Applications of Random Matrices to Small Cell Networks

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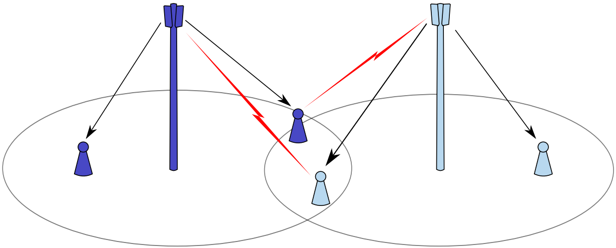
Outline

1. What are Small Cell Networks (SCNs)?
2. A general channel model for SCNs
3. Applications of RMT for the performance analysis of SCNs

"Globally, mobile data traffic will double every year through 2014, increasing 39 times between 2009 and 2014."

(Cisco, 2009)

How to increase the capacity of current cellular networks?



Options

More spectrum	$\rightarrow$	hardly available
New modulation/coding schemes	$\rightarrow$	not to be expected
Interference avoidance/cancellation	$\rightarrow$	less interference, less spectral efficiency
Cooperation (network MIMO)	$\rightarrow$	less interference, benefits only at cell edge
Cognitive radio	$\rightarrow$	exploit spectrum holes, does it work?

Will this be enough?

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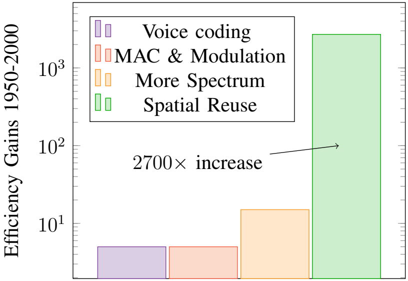
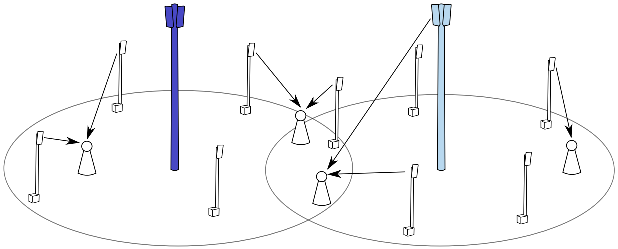


Source: ArrayComm & William Webb (Ofcom, 2007)

- Network densification seems only option to carry the forecasted traffic
- Increasing the macro cell density does not scale: too expensive to plan/deploy (CAPEX) and to maintain (OPEX)

Radical network design change required.

Small Cell Networks



Dense deployment of low-cost low-power BSs as additional capacity layer

Advantages

Collocate with existing street furniture	$\rightarrow$	no cell site acquisition
Use existing backhaul infrastructure	$\rightarrow$	reduced deployment cost
Self-organizing/optimization	$\rightarrow$	no planning or maintenance needed

SCNs can provide high network capacity *and* reduce CAPEX & OPEX.

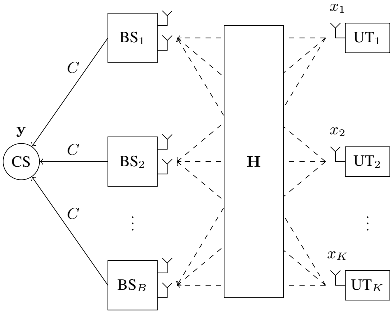
Challenges

- Line-of-sight channels to several base stations
- User mobility requires base station cooperation
- Path loss
- Imperfect channel knowledge
- Limited backhaul capacity

New tools for the performance analysis of SCNs are required.

The Role of RMT for Small Cell Networks

A general uplink channel model



$N \times K$ MIMO Channel:

$$\boxed{\mathbf{y} = \sqrt{\rho}\,\mathbf{H}\,\mathbf{x} + \mathbf{z}}$$

Gaussian Signaling

- $\mathbf{x} \sim \mathcal{CN}(0, \mathbf{I}_K)$
- $\rho \geq 0$

Rician fading channel with a variance profile: $\mathbf{H} = \mathbf{W} + \mathbf{A}$

- $[\mathbf{W}]_{ij} \sim \mathcal{CN}(0, \frac{\sigma_{ij}^2}{K})$
- $\sigma_{ij}^2 \geq 0$, variance profile $\rightarrow$ proportional to path loss
- $\mathbf{A}$, deterministic $\rightarrow$ LOS components

Noise, quantization error and correlated interference

- $\mathbf{Z} \triangleq \mathbb{E}\left[\mathbf{z}\mathbf{z}^{\mathsf{H}}\right] = \underbrace{\mathbf{I}_N}_{\text{thermal noise}} + \underbrace{\mathbf{Q}(C)}_{\text{quantization errors}} + \underbrace{\mathbf{\Delta}}_{\text{correlated interference}}, \quad \mathbf{z} \sim \mathcal{CN}(0, \mathbf{Z})$

Why RMT for small cells?

Mutual Information

$$\mathcal{I}(\rho) = \frac{1}{N} \log\det\left(\mathbf{I}_N + \rho \mathbf{Z}^{-\frac{1}{2}} \mathbf{H}\mathbf{H}^{\mathsf{H}} \mathbf{Z}^{-\frac{1}{2}}\right)$$

Ergodic Mutual Information

$$I(\rho) = \mathbb{E}_{\mathbf{H}}\left[\mathcal{I}(\rho)\right]$$

Outage Probability

$$P_{\text{out}} = \Pr\left[N\mathcal{I}(\rho) \leq R\right]$$

SINR at the output of the MMSE detector

$$\gamma_k = \mathbf{h}_k^{\mathsf{H}} \left(\frac{1}{\rho}\mathbf{Z} + \mathbf{H}_{[k]}\mathbf{H}_{[k]}^{\mathsf{H}}\right)^{-1} \mathbf{h}_k$$

RMT can provide close approximations of all these quantities.

Applications

Approximation of the ergodic mutual information

$$
\begin{aligned}
I(\rho) &= \mathbb{E}\left[\frac{1}{N}\log\det\left(\mathbf{I}_N + \rho\mathbf{Z}^{-\frac{1}{2}}\mathbf{H}\mathbf{H}^{\mathsf{H}}\mathbf{Z}^{-\frac{1}{2}}\right)\right] \\
&= \mathbb{E}\left[\frac{1}{N}\log\det\left(\mathbf{I}_N + \mathbf{Q}(C) + \boldsymbol{\Delta} + \rho\mathbf{H}\mathbf{H}^{\mathsf{H}}\right)\right] - \frac{1}{N}\log\det\mathbf{Z}
\end{aligned}
$$

Note that

$$
\mathbf{Q}(C) + \boldsymbol{\Delta} + \rho\mathbf{H}\mathbf{H}^{\mathsf{H}} = (\boldsymbol{\Gamma} + \boldsymbol{\Phi})(\boldsymbol{\Gamma} + \boldsymbol{\Phi})^{\mathsf{H}}
$$

where

$$
\begin{aligned}
\boldsymbol{\Gamma} &= [\sqrt{\rho}\mathbf{W}\ \mathbf{0}_{N\times N}] \\
\boldsymbol{\Phi} &= \left[\sqrt{\rho}\mathbf{A}\ \ (\mathbf{Q}(C) + \boldsymbol{\Delta})^{\frac{1}{2}}\right]\ .
\end{aligned}
$$

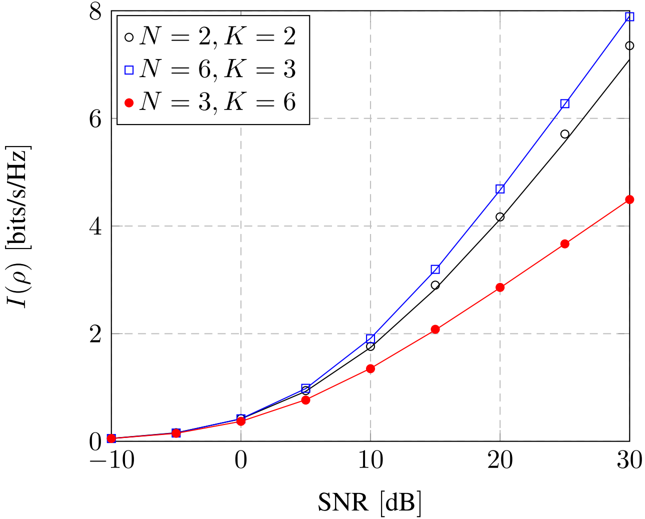
This is a Rician fading channel with a variance profile.

Theorem 4.1 (Hachem, Loubaton, Najim: AAP'2007)

Under some mild technical assumptions, the following limit holds true:

$$
I(\rho) - \bar{I}(\rho) \xrightarrow[N,K\to\infty]{} 0\ .
$$

Approximation of the ergodic mutual information: Numerical results



Approximation of the outage probability

Claim 1 (Hoydis, Kammoun, Najim, Debbah: ICC'11)

Let $\mathbf{Z} = \mathbf{I}_N$. Under some mild technical assumptions, the mutual information $\mathcal{I}(\rho)$ satisfies

$$\frac{N}{\Theta_{N,K}} \left(\mathcal{I}(\rho) - \bar{I}(\rho)\right) \xrightarrow[N,K\to\infty]{\mathcal{D}} \mathcal{N}(0,1)$$

where

$$\Theta_{N,K}^2 = -\log\det(\mathbf{J}_{N,K})\ .$$

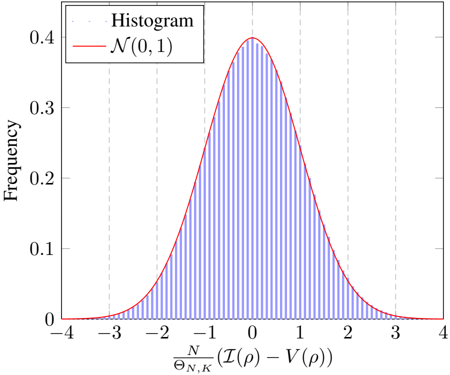
Remark

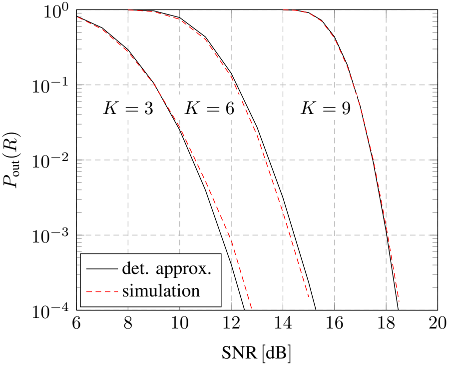
$\mathbf{J}_{N,K}$ is the Jacobian matrix of the fundamental equations of the random matrix model.

This result can be used to approximate the outage probability:

$$\boxed{P_{\text{out}}(R) \triangleq \Pr(N\mathcal{I}(\rho) < R) \approx 1 - Q\left(\frac{R - N\bar{I}(\rho)}{\Theta_{N,K}}\right)}$$

Approximation of the outage probability: Numerical Results





$$\boxed{N = 6, \quad K = 9}$$

Performance of linear receivers with imperfect CSI

Partial channel knowledge

$$\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}$$

- $\hat{\mathbf{H}}_{ij} \sim \mathcal{CN}(0, \frac{\hat{\sigma}_{ij}^2}{K})$, $\tilde{\mathbf{H}}_{ij} \sim \mathcal{CN}(0, \frac{\tilde{\sigma}_{ij}^2}{K})$
- $\hat{\mathbf{H}}_{ij}$ and $\tilde{\mathbf{H}}_{ij}$ mutually independent

SINR at the MMSE detector

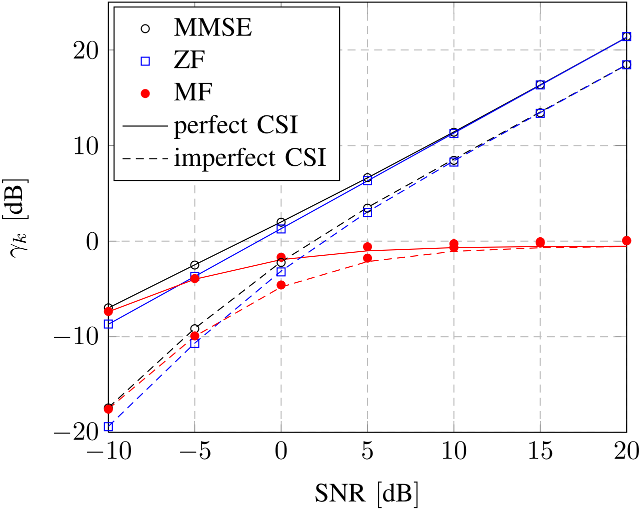
$$\hat{\gamma}_k = \frac{\left(\hat{\mathbf{h}}_k^{\mathsf{H}} \left(\hat{\mathbf{H}}_{[k]}\hat{\mathbf{H}}_{[k]}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-1} \hat{\mathbf{h}}_k\right)^2}{\left|\hat{\mathbf{h}}_k^{\mathsf{H}} \left(\hat{\mathbf{H}}_{[k]}\hat{\mathbf{H}}_{[k]}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-1} \tilde{\mathbf{h}}_k\right|^2 + \hat{\mathbf{h}}_k^{\mathsf{H}} \left(\hat{\mathbf{H}}_{[k]}\hat{\mathbf{H}}_{[k]}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-1} \left(\mathbf{H}_{[k]}\mathbf{H}_{[k]}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right) \left(\hat{\mathbf{H}}_{[k]}\hat{\mathbf{H}}_{[k]}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-1} \hat{\mathbf{h}}_k}$$

Corollary 1 (Hoydis, Kobayashi, Debbah: Asilomar'10)

Under some mild assumptions, the following limit holds true:

$$\hat{\gamma}_k - \overline{\gamma}_k \xrightarrow[N,K\to\infty]{\text{a.s.}} 0 \ .$$

Performance of linear receivers with imperfect CSI: Numerical results



$$\boxed{N = 12, \quad K = 16}$$

Optimal channel training

Partial channel knowledge through channel training

The channel **H** remains constant during T channel uses of which τ are used for channel estimation:

- $\hat{\sigma}_{ij}^2(\tau)$, concave increasing
- $\tilde{\sigma}_{ij}^2(\tau)$, convex decreasing

Goal: Maximization of the net ergodic achievable rate

$$\tau^* = \arg\max_{\tau} \left(1 - \frac{\tau}{T}\right) \mathbb{E}\left[\frac{1}{N} \log\det\left(\mathbf{I}_N + \rho \mathbf{Z}^{-\frac{1}{2}}(\tau)\hat{\mathbf{H}}\hat{\mathbf{H}}^{\mathsf{H}}\mathbf{Z}^{-\frac{1}{2}}(\tau)\right)\right]$$

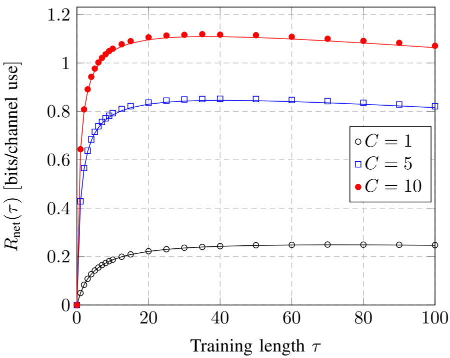
Solution: Maximize the deterministic equivalent approximation instead

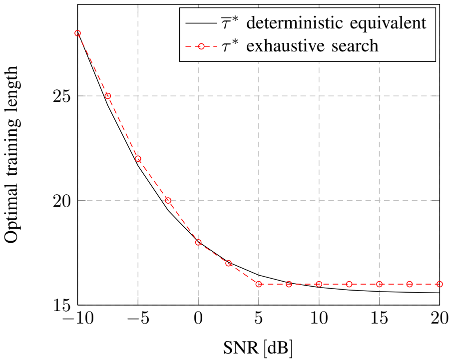
$$\overline{\tau}^* = \arg\max_{\tau} \left(1 - \frac{\tau}{T}\right) \hat{\overline{I}}(\rho, \tau)$$

Theorem 4 (Hoydis, Kobayashi, Debbah: IEEE Trans. SP'10)

$$\tau^* - \overline{\tau}^* \xrightarrow[N,K\to\infty]{} 0 \, .$$

Optimal channel training: Numerical Results





$$\boxed{N = 6, \quad K = 3}$$

Current work

Polynomial expansion detectors

- Closed-form expression of the asymptotic moments of the matrix $\mathbf{H}\mathbf{H}^{\mathsf{H}}$
- These results can be used to compute an approximation of the matrix

$$\left(\mathbf{H}\mathbf{H}^{\mathsf{H}} + \frac{1}{\rho}\mathbf{I}_N\right)^{-1} \approx \sum_{l=0}^{L} \lambda_l \left(\mathbf{H}\mathbf{H}^{\mathsf{H}}\right)^l$$

where the λ_l are only related to the asymptotic moments and can be precomputed.

Distributed Downlink Beamforming (Lakshminarayana, Hoydis, Debbah, Assad: PIMRC'10)

- Optimal downlink beamforming vectors can be computed by a distributed algorithm which requires the exchange of full CSI (Dahrouj, Yu: CISS'08).
- Proposed algorithm requires only exchange of statistical CSI.
- Significant reduction of message exchange over the backhaul network.

Conclusions

- Small cells are a promising network architecture to provide high capacity coverage.
- Smaller cell sizes pose many new challenges:
  - LOS channels
  - How to deal with imperfect CSI?
  - Cooperation of BSs necessary to handle user mobility
  - Limited backhaul capacity
- Asymptotic results of information-theoretic quantities for involved matrix models
- Close approximations for realistic system dimensions
- Simplify optimization problems
- Develop distributed algorithms

Thank you !