WORKSHOP PROGRAM & ABSTRACTS

Program

Monday, october 11th.

9 :30	Registration
9 :45-10 :15	Workshop opening speech, by Eric Moulines
10 :15-11 :00	Alan Edelman
11 :00-11 :30	Coffee break
11 :30-12 :15	Djalil Chafaï
12 :15-13 :00	Olivier Ledoit
13 :00-14 :30	Lunch break
14 :30-15 :15	Oleksiy Khorunzhiy
15 :15-16 :00	Florent Benaych
16 :00-16 :30	Coffee break
16 :30-17 :15	Øyvind Ryan
17 :15-18 :00	Philippe Loubaton

Tuesday, october 12th.

9 :30-10 :15	Folkmar Bornemann
10 :15-11 :00	Romain Couillet
11 :00-11 :30	Coffee break
11 :30-12 :15	Mylène Maïda
12 :15-13 :00	Malika Kharouf
13 :00-14 :30	Lunch break
14 :30-15 :00	Camille Mâle
15 :00-15 :30	Lu Wei
15 :30-16 :00	Pascal Vallet
16 :00-16 :30	Coffee break
16 :30-17 :15	Alain Rouault
17 :15-18 :00	Charles Bordenave

Wednesday, october 13th.

9 :30-10 :15	Dietrich von Rosen
10 :15-11 :00	Catherine Donati-Martin
11 :00-11 :30	Coffee break
11 :30-12 :15	Boaz Nadler
12 :15-13 :00	Jian-Feng Yao
13 :00-14 :30	Lunch break
14 :30-15 :00	Jakob Hoydis
15 :00-15 :30	Jean-Paul Ibrahim
15 :30-16 :00	Francisco Rubio
16 :00-16 :30	Coffee break
16 :30-17 :15	Jack Silverstein

Abstracts

Florent Benaych-George (Université Paris 6).

Title. Finite rank perturbations of random matrices and free probability theory.

Abstract. Let us consider a random Hermitian matrix X which empirical eigenvalue distribution tends to a limit distribution as the dimension tends to infinity and such that the extreme eigenvalues tend to the bounds of the support of the limit distribution (it is for example the case when the X is a Wigner matrix : in this case, the limit distribution is the semi-cricle law). We shall add a perturbation to X, and thus consider X+P, under the hypothesis is that the rank of P stays bounded as the dimension tends to infinity and that the eigenspaces of X and P are in generic position one to each other (it is for example the case when X is distributed according to the GUE).

Then, a natural question arises : how are the eigenvalues and the eigenvectors of X perturbed by the addition of P ?

This question had first been asked, for a quite close model, by Johnstone, and been solved, in a several particular cases, by Baik, Ben Arous, Péché, Féral, Capitaine and Donati-Martin.

We shall give a general answer, uncovering a remarkable phase transition phenomenon : the limit of the extreme eigenvalues of the perturbed matrix differs from the original matrix if and only if the eigenvalues of the perturbing matrix are above a certain critical threshold. We also examine the consequences of this eigenvalue phase transition on the associated eigenvectors and generalize our results to examine the case of multiplicative perturbations or of additive perturbations for the singular values of rectangular matrices.

Charles Bordenave (Université de Toulouse).

Title. Heavy-tailed random matrices and the Poisson Weighted Infinite Tree.

Abstract. Consider the hermitian matrix $H = (X_{i,j})_{1\le i,j\le n}$ with i.i.d. upper triangular entries $(X_{i,j})_{1\le i\le j\le n}$. We assume that the random variable $X_{1,1}$ is in the domain of attraction of an α stable law, $0 < \alpha < 2$. We will see that the empirical spectral distribution of H converges toward the average spectral measure of a random self-adjoint operator defined on Aldous' Poisson weighted infinite tree (PWIT).

Similarly, we consider the random matrix $M = (Y_{i,j})_{1\le i,j\le n}$ with i.i.d. entries $(Y_{i,j})_{1\le i,j\le n}$ in the domain of attraction of an α stable law, $0 < \alpha < 2$. We also show that the empirical spectral distribution of M converges toward a measure that can be expressed in term of an operator defined on the PWIT. This probability measure on C is radial and it has finite moments of any order.

This is a joint work with Pietro Caputo (Roma Tre) and Djalil Chafaï (Univ. Paris Est).

Folkmar Bornemann (Technische Universität München).

Title. Numerical Evaluation of Distribution Functions for Canonical Matrix Ensembles.

Abstract. We review our work on the numerical evaluation of distribution functions in random matrix theory based on the accurate computation of operator determinants. We explain efficient ways to address the orthogonal and symplectic ensembles, higher order gap probabilities, and joint probability distributions.

Djalil Chafaï (Université de Paris Est).

Title. Spectrum of Large Random Markov Chains.

Abstract. The set of nxn Markov matrices is a convex and compact polytope. The uniform law on this polytope is a multivariate Dirichlet distribution. It provides uniform random Markov matrices. In this talk, we will focus mainly on a circular law theorem for such random matrices. Joint work with Pietro Caputo and Charles Bordenave.

Romain Couillet (Supélec).

Title. Deterministic equivalents for unitary Haar matrices.

Abstract. In this presentation, we extend the deterministic equivalent approaches, based on the Stieltjes transform, used in random matrix models containing matrices with i.i.d. entries to random matrix models based on Haar matrices. We provide results that extend known results obtained by free probability theory. Namely, we propose a compact deterministic equivalent for the sum of independent $RWTW^*R$ matrices, with R, T Hermitian deterministic and W a Haar matrix, as well as a deterministic equivalent for matrix model XX^* where X is the entry-wise product of a Haar matrix and a deterministic matrix.

Catherine Donati-Martin (Université Paris 6).

Title. Large deviation for the largest eigenvalue of the Hermitian Brownian motion.

Abstract. We prove a LDP for the process of the largest eigenvalue for the Hermitian Brownian motion. By the contraction principle, we recover the large deviation principle at fixed time, i.e. for the GUE. This is a joint work with Mylène Maida.

Alan Edelman (Massachusetts Institute of Technology).

Title. A Numerical Linear Algebra View of the Tao-Vu Smallest Singular Value Limit and the SDO extension.

Abstract. We revisit the limiting distribution of the smallest singular value σ_n of an $n \times n$ random matrix with iid mean zero, variance 1 entries. Extensive computations performed in 1989 left little doubt that as n approaches infinity, $n\sigma_n$ approaches a limiting distribution independent of the distribution from which the entries are drawn. Twenty years later, Tao and Vu proved this conjecture (under a finite moment assumption) using an approach motivated by "property testing" from theoretical computer science.

We will dissect the result from a numerical analyst's perspective, starting with a reformulation of the intuition in terms of sampling a submatrix from the QR decomposition. We then explore data comparing the strength of bounds and accuracy of estimators to reality – which we hope will pique the interest of the experimentally-minded – and also argue a link (via bidiagonalization) to an underlying stochastic operator – an alternative approach which should interest the theoretically-oriented. This work is joint with Po-Ru Loh.

Jakob Hoydis (Supélec).

Title. Applications of Random Matrices to Small Cell Networks.

Abstract. "Small Cell Networks" (SCNs) is a radio network design concept based on the idea of a very dense deployment of low-cost low-power base stations that are substantially smaller than existing macro cell equipment. Smaller cell sizes cause significant changes to the wireless link. For example, due to lower antenna heights, the channels between base stations and user terminals are likely to contain strong line-of-sight components. In this talk, we present a class of random matrix models suited for the channel modeling of SCNs and discuss several applications of random matrix theory to this novel field of research.

Jean-Paul Ibrahim (Université de Toulouse).

Title. Large deviation properties for some perturbated sample covariance matrices.

Abstract. We study some large deviation asymptotic properties for perturbated sample covariance matrices when the population size is small with respect to the sample size. For this, we use classical large deviation arguments and discretisation technics. We finally get a universality property in the case of sub-Gaussian entries.

Malika Kharouf (Université de Paris ouest - Nanterre).

Title. A Central Limit Theorem for Information-Theoretic statistics of Gram random matrices.

Abstract. Let us consider a model of $N \times N$ Gram matrices $\Sigma_n \Sigma_n^*$ where the $N \times n$ random matrix Σ_n is given by

$$\Sigma_n = \frac{1}{\sqrt{n}} D_n^{1/2} X_n \tilde{D}_n^{1/2} + A_n,$$

matrices D_n and $\tilde{D}_n$ are respective $N \times N$ and $n \times n$ deterministic and diagonal, A_n is a $N \times n$ deterministic matrix with bounded spectral norm and the entries of the $N \times n$ random matrix X_n are i.i.d, centered and have a unit variance. In this work, we are interested in the study of the fluctuations of the random variable :

$$\mathcal{I}_n(\rho) = \frac{1}{N} \log\det\left(\Sigma_n \Sigma_n^* + \rho I_N\right),$$

where ρ is a real parameter. Under the assumption that $0 < \liminf \frac{N}{n} \leq \limsup \frac{N}{n} < \infty$, we prove that, when centered and properly rescaled, $\mathcal{I}_n(\rho)$ satisfies a Central Limit Theorem. An explicit expression of the variance is given and takes a simpler closed-form.

Oleksiy Khorunzhiy (Université de Versailles - Saint-Quentin).

Title. On sufficient and necessary condition for edge universality in wigner ensemble of random matrices.

Abstract. Abstrait : We study the asymptotic behavior of high moments of large Wigner random matrices whose entries are random variables with symmetric probability distribution. Our results allow us to put forward a conjecture that the existence of the twelfth moment of the matrix elements represent a sufficient and necessary condition for the asymptotic boundedness of the moments and eventually for the edge universality of the eigenvalue distribution of the Wigner ensemble of random matrices.

Olivier Ledoit (University of Zurich).

Title. Eigenvectors of some large sample covariance matrix ensembles.

Abstract. We consider sample covariance matrices $S_N = \frac{1}{p}\Sigma_N^{1/2} X_N X_N^* \Sigma_N^{1/2}$ where X_N is a $N \times p$ real or complex matrix with i.i.d. entries with finite 12^{th} moment and Σ_N is a $N \times N$ positive definite matrix. In addition we assume that the spectral measure of Σ_N almost surely converges to some limiting probability distribution as $N \to \infty$ and $p/N \to \gamma > 0$. We quantify the relationship between sample and population eigenvectors by studying the asymptotics of functionals of the type $\frac{1}{N}\mathrm{Tr}\left(g(\Sigma_N)(S_N - zI)^{-1}\right)$, where I is the identity matrix, g is a bounded function and z is a complex number. This is then used to compute the asymptotically optimal bias correction for sample eigenvalues, paving the way for a new generation of improved estimators of the covariance matrix and its inverse.

Philippe Loubaton (Université de Paris-Est).

Title. Exact separation of the eigenvalues of large dimension complex Gaussian Information plus Noise model

Abstract. We establish that the property called Exact Separation of Eigenvalues by Bai and Silverstein in 1998 holds in the context of complex Gaussian Information plus noise models. Our appraoch is based on Gaussian technics introduced by Haagerup-Thornbjorsen and used by Capitaine and Donati-Martin. This is a joint work with Pascal Vallet.

Mylène Maïda (Université de Paris-Sud).

Title. Large deviations of extreme eigenvalues of deformed random matrices and performance analysis

Abstract. This talk will consist on two quite different parts. We will first explain how large deviation results for a rank one perturbation of the Laguerre unitary ensemble allowed us to precisely study the performance of a test commonly used in MIMO contexts. This is joint work with P. Bianchi, M. Debbah and J. Najim.

Then, in a recent joint work with F. Benaych-Georges and A. Guionnet, we have obtained large deviation results for more general deformed models. We will describe these results and ask the question whether there could be further applications to wireless communications.

Camille Mâle (ENS Lyon).

Title. Norm of Polynomials in Large Random Matrices.

Abstract. Let us consider a family $\mathbf{X}_N = (X_1^{(N)}, \cdots, X_p^{(N)})$ of $N \times N$ independent random matrices from the Gaussian Unitary Ensemble (GUE). Haagerup and Thorbjørnsen proved that for any non commutative polynomial P the operator norm of the random matrix $P(X_1^{(N)}, \cdots, X_p^{(N)})$ converges when the size of the matrices goes to the infinity and described the limit by the operator norm of objects from free probability theory. In this talk we present sufficient conditions on (possibly random) matrices $\mathbf{Y}_N = (Y_1^{(N)}, \cdots, Y_q^{(N)})$, independent with $\mathbf{X}_N$, such that the later result holds for matrices of the form $P(\mathbf{X}_N, \mathbf{Y}_N, \mathbf{Y}_N^*)$.

By considering particular choices of matrices $\mathbf{Y}_N$ and polynomials P we can extend the class of matrices for which the Bai and Silverstein's "No eigenvalues outside the limiting spectra" phenomena holds.

Boaz Nadler (Weizmann Institute of Science, Israel).

Title. Non-parametric Signal Detection and RMT

Abstract. Determining the number of signals or sources from observed noisy data is a fundamental problem in signal processing and in other scientific fields. In this talk I'll describe various relations between non-parametric detection and random matrix theory. These connections, and in particular recent results from RMT are useful both for the performance analysis of existing algorithms as well as for the development of novel ones.

Alain Rouault (Université de Versailles - Saint Quentin).

Title. Truncations of Haar unitary matrices and bivariate Brownian bridge (joint work with Catherine Donati-Martin)

Abstract. Let U be a Haar distributed unitary matrix in $\mathbb{U}(n)$. We show that, after centering, the double index process

$$W^{(n)}(s,t) = \sum_{i \le \lfloor ns \rfloor, j \le \lfloor nt \rfloor} |U_{ij}|^2$$

converges in distribution to the bivariate tied-down Brownian bridge. The proof relies on the notion of second order freeness.

Dietrich von Rosen (Swedish University of Agricultural Sciences).

Title. High-dimensional analysis and estimation in general multivariate linear models

Abstract. When the number of variables compared with the number of observations is large this paper presents a new approach of estimating the parameters describing the mean structure in the Growth Curve model. An explicit estimator is obtained which is unbiased, consistent and asymptotically normally distributed. Its variance is also derived.

Francisco Rubio (Centre Tecnològic de Telecomunicacions de Catalunya).

Title. A CLT on the SNR of the diagonally loaded beamformer

Abstract. The Capon or minimum variance distorsionless response (MVDR) beamfomer is a prominent example of spatial filtering structure in sensor array signal processing. In practice, typical implementations are based on a regularized or diagonally-loaded version of the sample covariance matrix (SCM) estimator. Conventionally, performance evaluation of the beamformer relies on a measure of the signal-to-noise ratio (SNR) at the filter output. In this talk, we introduce a central limit theorem (CLT) result characterizing the output SNR performance of MVDR beamformer implementations employing diagonal loading for both spatially and temporally correlated Gaussian observations.

Øyvind Ryan (University of Oslo).

Title. On general criteria for when the spectrum of a combination of random matrices depends only on the spectra of the components.

Abstract. Gaussian matrices provide examples where combinations of random matrices have a spectrum which can be determined from that of the component matrices. Recently, it has been shown that a similar behaviour is the case for certain types of random Vandermonde matrices. We go through this parallel development for Vandermonde matrices, and also attempt to set the two cases into a common framework. In such a framework, deconvolution is possible, in that one can infer on the spectrum of one of the component matrices, based on knowledge of the structure in the others.

Jack Silverstein (North Carolina State University).

Title. Estimating Population Eigenvalues From Large Dimensional Sample Covariance Matrices.

Abstract. Let $B_n = (1/N)T_n^{1/2}X_nX_n^*T_n^{1/2}$ where $X_n = (X_{ij})$ is $n \times N$ with i.i.d. complex standardized entries, and $T_n^{1/2}$ is a Hermitian square root of the nonnegative definite Hermitian matrix T_n. This matrix can be viewed as the sample covariance matrix of N i.i.d. samples of the n dimensional random vector $T_n^{1/2}(X_n)_{\cdot 1}$, the latter having T_n for its population covariance matrix. Quite a bit is known about the behavior of the eigenvalues of B_n where n and N are large but on the same order of magnitude. These results are relevant in situations in multivariate analysis where the vector dimension is large, but the number of samples needed to adequatly approximate the population matrix (as prescribed in standard statistical procedures) cannot be attained. Work has been done in estimating the eigenvalues of T_n from those of B_n. This talk will introduce a method devised by X. Mestre, and will present an extension of his method to another ensemble of random matrices important in wireless communication.

Pascal Vallet (Université de Paris-Est).

Title. Eigenspace estimation for source localization using large random matrices.

Abstract. In this talk, we will consider the problem of eigenspace estimation for certain large "information plus noise" gaussian random matrices. This problem is motivated in signal processing with the context of source localization (MUSIC algorithm). In the case where the dimensions of the system (number of antennas M and number of samples N) tend to $+\infty$ at the same rate, it can be shown that the traditional estimator, based on the empirical covariance matrix of the observations, is not consistent. We will give here a new consistent estimator in the above asymptotic regime.

Jian-feng Yao (Université de Rennes 1 et IRMAR).

Title. on corrections of classical multivariate tests for high-dimensional data

Abstract. First a short introduction to the close link between random matrix theory and high-dimensional data analysis will be given while focusing on large sample covariance matrices, Marcenko-Pastur theorem and Bai-Silverstein theorem. The aim of the talk is to explain why traditional multivariate methods need to be corrected to cope with high-dimensional effect. To this end, recent developments on classical one-sample problem and two-samples problem will be presented.

Lu Wei (Aalto University, Finland).

Title. On the Demmel condition number distribution with applications in performance analysis of wireless communication systems.

Abstract. Knowledge of the Demmel Condition Number (DCN) distribution of Wishart matrices is key to understanding the performance of various communication technologies, particularly adaptive transmission, link adaptation, and spectrum sensing. In this talk, we study the distribution of the Demmel condition number of complex Wishart matrices. New exact and asymptotic expressions are derived for the probability density function and cumulative distribution function, which are very simple and valid for any matrix dimensions. These results are obtained by taking advantage of properties of the Mellin transform for products of independent random variables.