\right]^{-1}
\end{aligned}$$

admits a unique solution $(\delta, \widetilde{\delta})$ in $(0, +\infty)^2$ for any $\rho > 0$.

IV. Fluctuations of $\mathcal{I}(\rho)$: A Central Limit Theorem

In this section, we present a theorem about the asymptotic behavior of the fluctuations of the mutual information $\mathcal{I}(\rho)$. Before stating our result, we will first provide a brief overview of the recently studied scenarios.

A. Separable case

The case of a separable variance profile with Gaussian centered elements, i.e., $\mathbf{A} = \mathbf{0}$ and $\sigma_{ij}^2 = d_i\widetilde{d}_j$, was first studied using the replica method [7], where the functionals δ and $\widetilde{\delta}$ play a key role in the derivation of the asymptotic moments. More precisely, they arise during the computation of the cumulant generating function and are chosen to cancel the derivative of the first asymptotic moment. A saddle point approximation is then performed to approximate the other higher order asymptotic moments. In particular, the variance of the mutual information is given by

$$\begin{aligned}
\Theta_{N,n}^2 &= -\log(1 - \frac{\rho^2}{n^2}\mathrm{tr}\mathbf{D}^2\mathbf{T}^2\mathrm{tr}\widetilde{\mathbf{D}}^2\widetilde{\mathbf{T}}^2) \\
&= -\log\det\begin{bmatrix} 1 & \frac{\rho}{n}\mathrm{tr}\mathbf{D}^2\mathbf{T}^2 \\ \frac{\rho}{n}\mathrm{tr}\widetilde{\mathbf{D}}^2\widetilde{\mathbf{T}}^2 & 1 \end{bmatrix} \\
&\triangleq -\log\det(\mathbf{J}) \ .
\end{aligned}$$

Let $\Gamma : \mathbb{R}^2 \to \mathbb{R}^2\,(\delta, \widetilde{\delta}) \mapsto (\delta - \Gamma_1(\delta, \widetilde{\delta}), \widetilde{\delta} - \Gamma_2(\delta, \widetilde{\delta}))$, then one can easily verify that $\mathbf{J}$ is in fact the Jacobian matrix of the function Γ. In the replica method, the factor $\det(\mathbf{J})$ arises while applying the change of variables formula for the computation of a certain integral. In a later paper [9], the same result was given a rigorous mathematical proof.

B. General variance profile with zero mean entries

The case of a general variance profile with zero mean elements, i.e., $\mathbf{A} = \mathbf{0}$, has been studied in [12]. Taking a close look at the obtained second order results [12, Theorem 3.1], one can verify that the asymptotic variance $\Theta_{N,n}^2$ can be written again as $\Theta_{N,n}^2 = -\log\det(\mathbf{J})$, where $\mathbf{J}$ is the Jacobian of the function $\Gamma : \mathbb{R}^{n+N} \to \mathbb{R}^{n+N}$ $(\delta_1, \cdots, \delta_n, \widetilde{\delta}_1, \cdots \widetilde{\delta}_N) \mapsto (f_1, \cdots, f_n, \widetilde{f}_1, \cdots \widetilde{f}_N)$, where $f_j = \delta_j - \Gamma_j(\mathbf{\Delta}, \widetilde{\mathbf{\Delta}})$ $\forall j$ and $\widetilde{f}_i = \widetilde{\delta}_i - \widetilde{\Gamma}_i(\mathbf{\Delta}, \widetilde{\mathbf{\Delta}})$ $\forall i$.

[2] Such functions are known to be Stieltjes transforms of probability measures over $\mathbb{R}$ - see for instance [6, Proposition 2.2].

C. General variance profile with non zero mean entries

Based on the careful observation of the aforementioned results, we claim that the mutual information $\mathcal{I}(\rho)$ behaves asymptotically also for this more general random matrix model as a Gaussian random variable. This is our main result.

Theorem 3 (The CLT): Under some mild technical assumptions, the mutual information $\mathcal{I}(\rho)$ satisfies

$$\frac{N}{\Theta_{N,n}}\left(\mathcal{I}(\rho)-V(\rho)\right)\xrightarrow[N,n\to\infty]{\mathcal{D}}\mathcal{N}(0,1)$$

where $\mathcal{D}$ denotes convergence in distribution and the asymptotic variance $\Theta_{N,n}$ is given as

$$\Theta_{N,n}^2=-\log\det(\mathbf{J})\ . \tag{1}$$

Letting $\mathbf{a}_i$, $\mathbf{b}_i$ $\mathbf{t}_i$ and $\widetilde{\mathbf{t}}_i$ denote respectively the columns of $\mathbf{A}$, $\mathbf{A}^{\mathsf{H}}$, $\mathbf{T}$ and $\widetilde{\mathbf{T}}$, the matrix $\mathbf{J}$ takes the following form:

$$\mathbf{J}=\begin{pmatrix}\mathbf{J}_1 & \mathbf{J}_2\\ \mathbf{J}_3 & \mathbf{J}_4\end{pmatrix}$$

where

$$\begin{aligned}
[\mathbf{J}_1]_{\substack{k=1,\dots,n\\ m=1,\dots,n}} &= 1_{\{k,m\}}-\frac{1}{n(1+\delta_m)^2}\mathbf{a}_m^{\mathsf{H}}\mathbf{T}\mathbf{D}_k\mathbf{T}\mathbf{a}_m\\
[\mathbf{J}_2]_{\substack{k=1,\dots,n\\ m=1,\dots,N}} &= \frac{\rho}{n}\mathbf{t}_m^{\mathsf{H}}\mathbf{D}_k\mathbf{t}_m\\
[\mathbf{J}_3]_{\substack{k=1,\dots,N\\ m=1,\dots,n}} &= \frac{\rho}{n}\widetilde{\mathbf{t}}_m^{\mathsf{H}}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{t}}_m\\
[\mathbf{J}_4]_{\substack{k=1,\dots,N\\ m=1,\dots,N}} &= 1_{\{k,m\}}-\frac{1}{n(1+\widetilde{\delta}_m)^2}\mathbf{b}_m^{\mathsf{H}}\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{T}}\mathbf{b}_m
\end{aligned}$$

where $1_{\{k,m\}}=1$ for $k=m$ and zero otherwise.

As shown in the next section, Theorem 3 can be used to provide a tight approximation of the outage probability:

$$P_{\text{out}}(R)\triangleq\Pr(N\mathcal{I}(\rho)<R)\approx 1-Q\left(\frac{R-NV(\rho)}{\Theta_{N,n}}\right) \tag{2}$$

where $Q(x)$ is the Gaussian tail function.

Remark 1: One should notice the difference between the model of this paper, where the mutual information writes: $\mathcal{I}(\rho)=\frac{1}{N}\log\det\left(\mathbf{I}_N+\rho^{-1}(\mathbf{X}+\mathbf{A})(\mathbf{X}+\mathbf{A})^{\mathsf{H}}\right)$, where $\mathbf{X}$ is a centered random matrix with a variance profile and $\mathbf{A}$ is deterministic, and the model considered in [10]: $\mathcal{I}(\rho)=\frac{1}{N}\log\det\left(\mathbf{I}_N+\rho^{-1}\left(\mathbf{A}\mathbf{A}^{\mathsf{H}}+\mathbf{X}\mathbf{X}^{\mathsf{H}}\right)\right)$. Despite an apparently narrow difference between the two models, it turns out that the fluctuations of the former do not follow by far from those of the latter.

Outline of the proof of Theorem 3: In order to establish the CLT, one shall consider the following decomposition:

$$N(\mathcal{I}(\rho)-V(\rho))=N(\mathcal{I}(\rho)-\mathbb{E}\mathcal{I}(\rho))+N(\mathbb{E}\mathcal{I}(\rho)-V(\rho))$$

and treat each term of the right-hand side separately. Using Gaussian tools, such as the integration by part formula and Poincaré-Nash inequality (e.g. [9], [15]), one can prove that

$$N(\mathbb{E}\mathcal{I}(\rho)-V(\rho))\xrightarrow[N,n\to\infty]{}0\ .$$

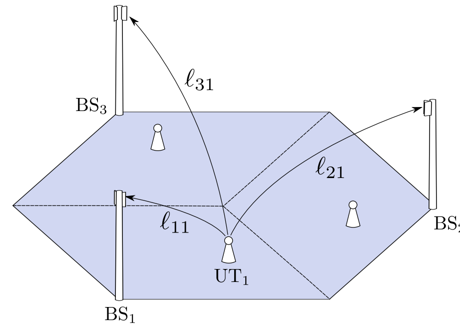

Fig. 2. Cellular example with $B=3$ BSs and $n=3$ UTs.

Now write

$$N(\mathcal{I}(\rho)-\mathbb{E}\mathcal{I}(\rho))\ =\ \sum_{i=1}^{n}(\mathbb{E}_i-\mathbb{E}_{i-1})N\mathcal{I}(\rho)\ \triangleq\ \sum_{i=1}^{n}\gamma_i$$

where $\mathbb{E}_i=\mathbb{E}(\cdot\mid\mathbf{h}_k;k\le i)$ is the conditional expectation with respect to the i first columns of matrix $\mathbf{H}$. Such a decomposition enables us to interpret $N(\mathcal{I}(\rho)-\mathbb{E}\mathcal{I}(\rho))$ as the sum of increments of a martingale. We can now rely on the CLT for the martingales, e.g. [16, Theorem 35.12]. According to this theorem, the crux of the proof is then to identify the deterministic equivalent of the quantity $\sum_{i=1}^{n}\mathbb{E}_{i+1}\gamma_i^2$:

$$\sum_{i=1}^{n}\mathbb{E}_{i+1}\gamma_i^2\approx\tilde{\Theta}_{N,n}^2 \tag{3}$$

(in the sense that the difference goes to zero in probability) and then to prove that this deterministic equivalent is an approximation of the variance:

$$\tilde{\Theta}_{N,n}^2-\Theta_{N,n}^2\xrightarrow[N,n\to\infty]{}0\ . \tag{4}$$

Once these facts are established, [16, Theorem 35.12] directly implies the required CLT. ■

This very streamlined outline of the proof must not hide the fact that (3) and (4) are very involved to obtain. It is, therefore, fortunate to have a general and straightforward formula to identify the variance (cf. (1)).

V. Numerical Results

We consider a cellular system consisting of $B=3$ BSs with $M=2$ antennas and $n=\{3,6,9\}$ UTs, as shown in Fig. 2. The UTs are uniformly distributed over the three cell sectors. The inverse path loss factor ℓ_{bk} between UT k and BS b is given as $\ell_{bk}=d_{bk}^{-3.6}$, where d_{bk} is the normalized distance between UT k and BS b. The Rician parameters κ_{bk} are drawn randomly from the unit interval while the phases ϕ_{mk}^{b} of the specular components are drawn randomly from the interval $[0,2\pi)$. The signal-to-noise-ratio is defined as $\text{SNR}=1/\rho$. We consider one random snapshot of user distributions and average over many different realizations of $\mathbf{H}$.

Fig. 3 shows the ergodic mutual information $I(\rho)$ versus SNR for $n=\{3,6,9\}$. The markers correspond to simulations while the solid lines correspond to the approximation by the deterministic equivalent $V(\rho)$. We observe an almost perfect

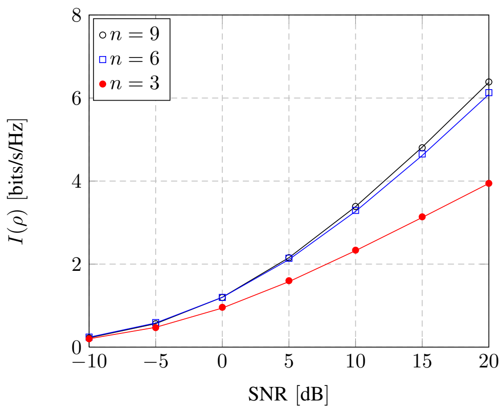

Fig. 3. Ergodic mutual information $I(\rho)$ versus SNR. Markers indicate simulation results, solid lines correspond to the deterministic equivalent approximation $V(\rho)$ as given by Theorem 1 (ii).

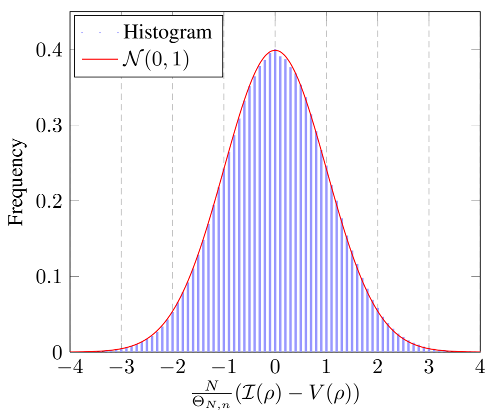

Fig. 4. Histogram of the random variable $\frac{N}{\Theta_{N,n}}(\mathcal{I}(\rho) - V(\rho))$ versus the Gaussian $\mathcal{N}(0,1)$ law. SNR $= 5\,$dB, $n = 9$.

overlap between both results. The validity of Theorem 3 is demonstrated in Fig. 4 which compares the histogram of the quantity $\frac{N}{\Theta_{N,n}}(\mathcal{I}(\rho) - V(\rho))$ against the normal law $\mathcal{N}(0,1)$. The fit between both results is surprisingly good for a rather small system size of 9 UTs. Finally, we present the performance of the approximation of the outage probability $P_{\text{out}}(R)$ as given by (2) in Fig. 5. We show the outage probability as a function of the SNR for a target rate of $R = n \times 3$ [nats/s/Hz]. The solid lines are calculated by Theorem 3 and (2), dashed lines corresponds to simulation results. The approximation is also very accurate for small system dimensions.

VI. Conclusions

We have derived a CLT of the fluctuations of the mutual information of a class of Rician fading MIMO channels and verified its accuracy for small system dimensions by simulations. The result is useful for the study of cooperative small-cell systems and can be used to provide a close approximation of the outage probability. The structural insight about the asymptotic variance of the mutual information provided in this work might be also helpful to derive similar results for other channel models.

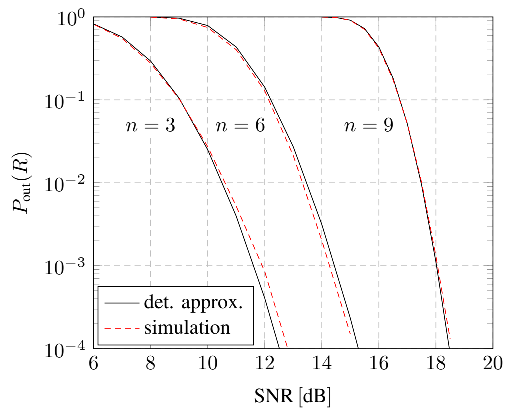

Fig. 5. Outage probability $P_{\text{out}}(R)$ versus SNR for $n = \{3, 6, 9\}$ and target rate $R = n \times 3$ [nats/s/Hz].

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