Analysis and Design of Energy-Efficient Heterogeneous Networks using Stochastic Geometry

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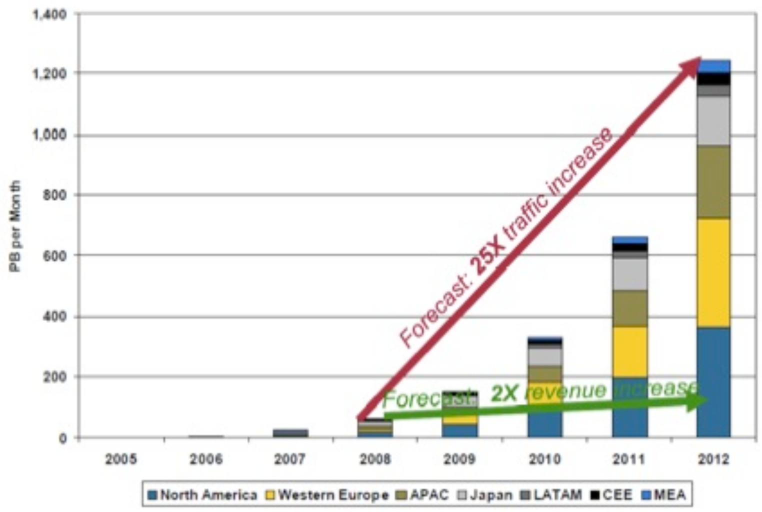
Wireless Networks: Facts & Trends

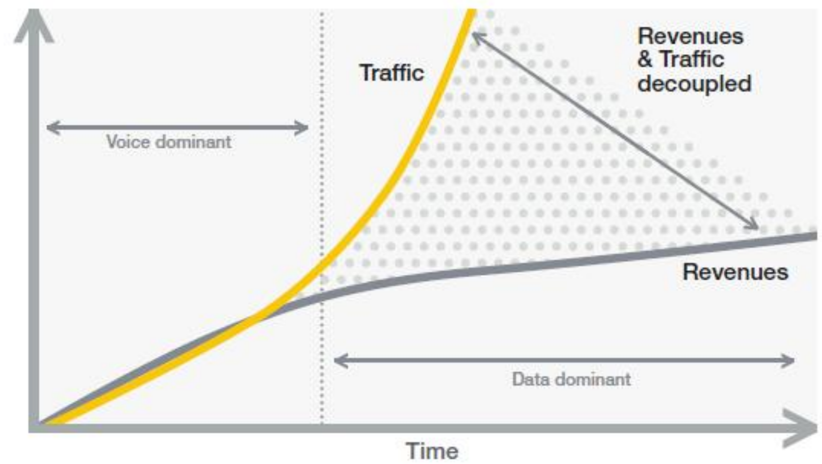
- Data traffic increasing **exponentially**
  - increases approx. 10x every 5 years ≈ 16 – 20 % increase of energy consumption/year.
- Voice traffic and data revenue increasing slowly
- ICT contributes an increasing share to the global greenhouse effect (2% in 2007)



Sources: Cisco, from Operators' network data and Analysts, 2008; Informa, 2008; and Pyramid, "Mobile data revenue will double by 2012," Dan Locke, Analyst Insight, 4/2008.

<u>Challenges</u>:

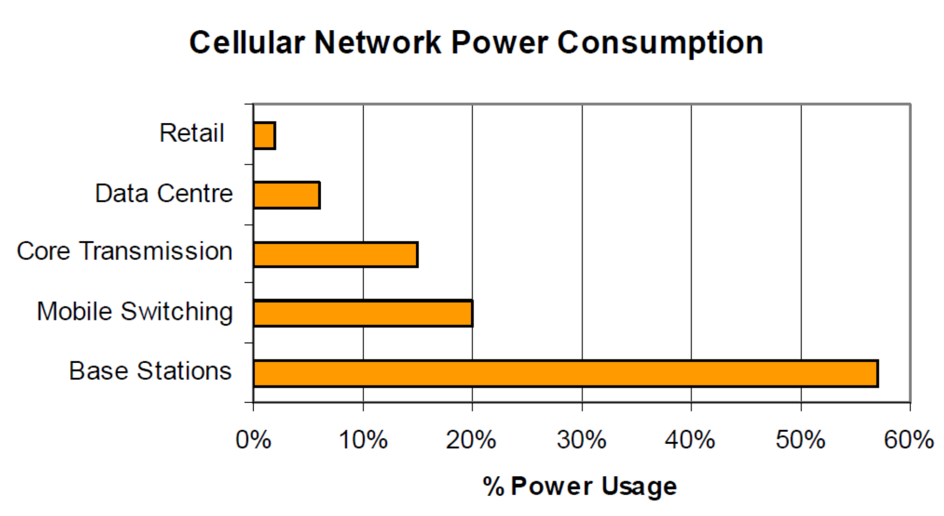
- How to satisfy these traffic demands?
- How to reduce the carbon footprint and power consumption?
- How to decrease €/bit (/Joule) exponentially?



Green Tech: Opportunities or Hype?

- 3% of the worldwide energy is consumed by the ICT infrastructure
  ≈ 2% of the worldwide CO_2 emissions
- 80% of the power is consumed in the RAN (specifically the base stations)
- Cellular Nets optimized mainly in terms of throughput and QoS
  - Concern only about UE battery lifetime.

- Cellular networks are energy greedy:



Power consumption of typical cellular network
(Source: Vodafone)

ICT results in significant increase in energy consumption

Outline

#1
Heterogeneous Nets

Paradigm Shift

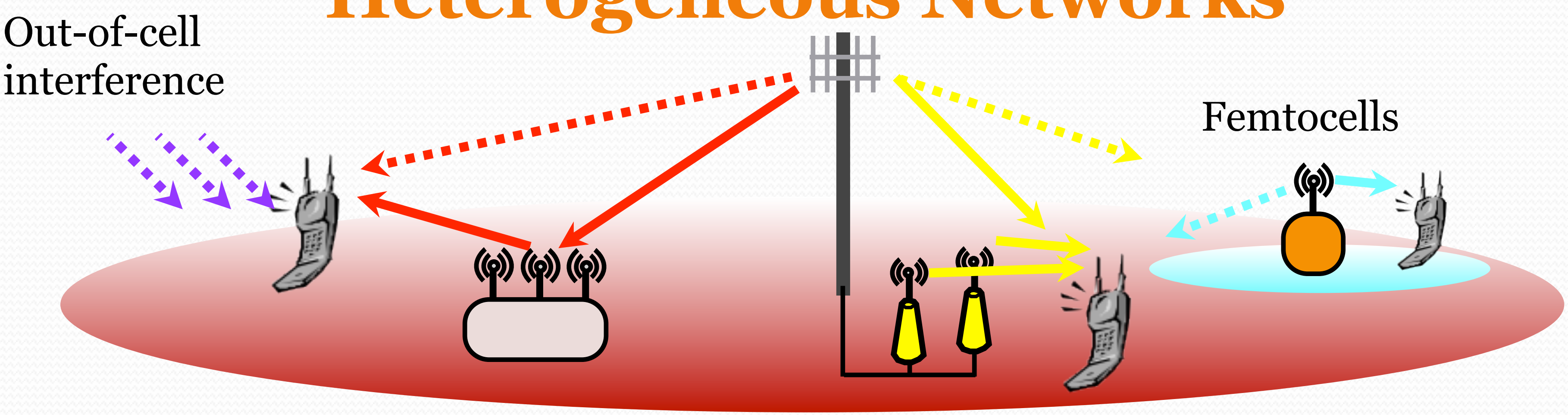
#2
Stochastic Geometry

Emerging Powerful Spatial Network Models

#3
Energy Efficiency

Optimizing bit/sec/Hz/m²/W

Heterogeneous Networks

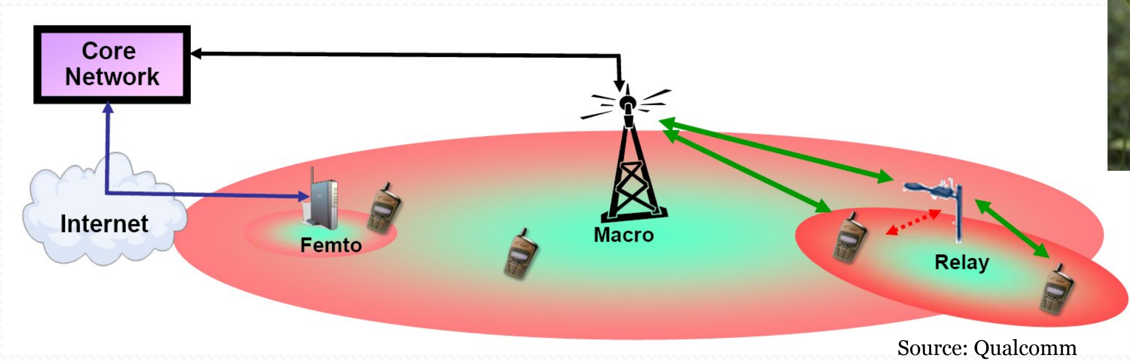


- **Traditional tower-mounted base stations**
  - *Expensive, 40W Tx power + high antenna gain, fast dedicated backhaul*
- **Picocells**
  - Short-range (~100m), 1-2W, low-cost, deployed, maintained and backhauled (X2 interface, usually fiber or wireless) by service provider, targeting traffic "hotspots" or dense areas.
- **Femtocells**
  - 100-200mW power, low cost, often user-installed, and backhaul (IP, e.g. DSL, coax). Licensed spectrum, cellular protocols, must interoperate with cellular network with minimal coordination
- **Distributed antennas (DAS), fixed relays, microcells, WiFi**

Cellular networks are becoming more complex

Is *Heterogeneity* the way to go?

Wireless Ecosystem

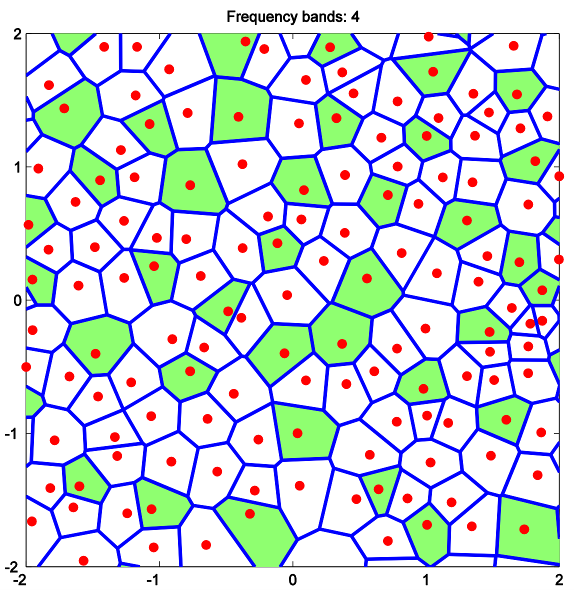


Source: Qualcomm

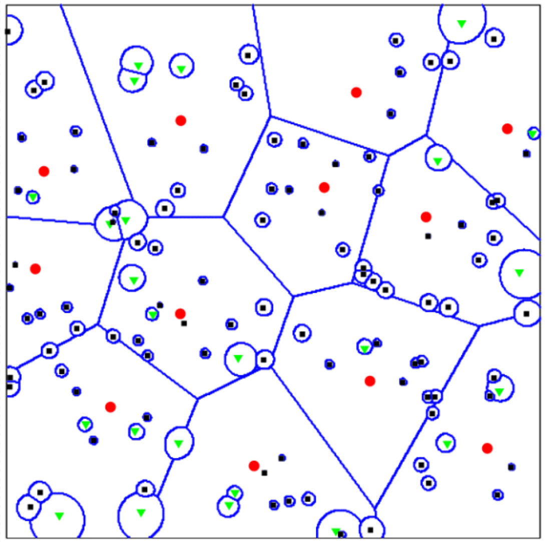
- *1-tier macrocellular net:* there is not enough peak rate
  - Average spectral efficiency over a fully loaded cell is about 2 bps/Hz at the Shannon limit (treating interference as noise)
- *Adding macrocell towers or buying more spectrum is completely out of the question*
- *Advanced PHY techniques offer logarithmic (at best) rate increase*
- **HetNet Paradigm:** Densify the network as much as possible (where data is demanded)
  - additional capacity and/or coverage
  - traffic offloading
  - decreased Tx power from reduced Tx-Rx distance (how about total energy per area??)
- Increase spatial reuse (i.e. # rate R links) per square meter

Modeling Emerging Cellular Networks:
The need for novel models

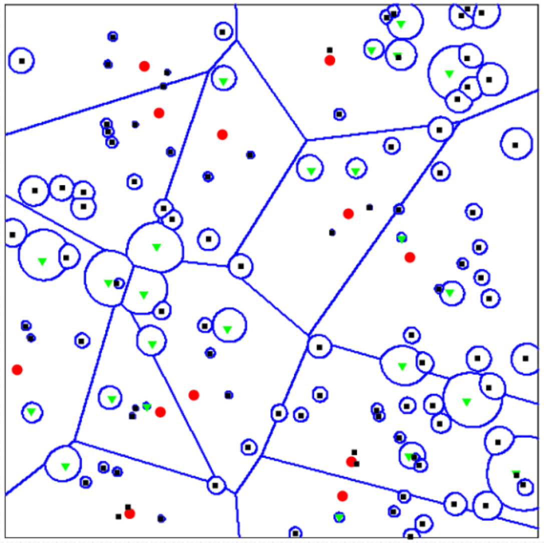
Network Topology and Coverage



Actual 4G macrocells



3-tier Hetnet Real data



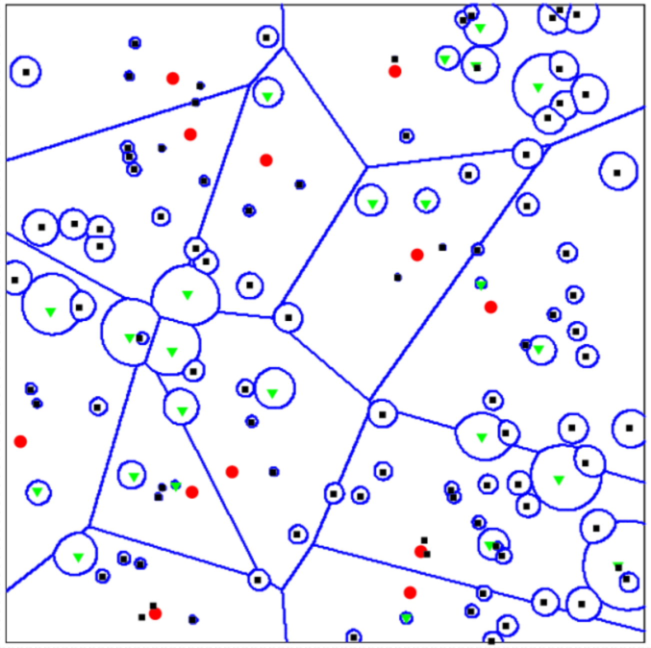
3-tier Hetnet PPP

Source: http://arxiv.org/abs/1103.2177

- **Claims**
  - Deterministic (i.e. grid) models are highly idealized
  - Fixed models are not scalable to a HetNet
  - Results based on traditional models are quite questionable

Heterogeneous Network Model

- *K* tiers of base stations
  BS locations taken from independent Poisson Point Processes (PPP)
  - Base Station Density: λ_i BS/area
  - Transmit Power: P_i Watts
  - SINR Target γ_i
  - Per-tier path loss exponent α_i



- *May include other quantities (traffic factor, bias, …)*
- *Clustering, repulsion, irregularity can be incorporated*
- *Intriguingly tractable model (as good as grid for typical cell)*

Baseline Concepts and Calculations

Poisson Point Process

- *Def*: A PPP with density λ is a random point set such that
  - The number of points $\mathcal{N}(\mathcal{A})$ for any bounded $\mathcal{A} \subset \mathbb{R}^d$ follows a Poisson distribution with
  $$\mathbb{E}(\mathcal{N}(\mathcal{A})) = \lambda \times \text{area of } (\mathcal{A})$$
  - $\mathcal{N}(\mathcal{A})$ and $\mathcal{N}(\mathcal{B})$ are independent if $\mathcal{A}$ and $\mathcal{B}$ are independent (disjoint sets)
- Basic properties
  - Superposition of two PPPs of densities λ_1 and λ_2 results in a PPP of density $\lambda_1 + \lambda_2$
  - Thinning: selecting a point of the process with probability p independently of the other points and discard it with probability $(1 - p)$ results in 2 indep. PPPs of intensity measures $p\lambda$ and $(1 - p)\lambda$
  - Conditional uniformity: Given $\mathcal{N}(\mathcal{A}) = n$, the locations of the n points in $\mathcal{A}$ are independent and uniformly distributed random variables

Simplest Example: One-Tier Downlink Model

- Building Block: received SINR at a typical Rx

$$\mathrm{SINR} = \frac{\rho S_0 d^{-\alpha}}{\sum_{i \in \Phi} \rho S_{i0} |X_i|^{-\alpha} + \eta}.$$

sums of a Poisson functional
(power law shot noise)

ρ	fixed transmit power
S_0	channel gain between typical Tx-Rx
S_{i0}	channel gain from i-th interferer
d	distance between source and destination
α	pathloss exponent ($\alpha > 2$)
η	noise power
$\lvert X_i \rvert$	distance from interferer i
$\Phi = \{X_i\}$	stochastic point process (node distribution)

Interference Geometry

- Interference power at a point $y \in \mathbb{R}^2$ in the network

$$I(y) = \sum_{x \in \mathcal{T}} P_x S_x \ell(\|y - x\|)$$

$\mathcal{T}$: set of all active Tx nodes, P_x: Tx power of node x, S_x the (power) fading coefficient, and $\ell(\cdot)$ the path loss function.

- Two main factors shape the interference
  - spatial distribution of the concurrently transmitting nodes
  - signal attenuation with distance (path loss law)

- Network geometry determines the distribution of the interference

- Interference can be viewed as a random field or as a shot noise process

$$I(t) = \sum_{i=1}^{\infty} g(t - x_i) = \sum_{x \in \Phi} g(t - x) \Rightarrow I(y) = \sum_{x \in \Phi} K_x g(y - x)$$

where K_x are i.i.d rv, $\Phi\{x_i\}$ is a stationary Poisson process and $g(x)$ is the impulse response

Interference Statistics

- No closed-form PDF for general α values (α = 1, 2, 4 only)
- Slyvnyak's Theorem: for PPP, distribution of $I(y)$ indep. of y
- Campbell's Theorem: Let $f(x)$:$\mathbb{R}^d \rightarrow [0,\infty)$ be a measurable function. Then for an intensity measure $\Lambda(x)$

$$\mathbb{E}\left(\sum_{x\in\Phi} f(x)\right) = \int_{\mathbb{R}^d} f(x)\Lambda(x)$$

- Probability Generating Functional (PGF) of a PPP

$$\mathbb{E}\left[\prod_{x\in\Phi} f(x)\right] = \exp\left(-\lambda\int_{\mathbb{R}^2}(1-f(x))\mathrm{d}x\right)$$

- I is a ***stable*** distribution with characteristic exponent $2/\alpha < 1$

$$\mathcal{L}_I(s) = \exp(-\lambda c_d \mathbb{E}(S^{\frac{2}{\alpha}})\Gamma(1-2/\alpha)s^{\frac{2}{\alpha}})$$

with c_d the volume of a d-dim. unit ball $b(0, 1)$, and $\ell(r) = r^{-\alpha}$

Coverage Probability

- Coverage probability (success probability)

$$\mathbb{P}[\mathsf{SINR} > T] = \mathbb{P}\left[\frac{S_0 d^{-\alpha}}{\eta + I} > T\right]$$

- For exponential fading S_0 with mean μ

$$\begin{aligned}\mathbb{P}[S_0 > Td^\alpha(\eta + I)] &= \mathbb{E}_I\left[\mathbb{P}[S_0 > Td^\alpha(\eta + I) \mid I]\right] \\ &= \mathbb{E}_I\left[\exp(-\mu T d^\alpha(\eta + I))\right] \\ &= e^{-\mu T d^\alpha \eta} \mathcal{L}_I(\mu T d^\alpha)\end{aligned}$$

Laplace transform of interference I

Laplace Transform of Interference

$$\mathcal{L}_I(s) = \mathbb{E}_I[e^{-sI}] = \mathbb{E}_{\Phi,S_i}[\exp(-s \sum_{i\in\Phi\setminus\{o\}} S_i D_i^{-\alpha})]$$

$$= \mathbb{E}_{\Phi,\{S_i\}}\left[\prod_{i\in\Phi\setminus\{o\}} \exp(-sS_i D_i^{-\alpha})\right]$$

$$= \mathbb{E}_{\Phi}\left[\prod_{i\in\Phi\setminus\{o\}} \mathbb{E}_S[\exp(-sSD_i^{-\alpha})]\right]$$

i.i.d. distribution of S_i and independence from point process Φ

From the PGF of PPP

$$= \exp\left(-2\pi\lambda \int_0^\infty \left(1 - \mathbb{E}_S[\exp(-sSv^{-\alpha})]\right) v\mathrm{d}v\right)$$

Performance Metrics

- Area Spectral Efficiency (network throughput)

$$\mathcal{T} = \lambda \mathbb{P}\left\{\mathrm{SINR} > \beta\right\} \log_2(1+\beta)$$

- Average data rate

$$\mathcal{R}(\lambda, \alpha) = \mathbb{E}\left\{\log(1+\mathrm{SINR})\right\} = -\int_0^\infty \log(1+\beta) d\mathcal{P}_s \ \mathrm{nats/Hz}$$

Energy Efficiency metrics

- Various proposals, mostly ad hoc or intuitive
- Commonly used metric:

EE = ratio of area spectral efficiency divided by total network power consumption

Energy Efficient Heterogeneous Cellular Networks

System Model

- *K*-tier HetNet
- Power consumption model:

Basic operating power RF output power

$$P_{\text{Het},i} = \lambda_i(P_{i0} + \Delta_{\text{i}} P_i)$$

slope of load-dependent power consumption

- <u>BS sleeping policies</u>:
  - ***Random sleeping***
    - *each BS operates with probability q (Bernoulli trials)*

power consumed in sleep mode

$$P_{\text{RS}} = \lambda_{\text{M}} q(P_{\text{M}0} + \Delta_{\text{m}} \beta P_{\text{M}}) + \lambda_{\text{M}}(1-q)P_{\text{sleep}}$$

  - ***Load-dependent sleeping***

    this strategic sleeping is modeled as a function s, such that if the activity level is x, it operates with probability s(x)

User Association Schemes

- *Location based scheme (LOC)*
  - the user connects to the BS corresponding to $\min(\{\kappa_i r_i\})$

biasing

relative distance to nearest BS

- *Average signal based scheme*
  - the user connects to the BS corresponding to $\max(\{\tau_i Q_i\})$

biasing

average received signal from each tier *i*

- *Instantaneous SINR based scheme (INS)*
  - the user connects to the BS which provides the maximum SINR (assuming $\gamma_i > 1$)

Coverage Probability (1/2)

- The coverage probability for a mobile user associated using the location based scheme is

$$\mathbb{P}_{\mathrm{LOC}} = \sum_{i=1}^{K} \int_{r=0}^{\infty} 2\lambda_i \pi r \exp(-r^2 c_i) \exp(-r^\alpha a_i)$$

where $a_i = \gamma\sigma^2 / P_{\mathrm{t,i}}$ *and* $c_i = \pi\lambda_i(1+\rho(\gamma,\alpha)) + \frac{\pi}{\kappa_i^2}\sum_{j\neq i}\lambda_j\big(1+\rho\big(\gamma\frac{P_{\mathrm{t,j}}}{P_{\mathrm{t,i}}}\frac{\kappa_i^\alpha}{\kappa_j^\alpha},\alpha\big)\big)$

- For $\alpha = 4$ and $\sigma^2 = 0$ we have the following simplifications

$$\mathbb{P}_{\mathrm{LOC},\alpha=4} = \sum_{i=1}^{K} \lambda_i G(b_i, a_i)$$

$$G(b,a) = 2\pi \int_{x=0}^{\infty} x e^{-ax^4} e^{-bx^2} dx = \frac{\pi^{3/2}}{\sqrt{a}} \exp(b^2/4a) Q(b/\sqrt{2a}).$$

$$\mathbb{P}_{\mathrm{LOC},\sigma^2=0} = \sum_{i=1}^{K} \frac{\lambda_i \kappa_i^2}{\sum_j \lambda_j \kappa_j^2 \big(1+\rho\big(\gamma\frac{P_{\mathrm{t,j}}}{P_{\mathrm{t,i}}}\frac{\kappa_i^\alpha}{\kappa_j^\alpha},\alpha\big)\big)}$$

where $\rho(\gamma,\alpha) = \gamma^{2/\alpha}\int_{\gamma^{-2/\alpha}}^{\infty}\frac{1}{1+u^{\alpha/2}}du$ $\quad b_i = \pi\lambda_i(1+\rho(\gamma,4)) + \frac{\pi}{\kappa_i^2}\sum_{j\neq i}\lambda_j\big(1+\rho\big(\gamma\frac{P_{\mathrm{t,j}}}{P_{\mathrm{t,i}}}\frac{\kappa_i^4}{\kappa_j^4},4\big)\big)$

Coverage Probability (2/2)

- The average signal based scheme is equivalent to the location based scheme with $\tau_i = \kappa_i^{\alpha}/P_{\mathrm{t,i}},\ \forall i.$

- The coverage probabilities for the instantaneous SINR user association scheme are

$$\mathbb{P}_{\mathrm{INS}} = \sum_{i=1}^{K} \lambda_i \int_{r=0}^{\infty} 2\pi r \exp(-(\sum_k \lambda_k P_{\mathrm{t,k}}^{2/\alpha})(\gamma_i/P_{\mathrm{t,i}})^{2/\alpha} C(\alpha) r^2) \exp(-(\gamma_i/P_{\mathrm{t,i}})\sigma^2 r^{\alpha}) dr$$

$$\mathbb{P}_{\mathrm{INS},\alpha=4} = \sum_{i=1}^{K} \lambda_i G(\sqrt{\gamma_i/P_{\mathrm{t,i}}} C(4) \sum_{i=1}^{K} \lambda_i P_{\mathrm{t,i}}, \gamma_i \sigma^2/P_i)$$

$$\mathbb{P}_{\mathrm{INS},\sigma^2=0} = \frac{\pi}{C(\alpha)} \frac{\sum_{i=1}^{K} \lambda_i P_i^{2/\alpha} \gamma_i^{-2/\alpha}}{\sum_{i=1}^{K} \lambda_i P_i^{2/\alpha}}$$

where $C(\alpha) = \frac{2\pi^2}{\alpha} \csc(2\pi/\alpha)$

Power Consumption Optimization

- With average signal based scheme, we have the following optimization problem

$$\mathcal{P}_{\text{SIG}} : \begin{cases} \min\limits_{\lambda_i, \forall i} & \sum_i \lambda_i (P_{i0} + \beta_i P_i) \\ \text{s.t.} & \mathbb{P}_{\text{OAP}} \geq \epsilon \end{cases}$$

- No-noise case: the BS density should be chosen as low as possible (in dense nets, shut down as many BSs as possible)
- When $\sigma^2 > 0$ the optimal "weighted" total density $S^\star = \sum_i \lambda_i^\star P_{\text{t,i}}^{2/\alpha}$ satisfies the fixed point equation

$$\epsilon = S^\star \int_{r=0}^{\infty} 2\pi r \exp(-r^2 \pi S^\star (1 + \rho(\gamma, \alpha))) \exp(-r^\alpha \gamma \sigma^2) dr$$

- The original optimization problem becomes a linear one *easy to compute!!*

$$\mathcal{P}_{\text{SIG}}^0 : \begin{cases} \min\limits_{\lambda_i, \forall i} & \sum_i \lambda_i (P_{i0} + \beta_i P_i) \\ \text{s.t.} & \sum_i \lambda_i P_{\text{t,i}}^{2/\alpha} = S^\star \end{cases}$$

Energy Efficiency Optimization (1/2)

- Assume K=2 (macro + femto/pico) and random sleeping

$$\mathcal{P}_{2\mathrm{T}} : \left\{ \max_{q} \quad \frac{(q^2\lambda_{\mathrm{M}}^2\sqrt{P_{\mathrm{t,m}}}+\lambda_F^2\sqrt{P_F})G(\pi(q\lambda_{\mathrm{M}}\sqrt{P_{\mathrm{t,m}}}+\lambda_F\sqrt{P_F})(1+\rho(\gamma,4)),\gamma\sigma^2)}{\lambda_{\mathrm{M}}q(P_{\mathrm{M}0}+\Delta_{\mathrm{m}}\beta P_{\mathrm{M}})+\lambda_{\mathrm{M}}(1-q)P_{\mathrm{sleep}}+\lambda_F(P_{F0}+\Delta_{\mathrm{f}}\beta P_F)} \log_2(1+\gamma) \right.$$

- For tractability, let α=4 and use Taylor approximation

$$\left\{ \max_{q} \quad \frac{(q^2\lambda_{\mathrm{M}}^2\sqrt{P_{\mathrm{t,m}}}+\lambda_F^2\sqrt{P_F})(\frac{\pi^{3/2}}{2}\frac{1}{\sqrt{\gamma\sigma^2}}-\frac{\pi}{2}\frac{\pi(q\lambda_{\mathrm{M}}\sqrt{P_{\mathrm{t,m}}}+\lambda_F\sqrt{P_F})(1+\rho(\gamma,4))}{\gamma\sigma^2})}{\lambda_{\mathrm{M}}q(P_{\mathrm{M}0}+\Delta_{\mathrm{m}}\beta P_{\mathrm{M}})+\lambda_{\mathrm{M}}(1-q)P_{\mathrm{sleep}}+\lambda_F(P_{F0}+\Delta_{\mathrm{f}}\beta P_F)} \log_2(1+\gamma) \right.$$

- Can be easily solved numerically (cubic equation in q)

- What happens for K > 2 and additional constraints (e.g. on ASE)??

Energy Efficiency Optimization (2/2)

- With instantaneous SINR user association

$$\mathcal{P}_{\text{INS}} : \left\{ \max_{q} \quad \frac{\log_2(1+\gamma)(\lambda_{\text{M}} q + \lambda_F)}{\lambda_{\text{M}}(qP_{\text{M}0} + q\Delta_M P_{\text{M}} + (1-q)P_{\text{sleep}}) + \lambda_F(P_{F0} + \Delta_F P_F)} \frac{\pi}{C(\alpha)} \frac{q\lambda_{\text{M}} P_{\text{t,m}}^{2/\alpha} \gamma_m^{-2/\alpha} + \lambda_F P_{\text{t,f}}^{2/\alpha} \gamma_f^{-2/\alpha}}{q\lambda_{\text{M}} P_{\text{t,m}}^{2/\alpha} + \lambda_F P_{\text{t,f}}^{2/\alpha}} \right.$$

- The optimal activation probability $q_{\text{INS}}^{\star}$ is given

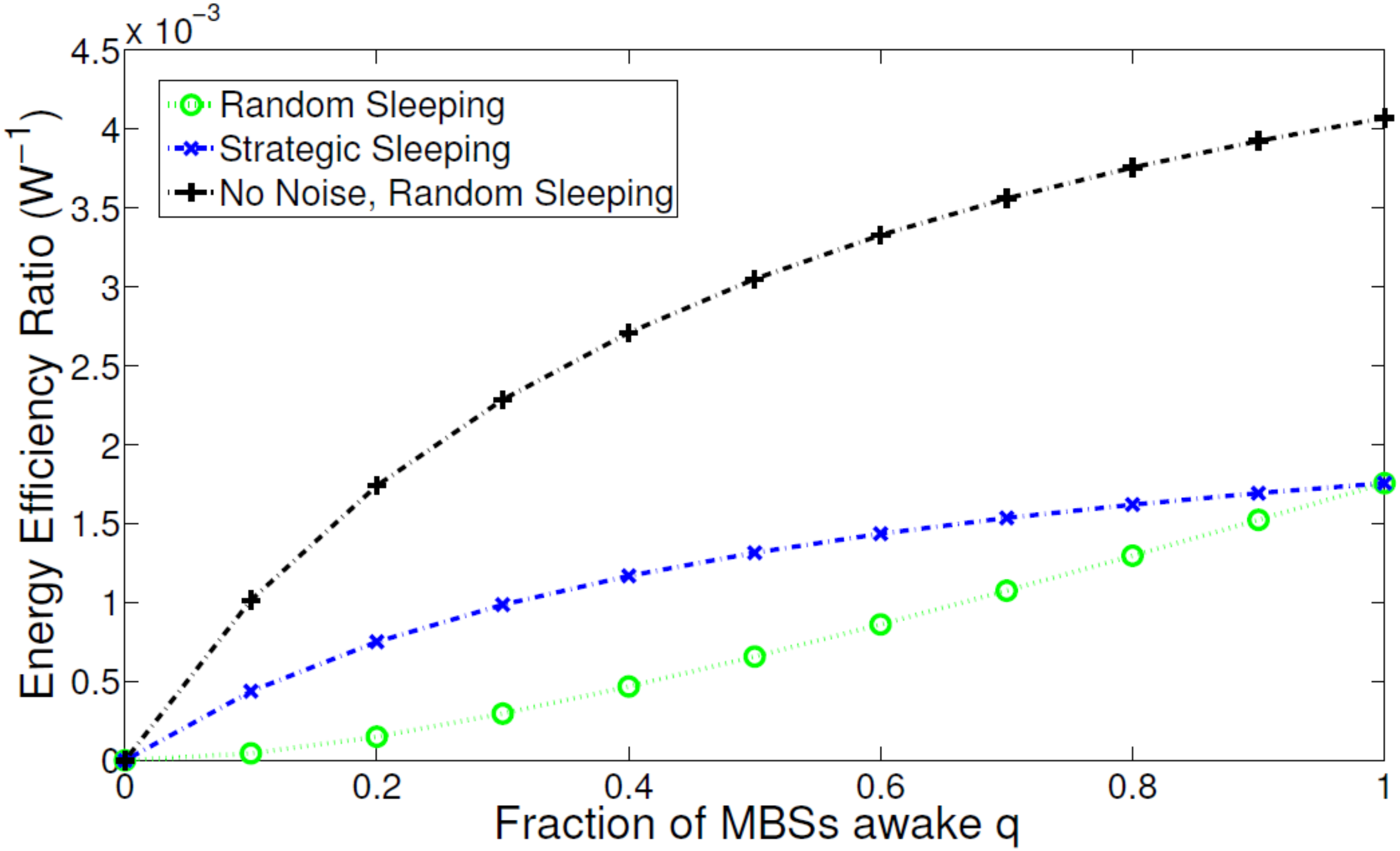
$$\max\{0, \min\{1, q'_{\text{INS}}\}\}$$

where

$$q'_{\text{INS}} = \frac{(C_1 - C_2) + -\sqrt{(C_2 - C_1)^2 - (B_2 - B_1)(C_2 B_1 - C_1 B_2)}}{B_2 - B_1}$$

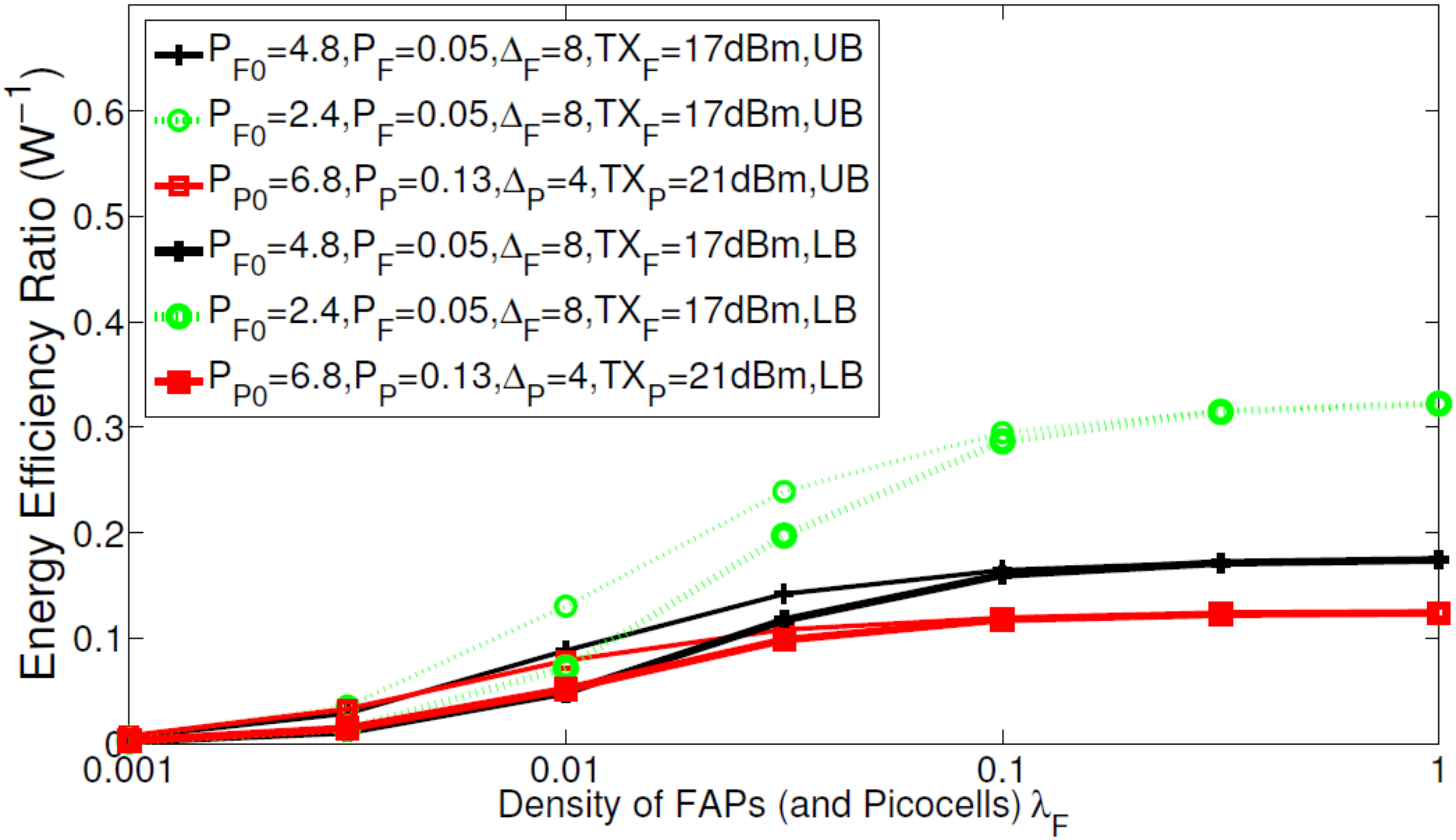
and $B_1 = \frac{\lambda_F}{\lambda_{\text{M}}}((\frac{P_F}{P_{\text{M}}}\frac{\gamma_M}{\gamma_F})^{2/\alpha} + 1)$, $B_2 = \frac{\lambda_F}{\lambda_{\text{M}}}(\frac{P_F}{P_{\text{M}}})^{2/\alpha} + \Delta_M \beta P_{\text{M}} + P_{\text{M}0} + (\lambda_F/\lambda_{\text{M}})\Delta_F P_F + (\lambda_F/\lambda_{\text{M}})P_{F0}$, $C_1 = \frac{\lambda_F}{\lambda_{\text{M}}}(\frac{P_F}{P_{\text{M}}}\frac{\gamma_M}{\gamma_F})^{2/\alpha}$, $C_2 = \frac{\lambda_F}{\lambda_{\text{M}}}(\frac{P_F}{P_{\text{M}}})^{2/\alpha}$.

Numerical Results



- Expectedly, load-dependent switch off performs better than random sleeping

Numerical Results



- Adding more small cell BSs increases the EE up to a limit (ceiling)

Parting Comments

- Cellular networks are undergoing a fundamental change
- Heterogeneous Networks are changing wireless communications in many ways … for good!
- Very large number of interesting problems that require new models, metrics, tools, and ways of thinking
- Stochastic geometry can be used to analyze tradeoffs (e.g. SE vs. EE) and to provide guidelines for optimizing large, dense networks
- Need for fundamental (Shannon-like) theory!! (some interesting results from thermodynamics)

Merci!