Asymptotic Analysis for Mutual Information of MIMO Rician Correlated Channels

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Outline

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1. Channel Model and Performance Criterion
   - MIMO Channel
   - Previous work
2. Study of the Average Mutual Information
   - Deterministic equivalent
   - Convergence rates
   - Expression of the Mutual Information
3. Optimal Transmit Covariance
   - Problem statement
   - Algorithm

General MIMO Channel

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$$

- Transmitted signal $\mathbf{x}$: $t \times 1$
- Received signal $\mathbf{y}$: $r \times 1$
- AWG Noise $\mathbf{n}$: $r \times 1$, with $E(\mathbf{n}\mathbf{n}^H) = \sigma^2 \mathbf{I}_r$
- Channel matrix $\mathbf{H}$: $r \times t$

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Fundamental system of equations

$$\mathbf{T}(\sigma^2) = \left[\sigma^2 \left(\mathbf{I}_r + \tilde{\delta}(\sigma^2)\mathbf{D}\right) + \mathbf{A}\left(\mathbf{I}_t + \delta(\sigma^2)\tilde{\mathbf{D}}\right)^{-1}\mathbf{A}^H\right]^{-1}$$

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- Final step (aim of our paper) :

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Outline

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Average Mutual Information

Bai-Silverstein's Assumptions : $\mathbf{A} = \mathbf{0}$, $\tilde{\mathbf{D}} = \mathbf{I}_t$

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Bai-Silverstein's Assumptions, Complex Gaussian Case

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Loubaton's Assumptions : **A** is $\mathbb{L}_2$ bounded

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Loubaton's Assumptions : Conjecture (known t.t. Moustakas et *al.* for **A=0**)

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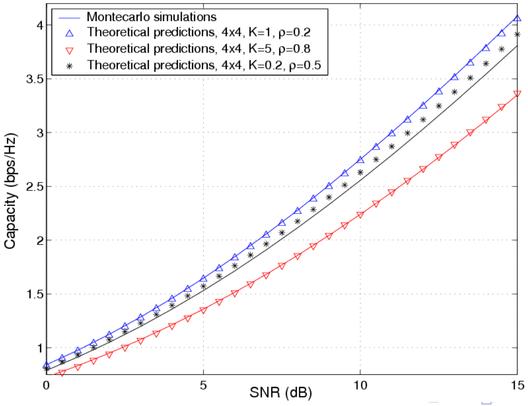
Theorem

$$\bar{I}(\sigma^2) = \log\det\left(\mathbf{I}_r + \tilde{\delta}(\sigma^2)\mathbf{D} + \frac{1}{\sigma^2}\mathbf{A}[\mathbf{I}_t + \delta(\sigma^2)\tilde{\mathbf{D}}]^{-1}\mathbf{A}^H\right)$$

$$+\log\det\left(\mathbf{I}_t + \delta(\sigma^2)\tilde{\mathbf{D}}\right) - \frac{t}{r}\sigma^2\delta(\sigma^2)\tilde{\delta}(\sigma^2)$$



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Reminder : the Ergodic Capacity Problem

Find the positive definite and normalized matrix $\mathbf{Q}$, solution of

$$\max_{\mathbf{Q}\geq 0, \frac{1}{t}Tr(\mathbf{Q})\leq 1} \mathbb{E}_{\mathbf{H}}\left[\log\det\left(\mathbf{I}_r + \frac{\mathbf{HQH}^H}{\sigma^2}\right)\right]$$

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$\Rightarrow$ Asymptotical methods are not useful for all these cases,
but we use them to solve the original case $\tilde{\mathbf{D}} = \mathbf{I}_t$

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$$\mathbf{H}\sqrt{\mathbf{Q}} = \mathbf{A}\sqrt{\mathbf{Q}} + \frac{1}{\sqrt{t}}\mathbf{D}^{\frac{1}{2}}\mathbf{X}\sqrt{\mathbf{Q}} \Rightarrow \text{Ergodic Capacity}$$

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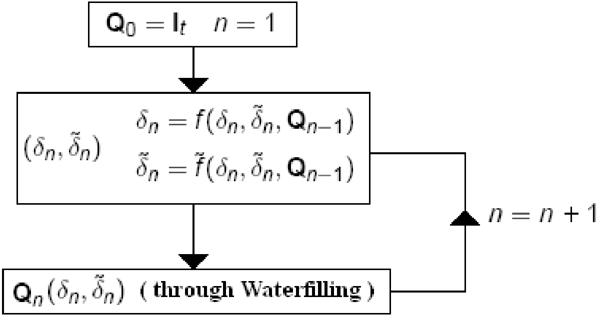

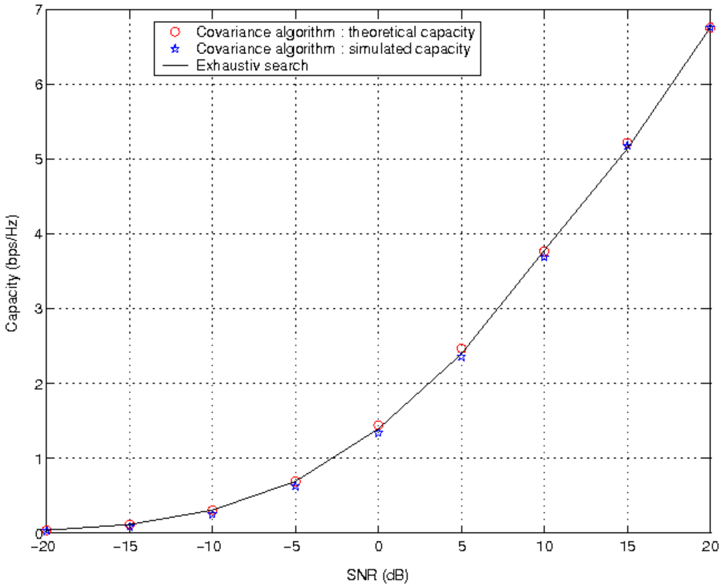
Covariance algorithm : theoretical capacity
Covariance algorithm : simulated capacity
Exhaustiv search
Capacity (bps/Hz)
SNR (dB)
0
1
2
3
4
5
6
7
−20
−15
−10
−5
0
5
10
15
20

Summary

We have addressed the Mutual Information Evaluation of Ricean Correlated Channel using Large Random Matrix Theory Tools, and proposed an algorithm to solve the problem of the optimal transmit covariance in original conditions.

- The ergodic mutual information has a simple approximation.
- The approximation is relevant for quite moderate values of the number of antennas.
- This theory proposed practical applications.

For the time being, we are obtaining new results about the outage capacity, by studying the variance of the error $I(\sigma^2) - \bar{I}(\sigma^2)$.

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Summary

We have addressed the Mutual Information Evaluation of Ricean Correlated Channel using Large Random Matrix Theory Tools, and proposed an algorithm to solve the problem of the optimal transmit covariance in original conditions.

- The ergodic mutual information has a simple approximation.
- The approximation is relevant for quite moderate values of the number of antennas.
- This theory proposed practical applications.

For the time being, we are obtaining new results about the outage capacity, by studying the variance of the error $I(\sigma^2) - \bar{I}(\sigma^2)$.