Non-Parametric Signal Detection and RMT

Boaz Nadler

Department of Computer Science and Applied Mathematics
The Weizmann Institute of Science

Joint work with Shira Kritchman
and with Federico Penna, Roberto Garello.

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Talk Outline

1. Signal Detection Problems
2. Connection to Random Matrix Theory
3. Implications

Detection of signals embedded in noise

Given a measurement system with p sensors (antennas / microphones / hyperspectral camera / etc)

Observe multivariate samples $\mathbf{x}(t_j) \in \mathbb{R}^p$ of the form

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \sigma\xi(t) \qquad (*)$$

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where

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- $\mathbf{A}$ is a $p \times K$ *fixed* unknown mixing matrix of rank K (steering matrix).

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- σ - noise level, $\xi \sim \mathcal{N}(0, \mathbf{I}_p)$.

Problem Formulation

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \sigma\xi(t) \qquad (*)$$

Assume $\mathbf{s}(t) \in \mathbb{R}^K$ is stationary random process with a full rank covariance matrix, and that mixing matrix $\mathbf{A}$ is of rank K (e.g., there are indeed K identifiable sources).

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Problem Formulation

Given n i.i.d. samples $\mathbf{x}_i$ from the model (*), estimate the number of sources K.

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- Analytical Chemistry / Chemometrics: **x** is the measured (logarithm of) spectra at p wavelengths, **s** - vector of concentrations of K chemical components. Eq. $(*)$ follows from Beer-Lambert's law.
- Signal Processing: **s** is a vector of K emitting sources, **x** - measurement at an array of p receivers (microphones, antennas, etc).

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- Analytical Chemistry / Chemometrics: $\mathbf{x}$ is the measured (logarithm of) spectra at p wavelengths, $\mathbf{s}$ - vector of concentrations of K chemical components. Eq. $(*)$ follows from Beer-Lambert's law.
- Signal Processing: $\mathbf{s}$ is a vector of K emitting sources, $\mathbf{x}$ - measurement at an array of p receivers (microphones, antennas, etc).
- Statistical Modeling at large : linear mixture models / factor models / error in variables / two-way table with multiplicative interactions.

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In signal processing -

a preliminary step before blind source separation / independent component analysis, direction of arrival estimation, many other parametric procedures whose number of parameters depends on K - the number of sources.

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In chemometrics -

process control, optimal number of latent variables in regression/calibration models, first step prior to self modeling curve resolution and many other estimation procedures.

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In this setting, assuming Gaussian signals, eigenvalues $\ell_1 \geq \ell_2 \geq \ldots \geq \ell_p$ of the sample covariance matrix

$$\mathbf{S}_n = \sum_i \mathbf{x}_i' \mathbf{x}_i$$

are *sufficient statistics* for eigenvalues of the population covariance matrix $\mathbf{\Sigma}$ [James 66', Muirhead 78'].

Detection of Signals as a model selection problem

Given K sources, the population covariance matrix of the observations has diagonal form

$$\mathbf{W}'\mathbf{\Sigma}\mathbf{W} = \sigma^2\mathbf{I}_p + diag(\lambda_1, \lambda_2, \ldots, \lambda_K, 0, \ldots, 0) \qquad (1)$$

Model Selection Problem: Given $\{\ell_j\}_{j=1}^{p}$, determine which model of the form (1) is most likely.

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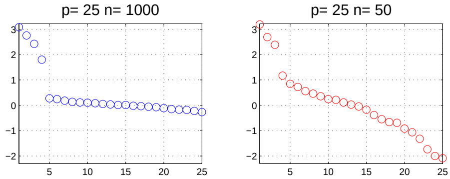


Eigenvalue Based Rank Estimation

Challenge: Distinguish between large yet insignificant noise eigenvalues and small yet significant signal eigenvalues.

Problem Parameters: n - number of samples, p - dimensionality, σ - noise level.

In classical array processing $p/n \ll 1$,

In chemometrics and in some modern radar/sonar systems: $p/n = O(1)$ and often $p/n \gg 1$.

Previous Approaches

Can be divided into 2 main disciplines,

- Nested Hypothesis Tests (with various test statistics)
- Information Theoretic Criteria (BIC, MDL, AIC, etc)

Previous Approaches

The "godfathers"

- Bartlett (1940's), Lawley (1950's) - likelihood ratio tests, tests for sphericity, assume Gaussian observations, asymptotic expansions for p fixed, $n \to \infty$.

$$T = \frac{\left(\prod \ell_i\right)^{1/p}}{\frac{1}{p}\sum \ell_i}$$

This statistic does not work well when p is of the same order of n and is undefined if $p > n$.

Previous Approaches

In chemometrics ($p \gg n$):

- Malinowski F-test, 1977, 1980, 1987-1990, *Analytical Chemistry, J. Chemometrics*.
- **Faber and Kowalski, Modification of Malinowski's F-test, J. Chemometrics, 1997.**
- Faber, Buydens and Kateman, Aspects of pseudorank estimation methods based on the eigenvalues of principal component analysis of random matrices, 1994.
- at least 15 other papers describing and comparing various algorithms.

Previous Approaches

In signal processing and statistics literature:

- Wax & Kailath, *Detection of signals by information theoretic criteria*, 85'
- Zhao, Krishnaiah and Bai. JMVA, 1986
- Fishler, Grosmann and Messer, IEEE Sig. Proc. 2002.
- P-J. Chung, J.F. Böhme, C.F. Mecklenbraüker and A.O. Hero, 2007.
- many other papers in signal processing and in statistics.
- Silverstein & Combettes, 1992
- Schott, *A high-dimensional test for the equality of the smallest eigenvalues of a covariance matrix*, JMVA, 2006.
- Rao & Edelman, *Sample eigenvalue based detection of high-dimensional signals in white noise using relatively few samples*. 2007.

Detection of Signals by Information Theoretic Criteria

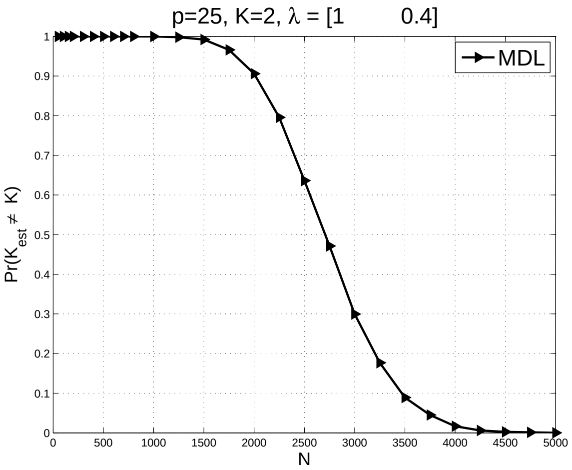
$$\hat{k}_{MDL} = \arg\min \quad -(p-k)n\left[\frac{1}{p-k}\log\left(\prod_{k+1}^{p}\ell_j\right) - \log\frac{\sum_{k+1}^{p}\ell_j}{p-k}\right] + \frac{1}{2}k(2p-k)\log n$$

$$\hat{k}_{AIC} = \arg\min \quad -(p-k)n\left[\frac{1}{p-k}\log\left(\prod_{k+1}^{p}\ell_j\right) - \log\frac{\sum_{k+1}^{p}\ell_j}{p-k}\right] + k(2p-k)$$

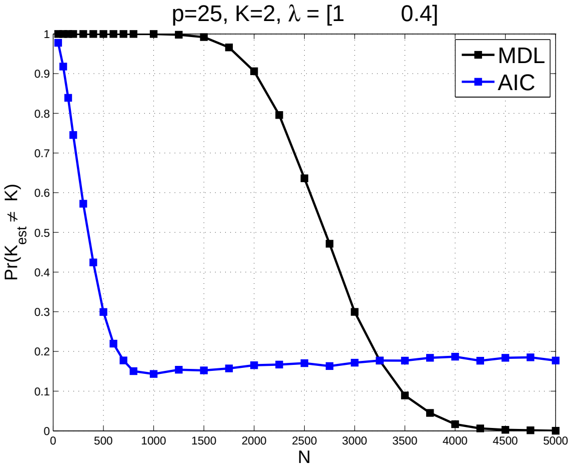
MDL estimator became the standard tool in signal processing.

WAX & KAILATH 1985.

Example: MDL consistent



Example: AIC not consistent



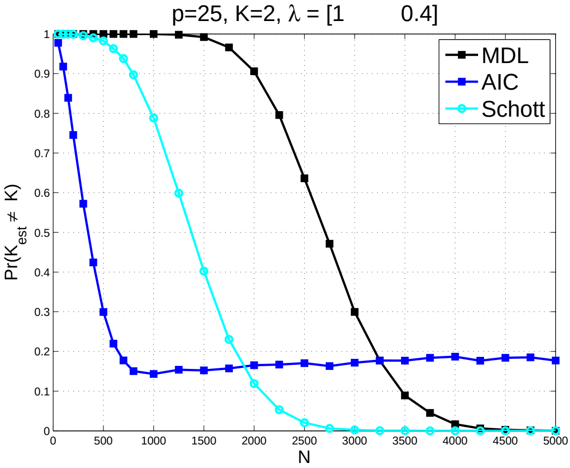
Some Modern Approaches:

Rao & Edelman (2007) and Schott (2006)

$$T = \frac{\sum \ell_j^2}{(\sum \ell_j)^2}$$

This statistic can be used for sphericity test for all values of p/n (Ledoit & Wolf).

Example:


p=25, K=2, λ = [1 0.4]
MDL
AIC
Schott
Pr(K_est ≠ K)
N
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
500
1000
1500
2000
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Detection of Signals - Theoretical Questions

- Which Test Statistic to use ?
- Why does AIC overestimate number of signals ?
- Detection Performance ?
- Known vs. unknown noise level
- Non-parametric vs fully parametric setting.

KEY TOOLS

- Behavior of noise eigenvalues.

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TOOLS:

- RMT

- Matrix Perturbation Theory

Which Test Statistic ?

Consider the case of two (nearly) *simple* hypothesis

$$\mathcal{H}_0 : \mathbf{\Sigma} = \mathbf{I} \quad vs. \quad \mathcal{H}_1 : \mathbf{W}'\mathbf{\Sigma}\mathbf{W} = \mathbf{I} + diag(\lambda, 0, \ldots, 0)$$

with λ - known. What is unknown is the basis which makes $\mathbf{\Sigma}$ diagonal in $\mathcal{H}_1$.
Neyman-Pearson: optimal method is likelihood ratio test

$$\frac{p(\ell_1, \ldots, \ell_p | \mathcal{H}_1)}{p(\ell_1, \ldots, \ell_p | \mathcal{H}_0)} \gtrless C(\alpha)$$

Which Test Statistic ?

From multivariate analysis (Muirhead 78')

$$p(\ell_1, \ldots, \ell_p | \Sigma) = C_{n,p} \prod \ell_i^{(n-p-1)/2} \prod_{i<j} (\ell_i - \ell_j) \; {}_0F_0(-\frac{1}{2} nL, \Sigma^{-1})$$

${}_0F_0$ - hypergeometric function with matrix argument.
Key point: asymptotically in n, for p fixed,

$$\log \left(\frac{p(\ell_1, \ldots, \ell_p | \mathcal{H}_1)}{p(\ell_1, \ldots, \ell_p | \mathcal{H}_0)} \right) \approx n(\ell_1 - h(\lambda)) + O(\sum c_{1j}/(\ell_1 - \ell_j))$$

Asymptotically, should only look at *largest* eigenvalue !

Roy's Largest Eigenvalue Test

If $\ell_1 > \sigma^2 \cdot th(\alpha)$ accept $\mathcal{H}_1$ – signal is present.

$th(\alpha)$ - found by distribution of ℓ_1 under the null of no signals. Then, $S_n = 1/n \sum_i \mathbf{x}_i \mathbf{x}_i'$ is a scaled Wishart matrix.

Theorem: [Johansson 00', Johnstone 01',El-Karoui 07'] As $p, n \to \infty$

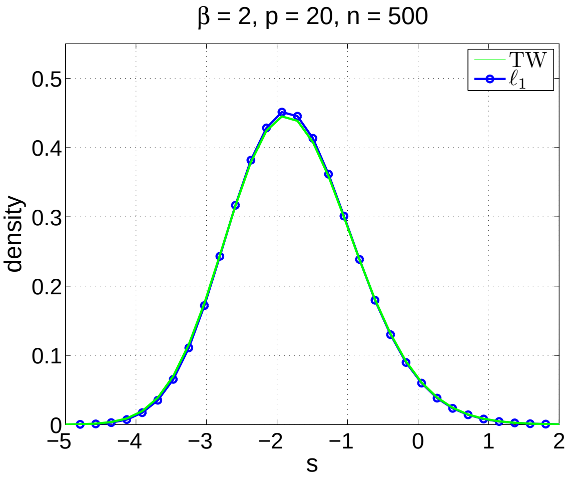
$$\boxed{\Pr\{\ell_1 < \sigma^2 (\mu_{n,p} + s\sigma_{n,p})\} \to TW_\beta(s)}$$

where TW_β - Tracy-Widom distribution of order β

$\beta = 1$ - real valued noise, $\beta = 2$ - complex valued noise.

For any confidence level α can invert TW distribution to obtain threshold $s(\alpha)$.

Largest Eigenvalue Distribution



Unknown Noise Level

Standard approach: replace σ^2 by its ML estimate

$$\hat{\sigma}^2 = \frac{1}{p}\sum_{j=1}^{p} \ell_j$$

GLRT:

$$U = \frac{\ell_1}{\frac{1}{p}\sum_j \ell_j} > th(\alpha)$$

This test statistic plays a role in:

- Signal Detection [Besson & Scharf 06', Kritchman & N. 08, Bianchi et al. 09']
- Two-way models of interaction [Johnson & Graybill, 72']
- Models for Quantum Information Channels.

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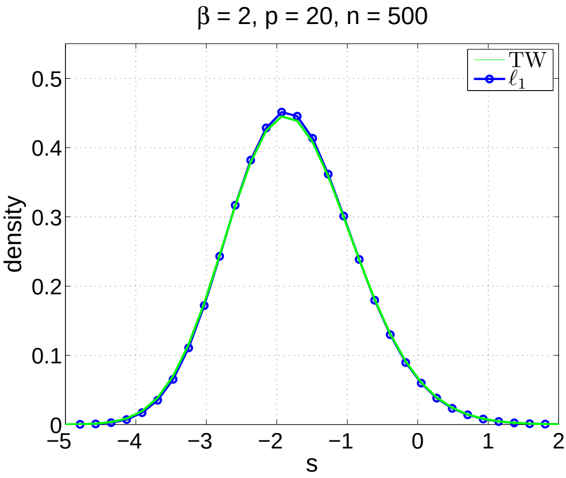
Unknown Noise Level

In principle, can use same threshold $th(\alpha)$, since:

Theorem:[Bianchi et. al.] As $p, n \to \infty$

$$\Pr[U < \mu_{n,p} + s\sigma_{n,p}] \to TW_\beta(s)$$

Ratio Distribution



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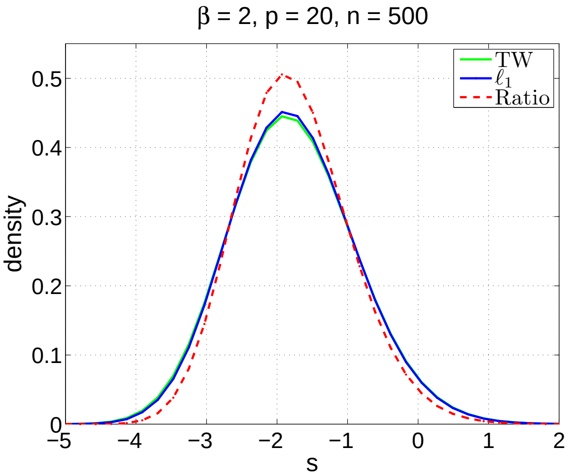

Ratio Distribution

Definition:

$$F_{np}(s) = \Pr\left[\frac{\ell_1 - \mu_{np}}{\sigma_{np}} < s\right], \quad H_{np}(s) = \Pr\left[\frac{U - \mu_{np}}{\sigma_{np}} < s\right].$$

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Key property: U and $T = \frac{1}{p}\sum_j \ell_j$ are independent.

Ratio Distribution

Since $\ell_1 = U \cdot T$,

$$\Pr[\ell_1 < x] = \int_0^{px} \Pr\left[U < \frac{x}{t}\right] p_T(t)dt$$

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L.H.S. $=$ approximately TW
R.H.S. $=$ convolution of required function with χ^2 density.

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Ratio Distribution

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(i) uniformly in p and s, $H_{np}(s)$ is a smooth function with bounded third derivative, $|H'''_{np}(s)| < C$.

(ii)

$$|F'_{np}(s) - TW'_{\beta}(s)| \to 0 \qquad \text{and} \qquad |F''_{np}(s) - TW''_{\beta}(s)| \to 0 \quad (2)$$

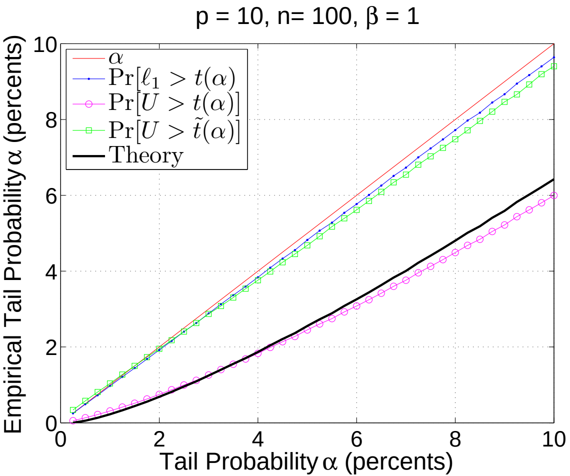
Ratio Distribution

Theorem: In the joint limit $p, n \to \infty$,

$$
\begin{aligned}
H_{np}(s) - TW_\beta(s) &= [F_{np}(s) - TW_\beta(s)] - \\
&\quad \frac{1}{2}\left(\frac{2}{\beta np}\right)\left(\frac{\mu_{np}}{\sigma_{np}}\right)^2 TW''_\beta(s) + o(p^{-2/3}).
\end{aligned}
$$

[N., to appear in JMVA]

Ratio Distribution Tail Probabilities



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$$1 - TW_2(s) \approx 5\%$$

Then

$$\frac{|TW_2''(s)|}{1 - TW_2(s)} \approx 7 \quad \text{and} \quad \frac{1}{np}\left(\frac{\mu_{np}}{\sigma_{np}}\right)^2 \approx 1/p^{2/3}.$$

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Hence, for a 10% relative error,

$$\frac{1}{2}\frac{2}{\beta np}\left(\frac{\mu_{np}}{\sigma_{np}}\right)^2 \frac{|TW_2''(s)|}{1 - TW_2(s)} \leq 0.1$$

we need $p \gtrsim (35)^{3/2} \approx 200$.

Detection of Signals - Theoretical Questions

- Which Test Statistic to use ?
- **Why does AIC overestimate number of signals**
- Detection Performance ?
- Known vs. unknown noise level
- Non-parametric vs fully parametric setting.

Overestimation Probability of AIC (no signals)

AIC overestimates number of signals when

$$-\ln \mathcal{L}_0 > -\ln \mathcal{L}_1 + \frac{2p-1}{n}$$

where

$$\mathcal{L}_k = \frac{\prod_{i>k} \ell_i}{\left(\frac{\sum_{i>k} \ell_i}{p-k}\right)}$$

Overestimation Probability of AIC

Let ξ be solution of

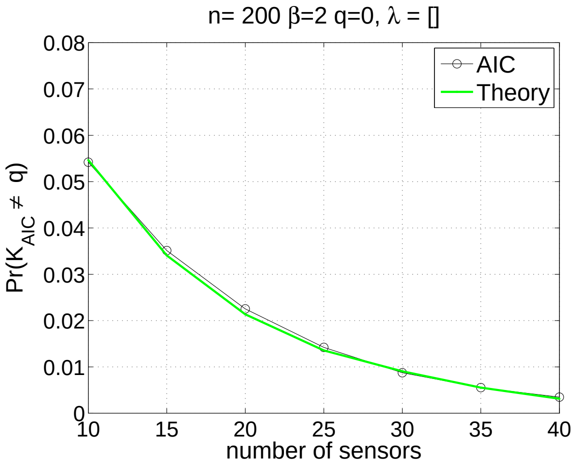
$$-\ln(1+\xi) - (p-1)\ln(1 - \frac{\xi}{p-1}) = \frac{2p-1}{n}$$

Lemma:

$$\Pr[k_{\text{AIC}} > 0] = \Pr[U > 1 + \xi] + O\left(\frac{1}{n}\right)$$

[N., IEEE-TSP 10']

AIC Overestimation Probability



AIC Overestimation Probability

Since

$$\xi = 2\sqrt{\frac{p-1/2}{n}}\sqrt{1-\frac{1}{p}}\left(1+O\left(\frac{1}{n^{1/2}}\right)\right)$$

AIC penalty is not sufficiently strong.

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Legacy System:

$$\hat{k}_{AIC} = \arg\min \qquad -\ln \mathcal{L}_k + \frac{2Ck}{n}\left(p+1-\frac{k+1}{2}\right)$$

If $C = 1$ penalty comparable to original AIC.

Modified AIC

Theorem: Modified AIC estimator with $C = 2$ has negligible overestimation probability, which for large n is exponential small in p.

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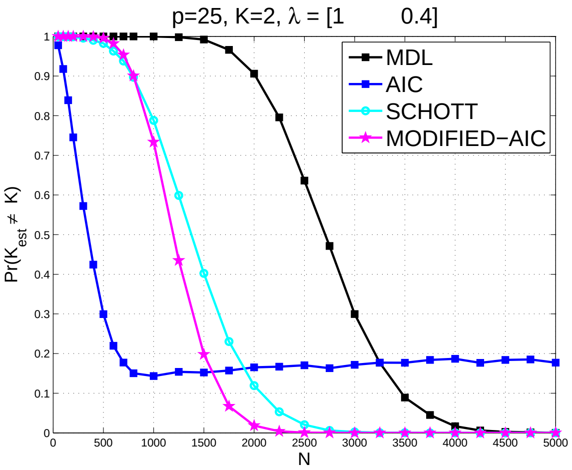
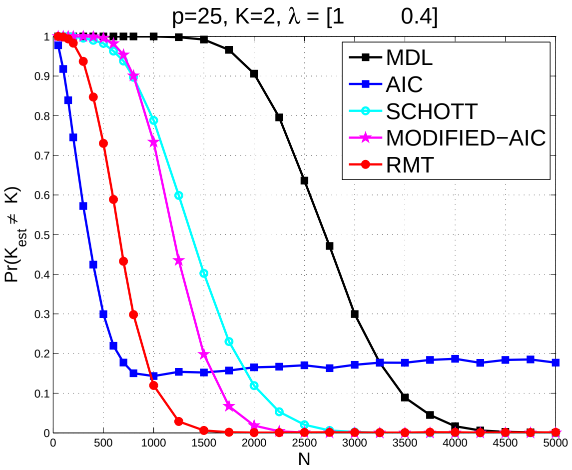
Basic idea: use non-asymptotic bound of Ledoux, valid for all p, n (complex)

$$\Pr[\ell_1 > (1 + \sqrt{p/n})^2 + \epsilon] \leq e^{-nJ_{LAG}(\epsilon)}$$

Modified AIC



Example:


p=25, K=2, λ = [1 0.4]
MDL
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$Pr(K_{est} \neq K)$
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- **Detection Performance ?**
- Known vs. unknown noise level
- Non-parametric vs fully parametric setting.

Detection Performance

Depends on behavior of largest signal eigenvalue in presence of noise. With p fixed, $n \to \infty$,

$$P_d^{Roy} \approx Q\left[\sqrt{n}\left(\frac{th(\alpha)}{1+\lambda/\sigma^2} - \frac{p-1}{N\lambda/\sigma^2} - 1\right)\right]$$

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$$P_d^{GLRT} \approx Q\left[\sqrt{n}\left(\tilde{th}(\alpha)(\frac{1}{1+\lambda/\sigma^2} - \frac{1}{n\lambda/\sigma^2}) - \frac{p-1}{N\lambda/\sigma^2} - 1\right)\right]$$

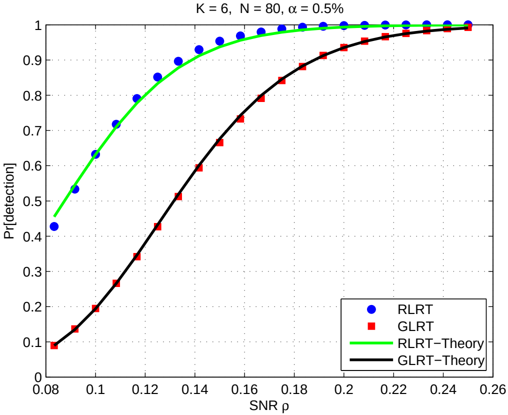
where $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-x^2/2} dx$,

$$\tilde{th}(\alpha) = \frac{p-1}{p - th_U(\alpha)} th_U(\alpha) > th(\alpha)$$

[Kritchman & N. 09']
[N, Penna, Garello, submitted]

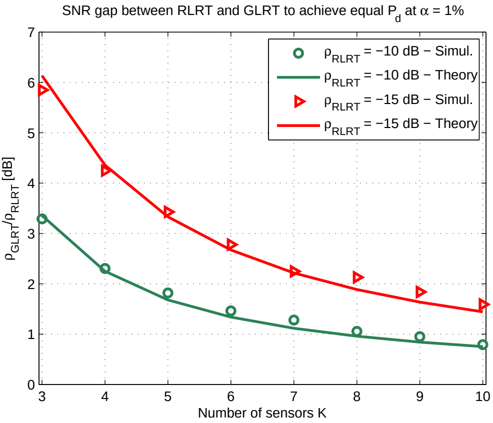
Known vs. Unknown Noise Level

Difference can be large:



Known vs. Unknown Noise Level

Difference can be large (several dB)



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Non-Parametric vs. Parametric Detection

In signal array processing, the array manifold is typically known.
For Uniform Linear Array

$$\mathbf{a}(\theta) = \left[1 e^{i\pi \sin\theta} e^{2i\pi \sin(\theta)} \ldots e^{(p-1)i\pi \sin\theta} \right]$$

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Instead of largest eigenvalue of sample covariance matrix, maximal correlation of $\mathbf{x}$ with $\mathbf{a}(\theta)$

Non-Parametric vs. Parametric Detection

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Instead of largest eigenvalue of sample covariance matrix, maximal correlation of $\mathbf{x}$ with $\mathbf{a}(\theta)$

Theorem: Parametric Signal Detection requires

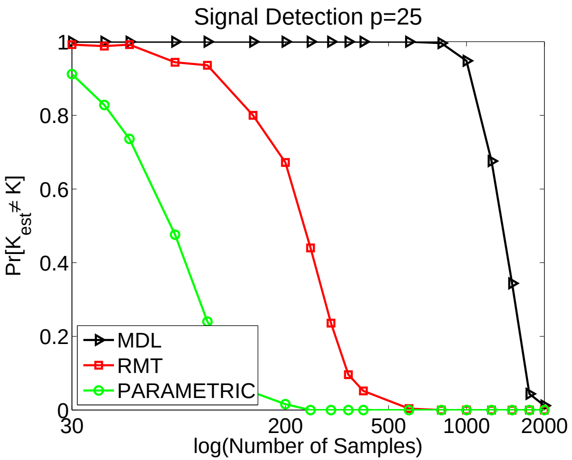
$$SNR \gg \sqrt{\frac{2 \ln p}{n}}$$

instead of

$$SNR \gg \sqrt{\frac{p}{n}}$$

[Arkind & N., 10']

Example:


Signal Detection p=25
Pr[K_est ≠ K]
log(Number of Samples)
0
0.2
0.4
0.6
0.8
1
30
200
500
1000
2000
MDL
RMT
PARAMETRIC

Summary and Open Questions

Open Questions:

- Detection in unknown noise environments
- Non-Gaussian signals
- Sparse covariance matrices

http://www.wisdom.weizmann.ac.il/∼nadler/

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C'est Tou / Merci Beaucoup