Green Wireless Networking with Energy Harvesting Nodes

Wireless Communications & Networking Laboratory
WCAN@PSU

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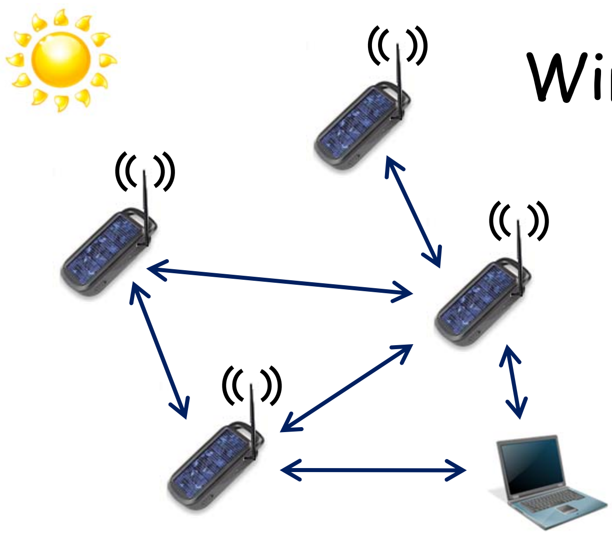
Introduction

- **Wireless networking with energy harvesting nodes:**
  - Green, self-sufficient nodes,
  - Extended network lifetime,
  - Smaller nodes with smaller batteries.

A relatively new field with increasing interest.

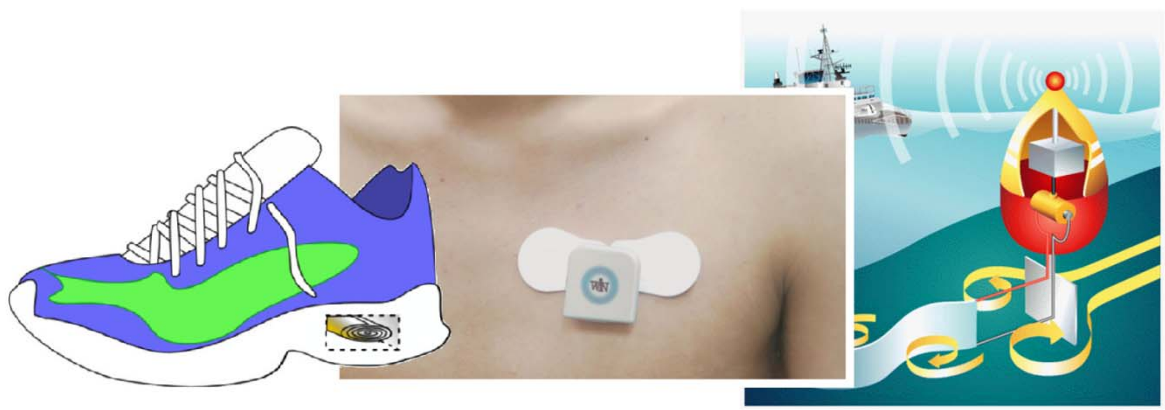
Some Applications



Wireless sensor networks

Green communications

Various other applications



Harvesting Energy

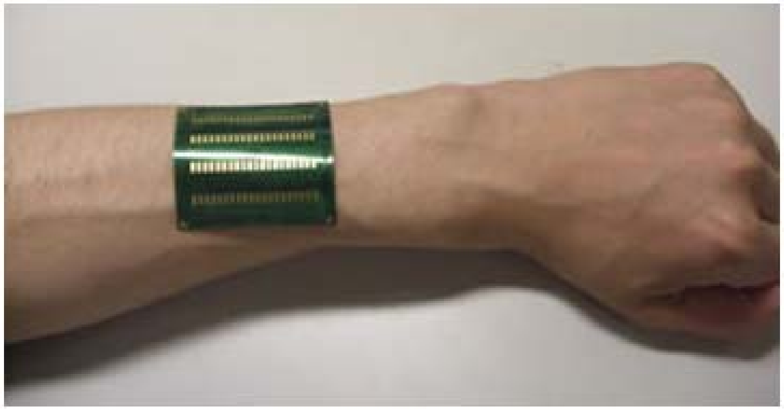
- Wind turbines
  50-750kW, intermittent

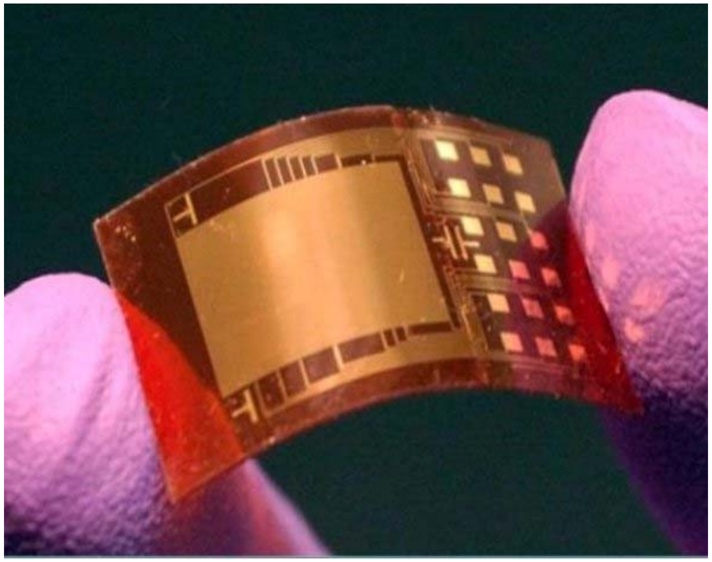
- Photovoltaic Cells
  Abundant solar energy

Image Credits: (top) http://www.popsci.com/files/imagecache/article_image_large/articles/GreenMountainWindFarm_Fluvanna_2004.jpg
(bottom) http://edu.glogster.com/media/4/33/80/5/33800548.jpg

Harvesting Energy

- Fujitsu's hybrid device utilizing **heat or light.**





- Nanogenerators built at Georgia Tech, utilizing **strain**

Image Credits: (top) http://www.fujitsu.com/global/news/pr/archives/month/2010/20101209-01.html
(bottom) http://www.zeitnews.org/nanotechnology/squeeze-power-first-practical-nanogenerator-developed.html

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Motivation

- **New Wireless Network Design Challenge:**

  A **set of energy feasibility constraints** based on harvests govern the communication resources.

- **Design question:**

  When and at what rate/power should a "rechargeable" (energy harvesting) node transmit?

- **Optimality? Throughput; Delivery Delay**

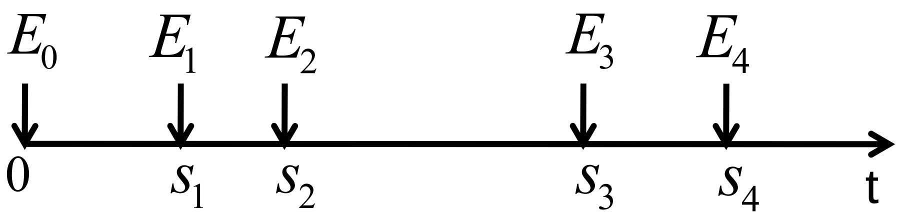
1. Short-term Throughput Maximization (STTM)

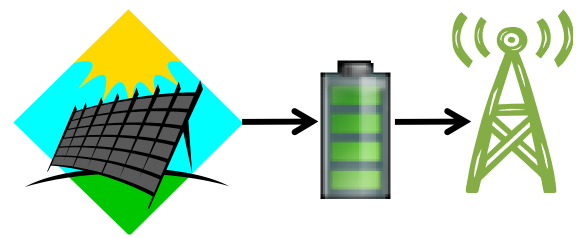
- Maximize the throughput of an energy harvesting transmitter by **deadline** T.
- Find optimal power allocation/transmission policy that departs maximum number of bits in a given duration.

- Up to a certain amount of energy can be stored by the transmitter → **BATTERY CAPACITY**

System Model

- Energy arrivals of energy E_i at times s_i



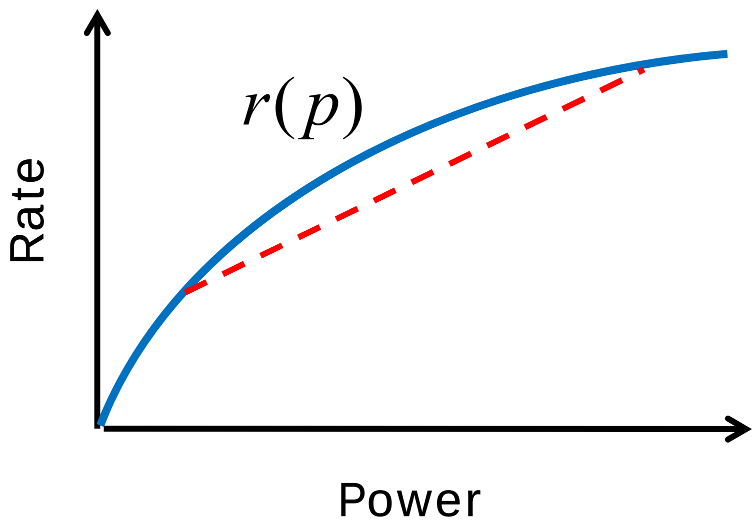


- Arrivals known **non-causally** by transmitter,
- Stored in a **finite battery** of capacity $E_{\max}$,
- Design parameter: **power** $\rightarrow$ **rate** $r(p)$.

Power-Rate Function

- **Strictly concave**

Example: AWGN Channel, $r(P) = \frac{1}{2}\log(1+\frac{P}{N})$



- Power allocation function: $p(t)$
- Energy consumed: $\int_0^T p(t)dt$
- Short-term throughput:

$$\int_0^T r(p(t))dt$$

Energy Constraints

(Energy arrivals of E_i at times s_i)

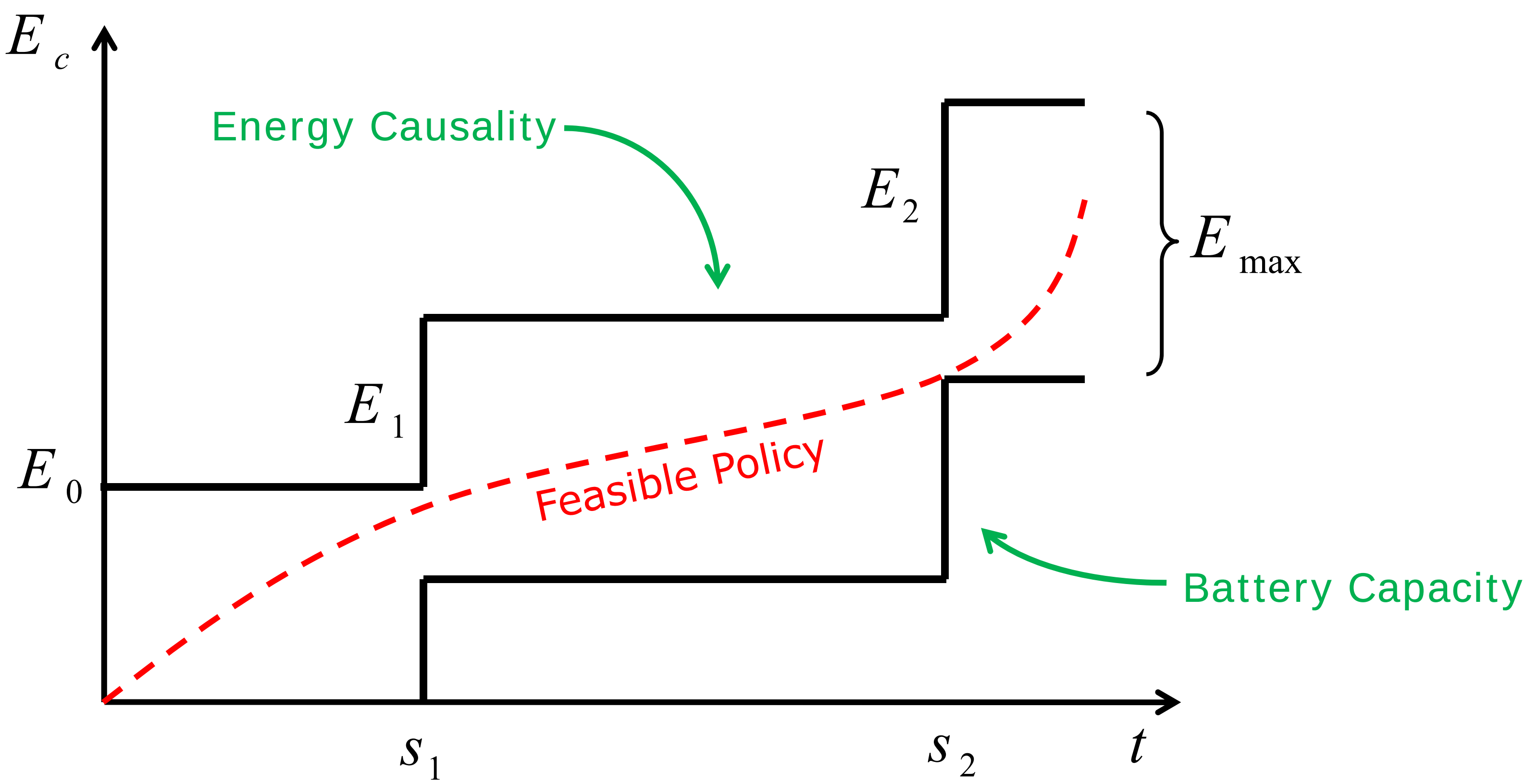
- **Energy Causality:** $\sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \geq 0 \qquad s_{n-1} \leq t' \leq s_n$

- **Battery Capacity:** $\sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{\max} \qquad s_{n-1} \leq t' \leq s_n$

- **Set of energy-feasible power allocations**

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \leq \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \leq E_{\max}, \forall n > 0, s_{n-1} \leq t' \leq s_n \right\}$$

Energy "Tunnel"



Short-Term Throughput Maximization Problem

- Maximize total number of transmitted bits by deadline **T**

$$\max_{p(t)} \int_0^T r(p(t))dt, \quad s.t. \quad p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

- Convex constraint set, concave maximization problem

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Necessary Conditions for Optimality of a transmission policy

- **Property 1:** Transmission power remains constant between arrivals.
- **Property 2:** Battery never overflows.
- **Property 3:** The change in power level at an energy arrival instant has to be non-negative (non-positive) if the battery is depleted (full) at that time instant.
- **Property 4:** Battery is depleted at the end of transmission.

Necessary Conditions for Optimality

Implications of Properties 1-4:

- **Structure of optimal policy:** (Property 1)

$$p(t) = \begin{cases} p_n & i_{n-1} < t < i_n \\ 0 & t > T \end{cases}, \qquad i_n \in \{s_n\}, \quad p_n \text{ constant}$$

- For power to increase or decrease, policy must meet the upper or lower boundary of the tunnel respectively (Property 3)
- At termination step, battery is depleted (Property 4).

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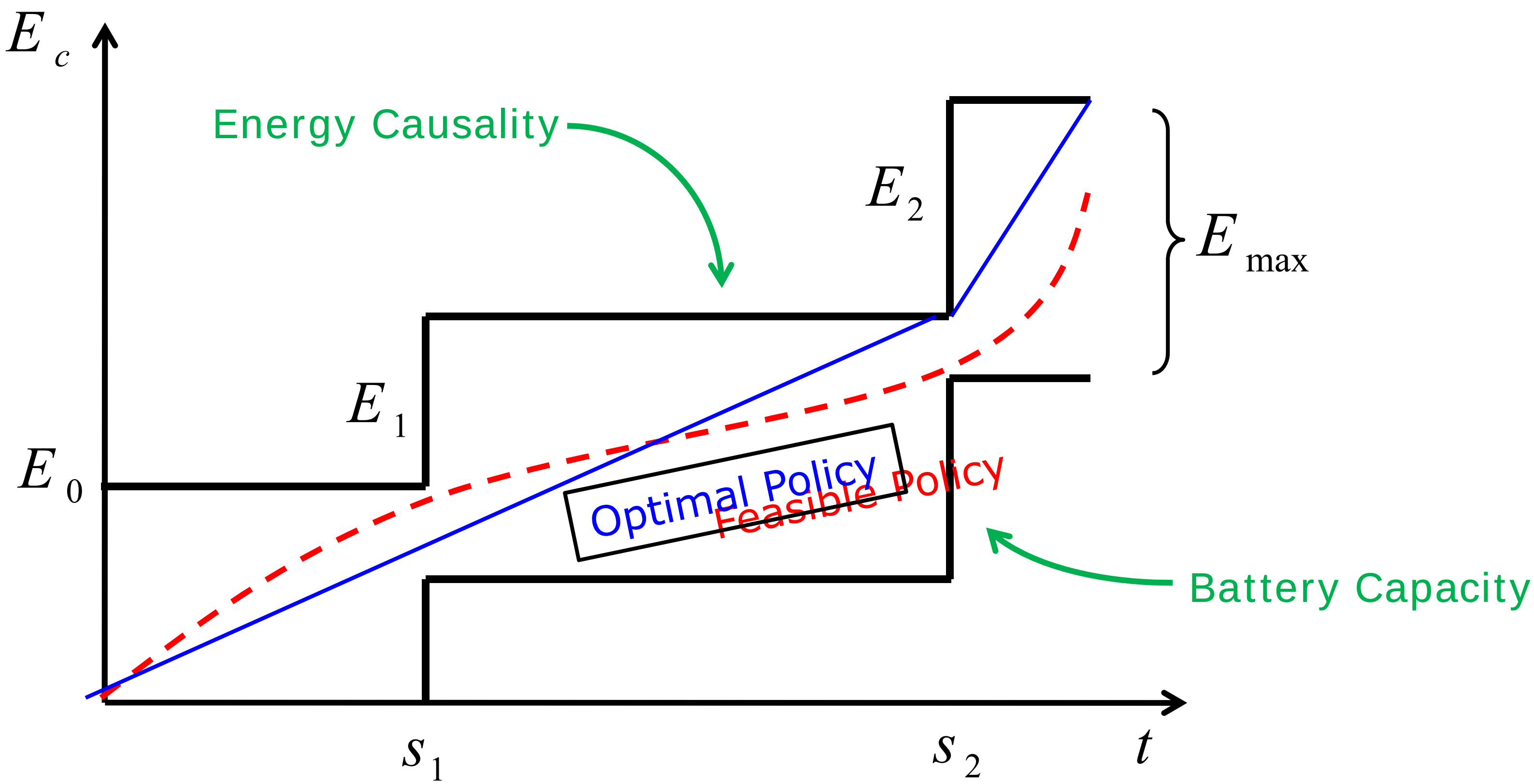
Energy “Tunnel”



Shortest Path Interpretation

- **Optimal policy is identical for any concave power-rate function!**
- Let $r(p) = -\sqrt{p^2+1}$, then the problem solved becomes:

$$\max_{p(t)} \int_0^T -\sqrt{p^2(t)+1}\,dt \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

$$= \min_{p(t)} \underbrace{\int_0^T \sqrt{p^2(t)+1}\,dt}_{} \qquad s.t.\ \ p(t) \in \mathfrak{P}$$

length of policy path in energy tunnel

> $\Rightarrow$ The **throughput maximizing policy** yields the **shortest path** through the energy tunnel for any concave power-rate function.

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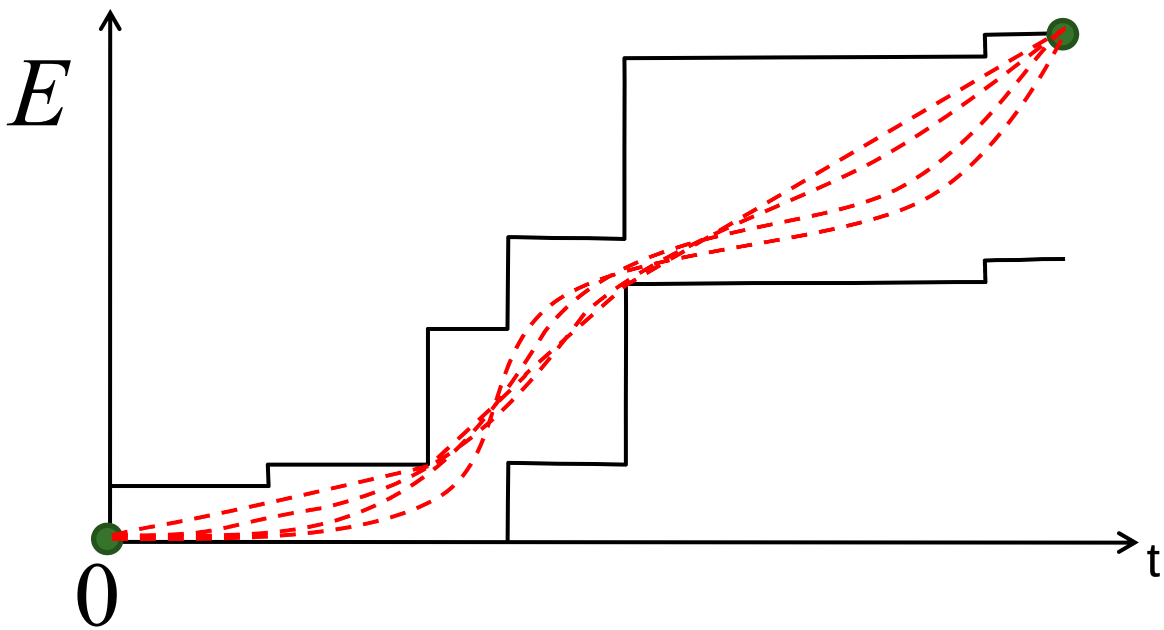
Shortest Path Interpretation

- **Property 1:** Constant power is better than any other alternative
- **Shortest path** between two points is a line (constant slope)



Throughput Maximizing Algorithm (TMA)

- Knowing the structure of the policy, we can construct an iterative algorithm to get the tightest string in the tunnel.
- Note: After a step (p_1, i_1) is determined, the rest of the policy is the solution to a ***shifted problem*** with shifted arrivals and deadline:

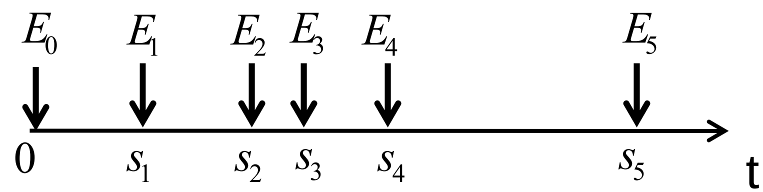
$$E_0' = \sum_{k=0}^{n_1} E_k - i_1 . p_1, \quad T' = T - i_1, \quad n'_{\max} = n_{\max} - n_1,$$

$$E_n' = E_{n+n_1}, \quad s_n' = s_{n+n_1} - i_1, \quad \text{for } n = 0, \ldots, n'_{\max}$$

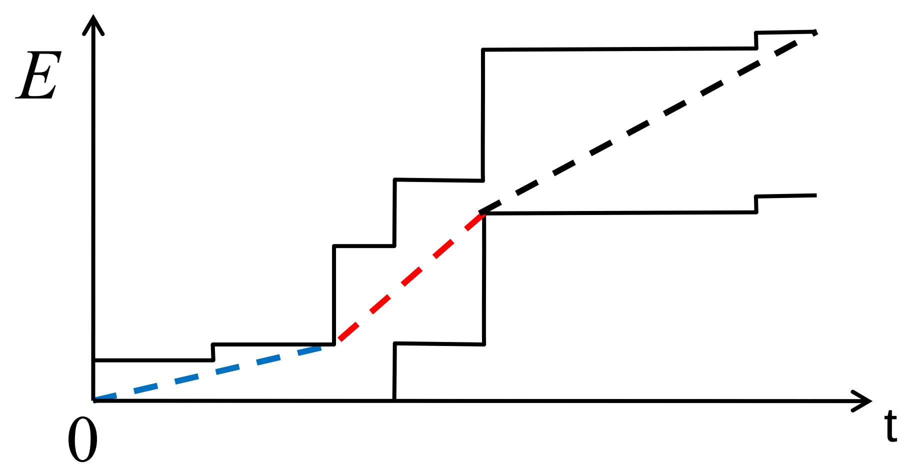
- **Essentially, the algorithm compares and find the tightest segment that hits the upper or lower wall staying feasible all along.**

Sample Run of the TMA

Energy harvesting scenario



Energy-feasible tunnel with optimal transmission policy



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Alternative Solution

- Transmission power constant within each epoch:

$$p(t) = \{ p_i \qquad t \in \text{epoch } \ i, \ i = 1,\ldots, N + M + 1 \}$$

- STTM problem expressed with above notation

$$\max_{p_i} \sum_{i=1}^{M+N+1} L_i . r(p_i)$$

(L_i: length of epoch i)

$$s.t. \quad 0 \le \sum_{i=1}^{l} E_i - L_i p_i \le E_{\max} \quad \forall l$$

Energy constraints: sufficient to check arrivals only

Water-filling approach

- Lagrangian function for STTM:

$$\max_{p_i} \sum_{i=1}^{M+N+1} L_i . r(p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right)$$

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0 \quad \forall j$$

$$\mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0 \quad \forall j$$

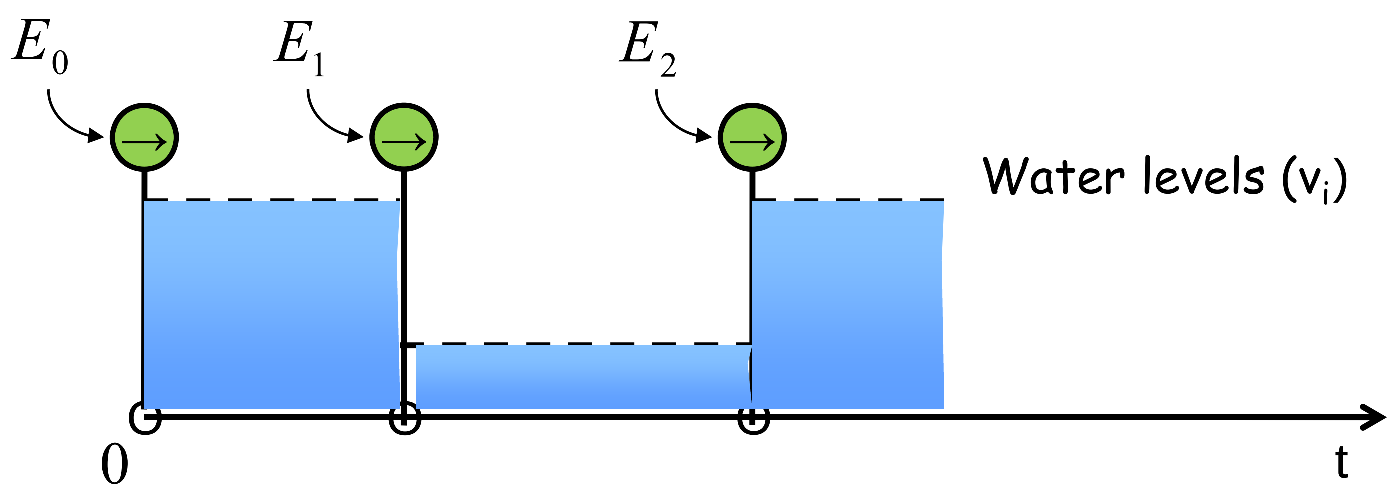
(Complementary slackness conditions)

- **Solution:** *constrained* water-filling

$$p_i^* = \frac{1}{\sum_{j=i}^{M+N+1} \lambda_j - \mu_j}$$

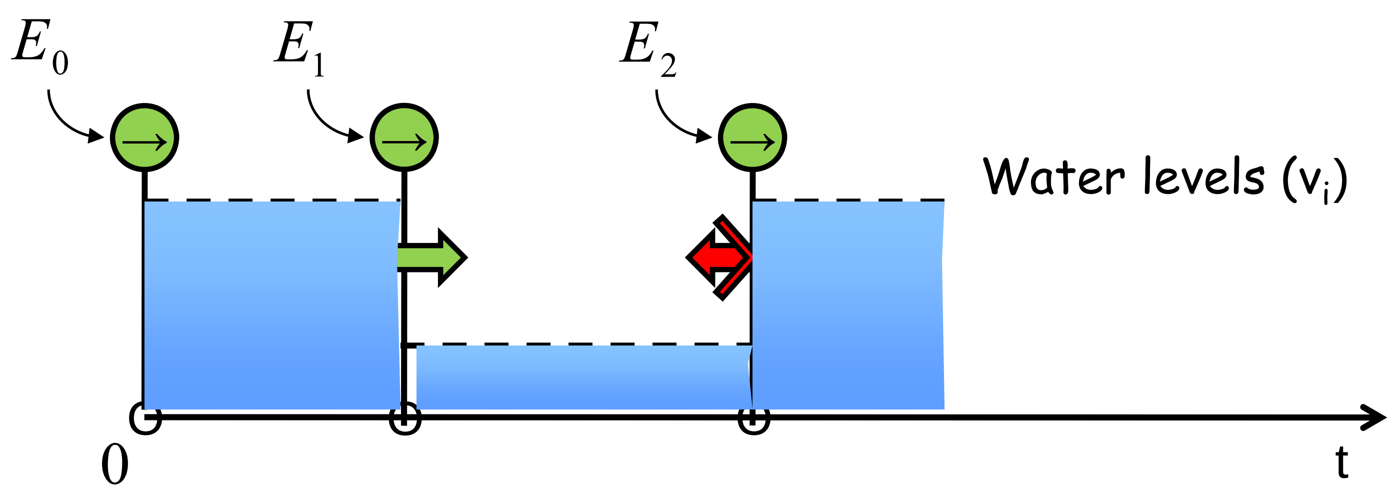
Directional Water-Filling

- Harvested energies filled into epochs individually



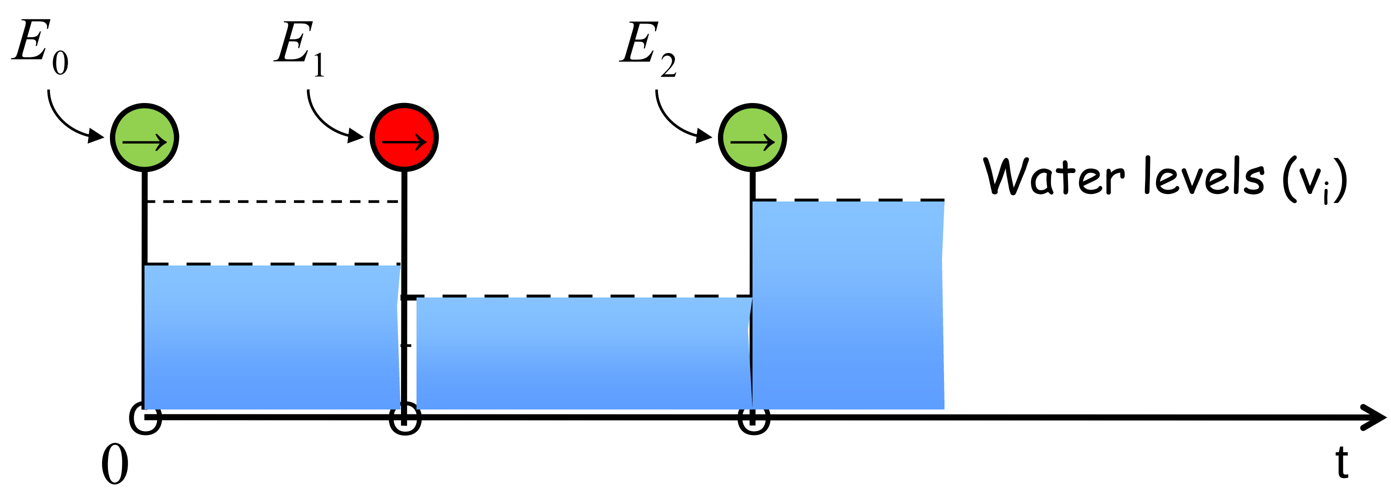
Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time



Directional Water-Filling

- Harvested energies filled into epochs individually
- Constraints:
  - **Energy Causality**: water-flow only forward in time
  - **Battery Capacity**: water-flow limited to E_{max} by taps



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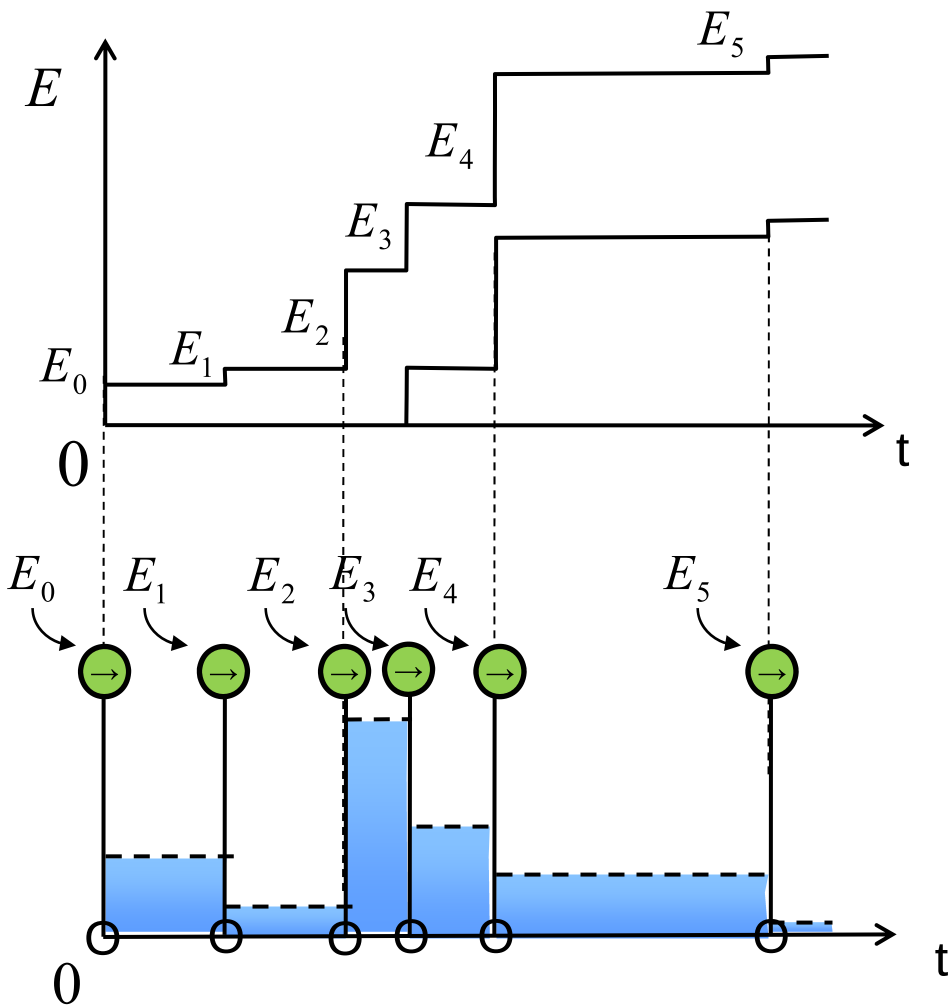
Directional Water-Filling



- Energy tunnel and directional water-filling approaches yield the same policy

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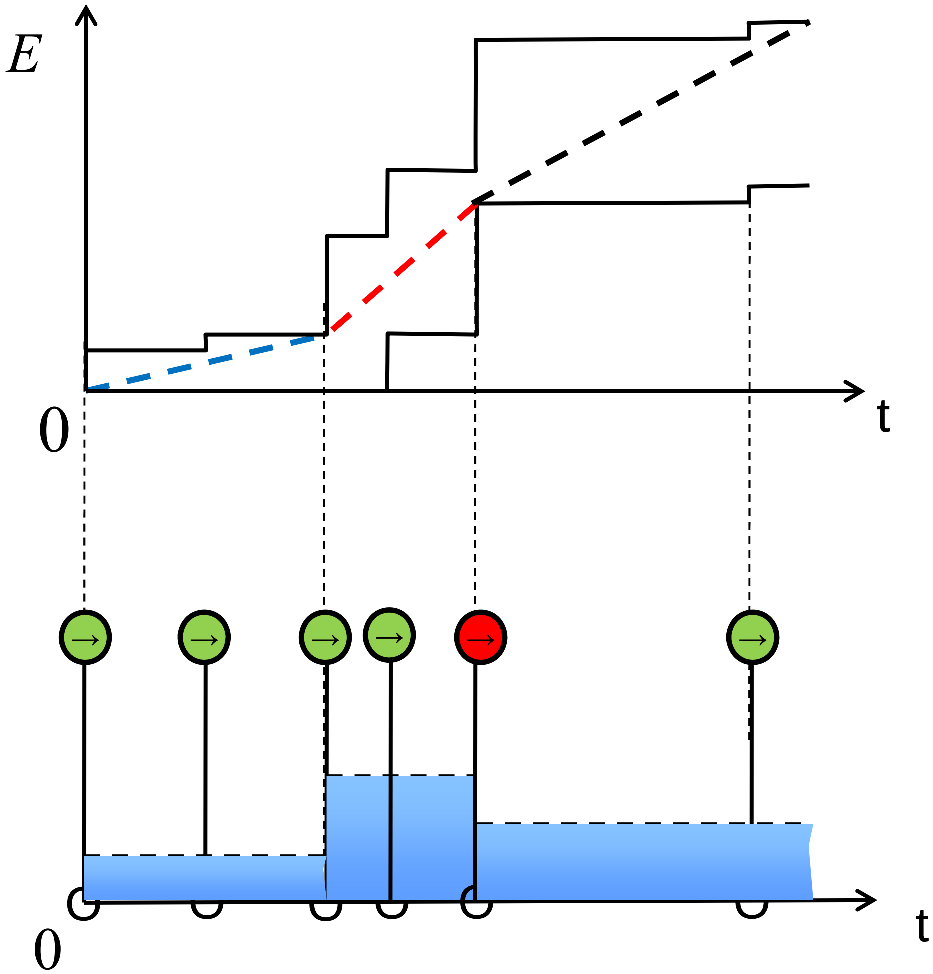
Directional Water-Filling



- **Energy tunnel** and **directional water-filling** approaches yield the same policy

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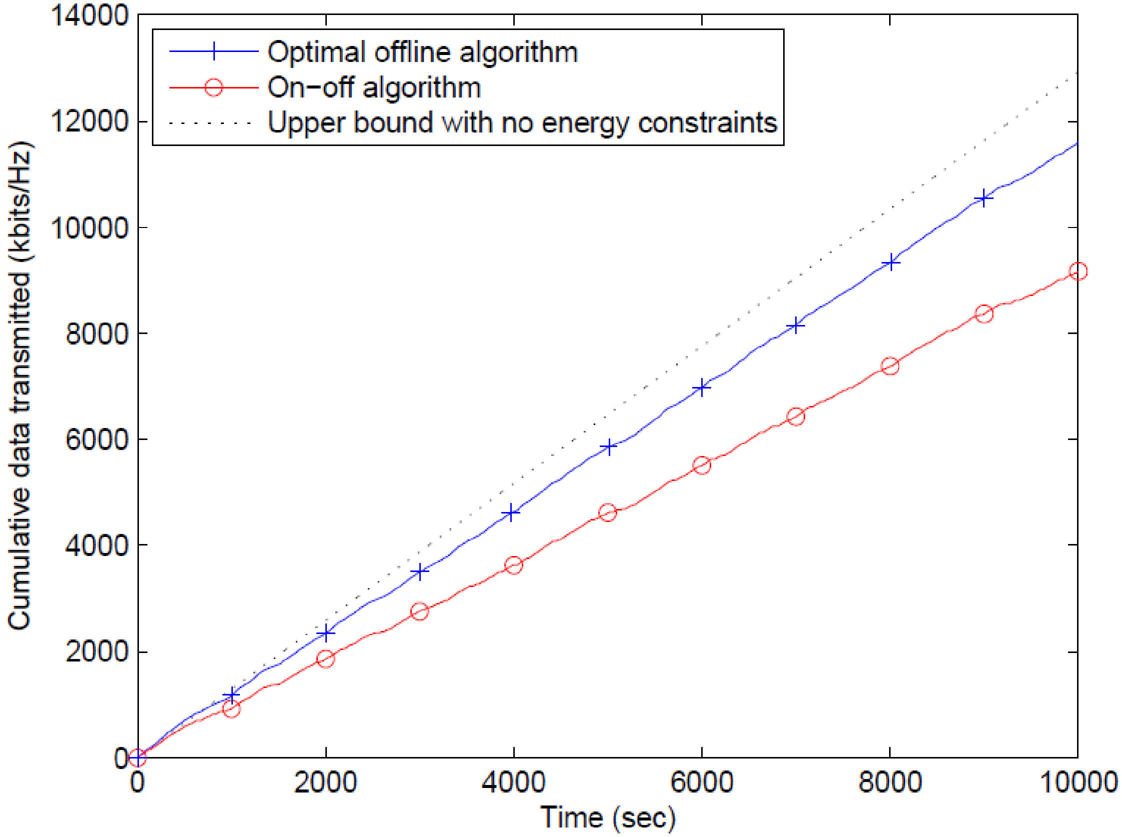
Simulation Results



- Improvement of optimal algorithm over an *on-off transmitter* in a simulation with truncated Gaussian arrivals.

2. Transmission Completion Time Minimization (TCTM)

- Given the total number of bits to send as B, finalize the transmission in the shortest time possible.

$$\min_{p(t)} T \quad s.t. \quad B - \int_0^T r(p(t))dt \le 0, \;\; p(t) \in \mathfrak{P}$$

$$\mathfrak{P} = \left\{ p(t) \,\middle|\, 0 \le \sum_{k=0}^{n-1} E_k - \int_0^{t'} p(t)dt \le E_{\max}, \forall n > 0, s_{n-1} \le t' \le s_n \right\}$$

Relationship of STTM and TCTM problems

- Lagrangian dual of TCTM problem becomes:

$$\max_{u\geq 0}\left(\min_{p(t)\in\mathfrak{P},T} T+u\left(B-\int_0^T r(p(t))dt\right)\right)$$

$$=\max_{u\geq 0}\left(\min_{T}\left(T+uB-u.\underbrace{\max_{p(t)\in\mathfrak{P}}\int_0^T r(p(t))dt}_{\textbf{STTM problem for deadline } T}\right)\right)$$

Relationship of STTM and TCTM problems

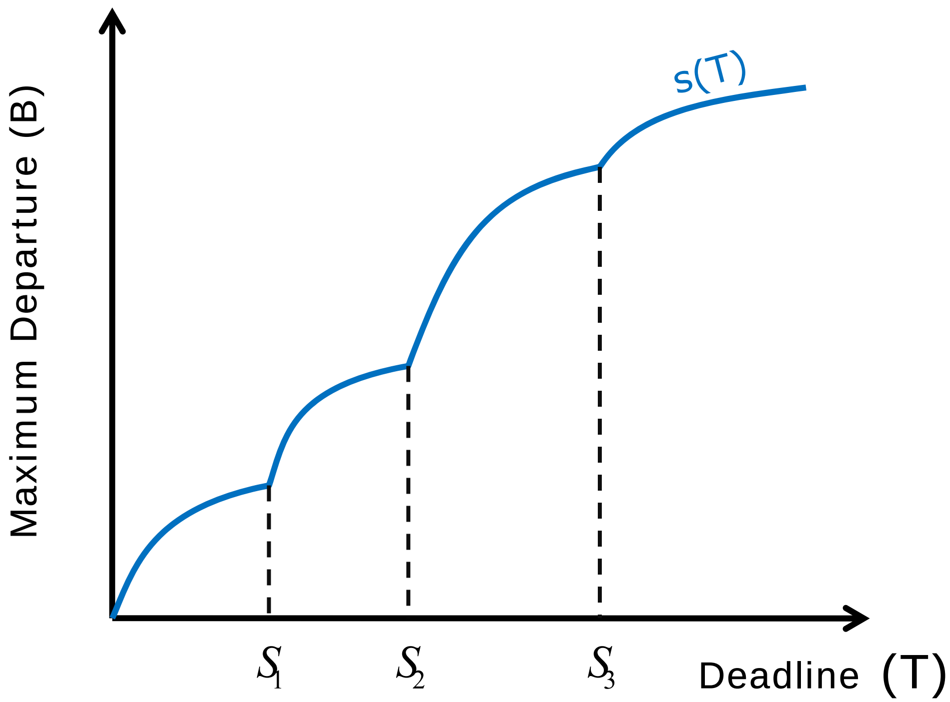
- Optimal allocations are identical:

STTM's solution for deadline T departing B bits	$\equiv$	TCTM's solution for departing B bits in time T

- STTM solution can be used to solve the TCTM problem

Maximum Service Curve

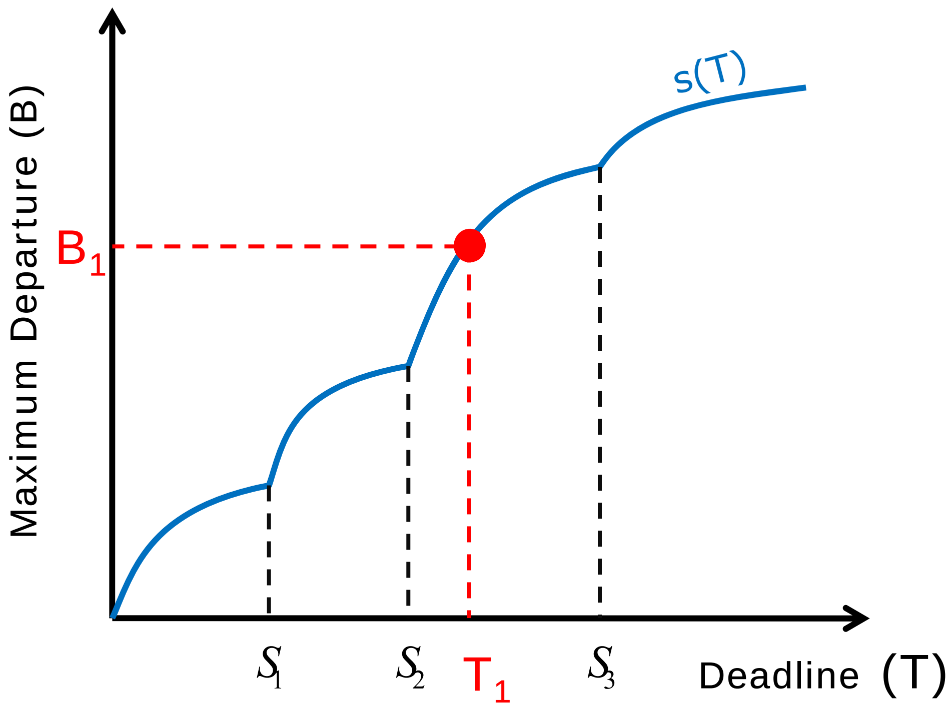
$$s(T) = \max_{p(t)} \int_0^T r(p(t))dt, \quad s.t. \quad p(t) \in \mathfrak{P}$$



- Maximum number of bits that can be sent in time T.
- Each point calculated by solving the corresponding STTM problem.

Maximum Service Curve

- Continuous, monotone increasing, **invertible**



- Optimal allocation for TCTM with B_1 bits

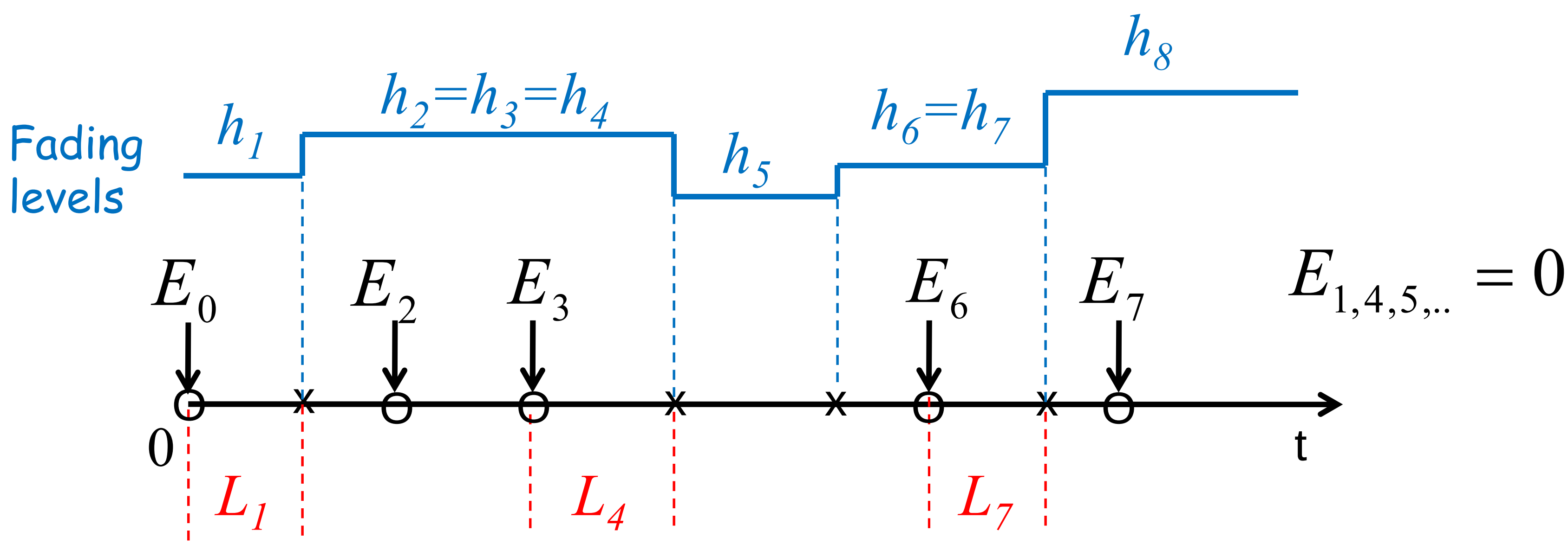
$$=$$

Optimal allocation for STTM with deadline T_1

3. Extension to Fading Channels

- Find the short-term throughput maximizing and transmission completion time minimizing power allocations in a fading channel with non-causally known channel states.

System Model



- AWGN Channel with fading h: $R(P,h) = \frac{1}{2}\log(1+h.P)$
- Each "epoch" defined as the interval between two "events".
- Fading states and harvests known **non-causally**

STTM Problem with Fading

- Transmission power constant within each epoch:

$$p(t) = \{p_i \qquad t \in \text{epoch } i, \ i = 1, \ldots, N + M + 1\}$$

- Maximize total number of transmitted bits by a deadline T

$$\max_{p_i} \sum_{i=1}^{M+N+1} \frac{L_i}{2} \log(1 + h_i p_i)$$

$$s.t. \quad 0 \le \sum_{i=1}^{l} E_i - L_i p_i \le E_{\max} \quad \forall l$$

STTM Problem with Fading

- Lagrangian of the STTM problem

$$\max_{p_i} \sum_{i=1}^{M+N+1} \frac{L_i}{2} \log(1+h_i p_i) - \sum_{j=1}^{M+N+1} \lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) - \sum_{j=1}^{M+N+1} \mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) + \sum_{i=1}^{M+N+1} \eta_i p_i$$

$$\lambda_j \left(\sum_{i=1}^{j} L_i p_i - E_i \right) = 0 \quad \forall j$$

$$\mu_j \left(\sum_{i=1}^{j} E_i - L_i p_i - E_{\max} \right) = 0 \quad \forall j$$

$$\eta_j p_j = 0 \quad \forall j$$

(Complementary slackness conditions)

- **Solution:** *constrained* water-filling with fading levels: $p_i^* = \left[v_i - \frac{1}{h_i} \right]^+, \quad v_i = \frac{1}{\sum_{j=i}^{M+N+1} \lambda_j - \mu_j}$

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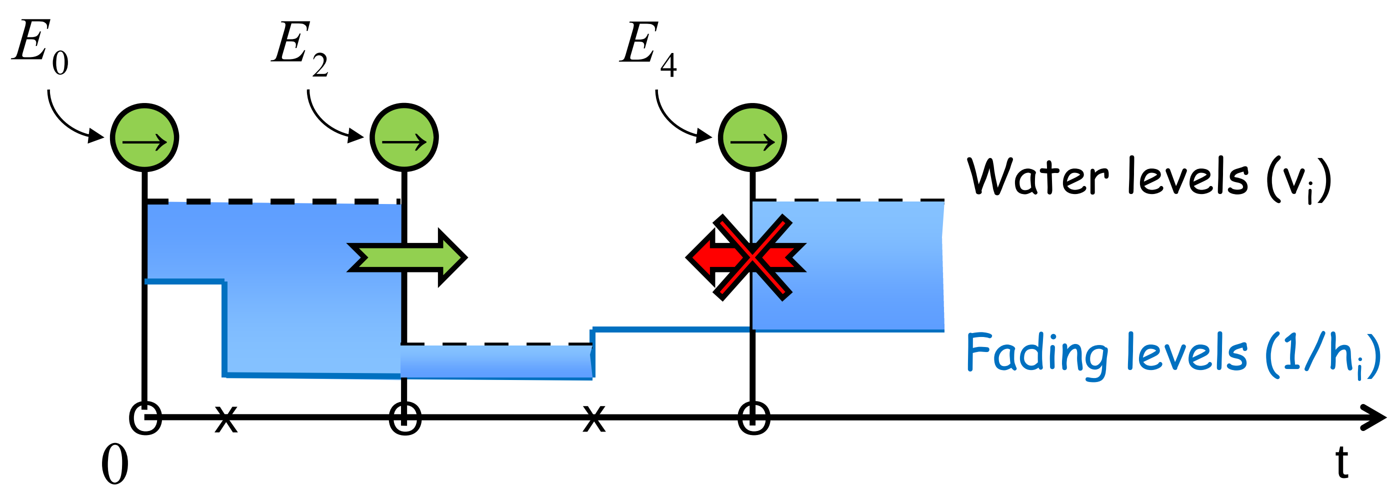
Directional Water-Filling

- Same directional water filling model with added fading levels.
  - Directional water flow **(Energy causality)**
  - Limited water flow **(Battery capacity)**



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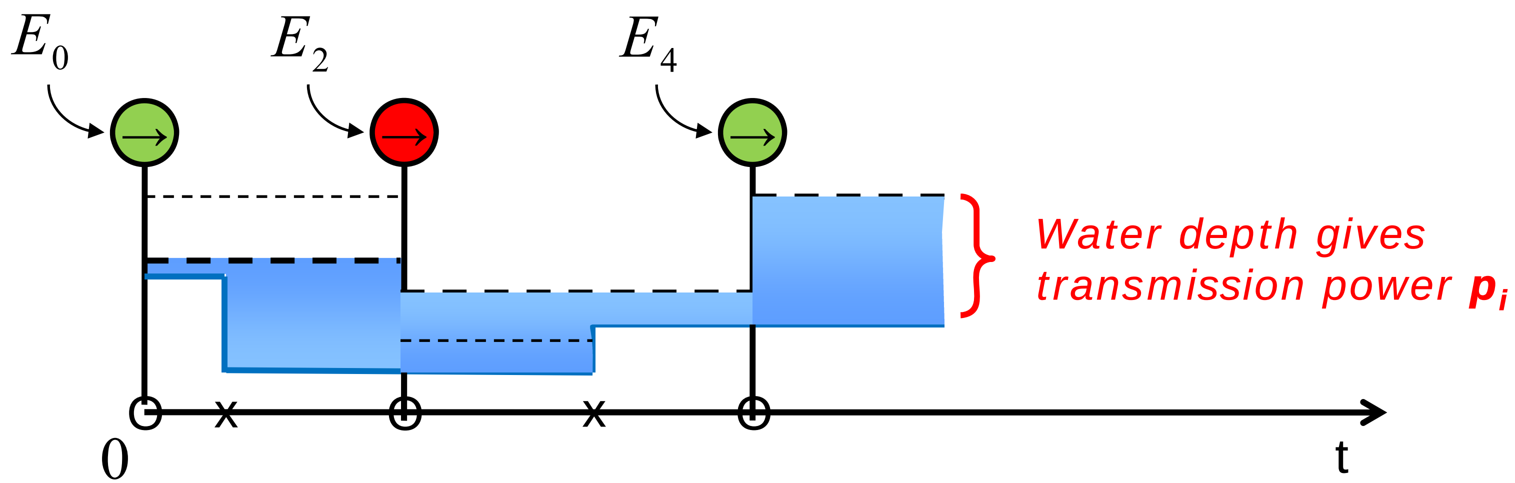

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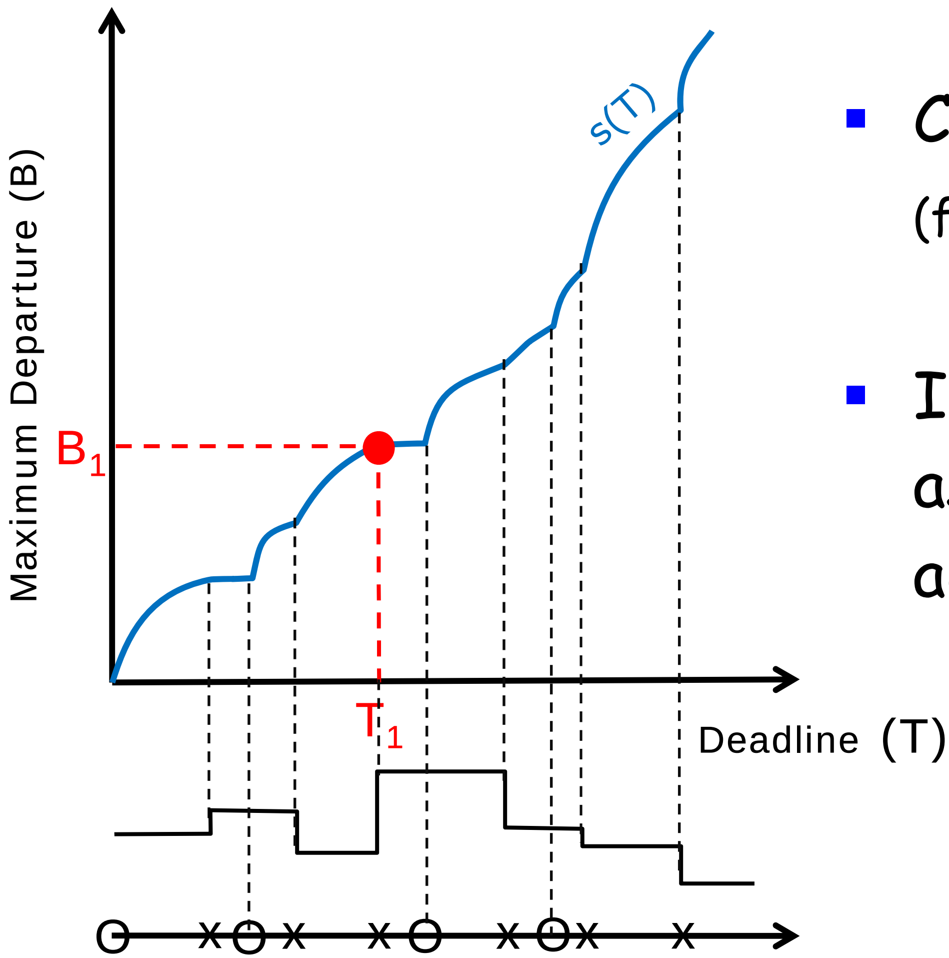
Maximum Service Curve



- Continuous, **non-decreasing**

  (flat regions when fading is severe)

- Inverse can be considered as the **smallest** T that achieves B_1

4. Online Algorithms

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Optimal online policy can be found using dynamic programming

- States of the system: fade level: $\boldsymbol{h}$, battery energy: $\boldsymbol{e}$

$$J_g(e,h,t) = E\left[\int_t^T \frac{1}{2}\log(1+h(\tau)g(e,h,\tau))d\tau\right]$$

$$J(e,h,t) = \sup_g J_g$$

- Quantizing time by δ, $g^*(e,h,k\delta)$ can be found by iteratively solving

$$\max_{g(e,h,t)}\left(\frac{\delta}{2}\log(1+h.g(e,h,t)) + J(e',h',t+\delta)\right)$$

$$\begin{pmatrix} e' = e + \delta(-g(e,h,t) + P_{avg}) \\ h' = E[h(t+\delta) \mid h(t)] \end{pmatrix}$$

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Online Algorithms

Constant Water Level

- A cutoff fading level h_0 is determined by the **average harvested power P_{avg}** as:

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh = P_{avg}$$

$f(h)$: Fading distribution

- Transmitter uses the corresponding water-filling power if available, goes silent otherwise

$$p_i = \left(\frac{1}{h_0}-\frac{1}{h_i}\right)^{+}$$

Online Algorithms

Energy Adaptive Water-Filling

- Cutoff fade level h_0 determined from **current energy** as:

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh = E_{current}$$

- Transmission power determined by water-filling expression:

$$p_i = \left(\frac{1}{h_0}-\frac{1}{h_i}\right)^{+}$$

- Sub-optimal, but requires **only** fading statistics.

Online Algorithms

Time-Energy Adaptive Water-filling

- h_0 determined by **remaining energy scaled by remaining time** as

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh=\frac{E_{current}}{T-t}$$

Hybrid Adaptive Water-filling

- h_0 determined similarly but by **adding average received power**

$$\int_{h_0}^{\infty}\left(\frac{1}{h_0}-\frac{1}{h}\right)f(h)dh=\frac{E_{current}}{T-t}+P_{avg}$$

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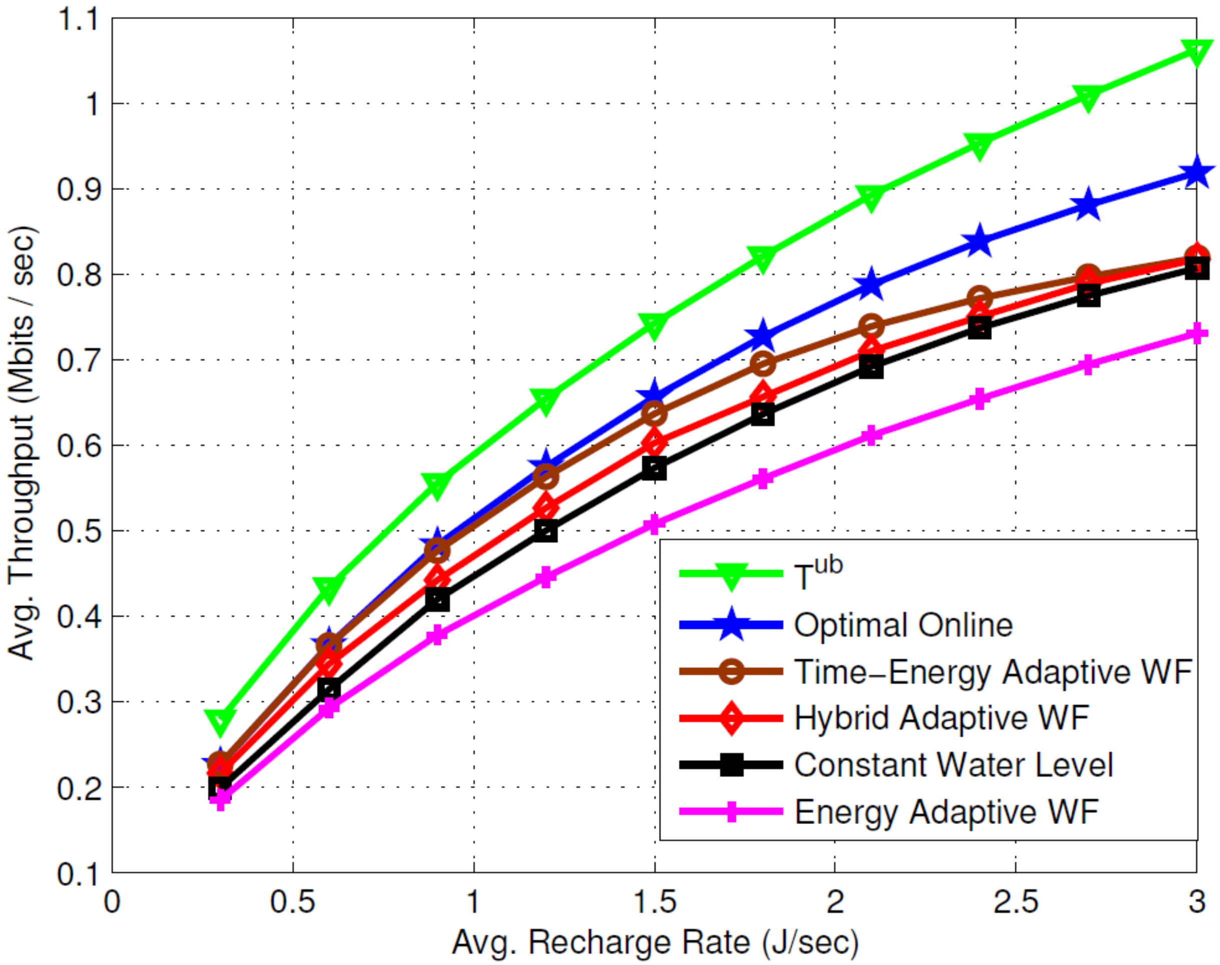
Simulations



Performances of the policies w.r.t. **energy arrival rates** under:

- unit mean Rayleigh fading
- T = 10 sec
- E_{max} = 10 J.

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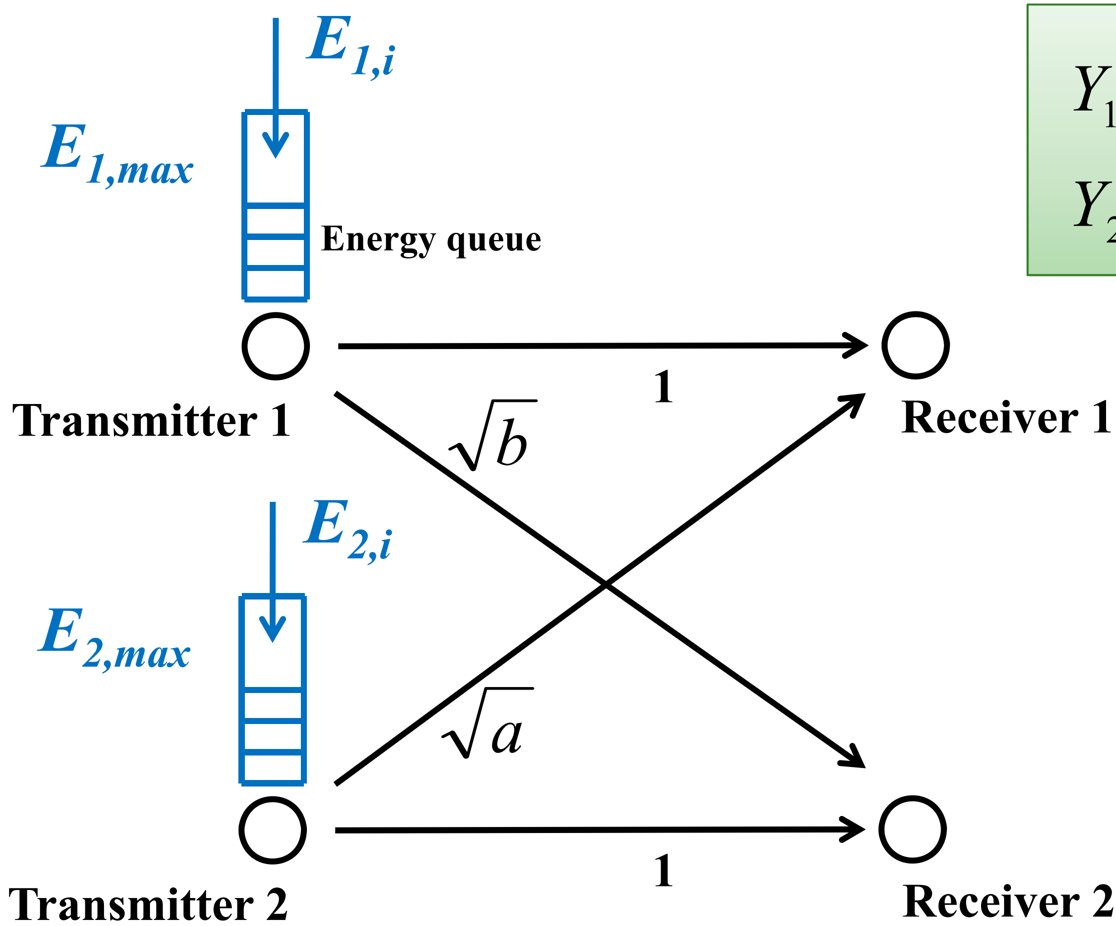
5. Gaussian IC with EH Transmitter



$$Y_1 = X_1 + \sqrt{a}X_2 + Z_1$$
$$Y_2 = \sqrt{b}X_1 + X_2 + Z_2$$

Problem Definition

- **Sum-Throughput Maximization Problem:**

Find optimal transmission power/rate policies that maximize the total amount of data transmitted to both receivers by a deadline $T=N\tau$.

$$\max_{\mathbf{p}_1 \geq 0, \mathbf{p}_2 \geq 0} \sum_{i=1}^{N} \tau \cdot r(p_{1,i}, p_{2,i})$$

$$s.t. \quad 0 \leq \sum_{i=1}^{n} E_{j,i} - \tau \cdot p_{j,i} \leq E_{j,\max}$$

$$0 \leq \sum_{i=1}^{n} B_{j,i} - \tau \cdot r_j(p_{1,i}, p_{2,i}) \quad\quad j = 1,2 \quad n = 1, \ldots, N$$

Concavity of sum-rate

- **Claim:** $r(p_1, p_2)$ is jointly concave in p_1 and p_2

Given any transmission scheme achieving a sum-rate ***r(p₁,p₂)***, one can utilize time-sharing to construct concave sum-rate:

$$r^*(p_1, p_2) = \max \left\{ \begin{array}{l} r(p_1, p_2), \\ \left\{ \begin{array}{l} \lambda \cdot r(p_1', p_2') + (1-\lambda) \cdot r(p_1'', p_2'') \\ s.t.\ \ \lambda \cdot p_j' + (1-\lambda) \cdot p_j'' = p_j,\ 0 \le \lambda \le 1,\ p_j', p_2'' \ge 0 \end{array} \right\} \end{array} \right\}$$

Alternating Maximization (Cyclic Coordinate Descent)

- Alternating maximization method among the two users converge to the optimal transmission

$$\mathbf{p}_1^{\mathbf{k}} = \arg\max_{\mathbf{p}_1 \geq 0} \sum_{i=1}^{N} \tau \cdot r(p_{1,i}, p_{2,i}^{k-1})$$

$$s.t. \quad 0 \leq \sum_{i=1}^{n} E_{1,i} - \tau \cdot p_{1,i} \leq E_{1,\max}$$

$$\mathbf{p}_2^{\mathbf{k}} = \arg\max_{\mathbf{p}_2 \geq 0} \sum_{i=1}^{N} \tau \cdot r(p_{1,i}^{k}, p_{2,i})$$

$$s.t. \quad 0 \leq \sum_{i=1}^{n} E_{2,i} - \tau \cdot p_{2,i} \leq E_{2,\max}$$

Only constraints corresponding to the optimized user are relevant

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Generalized Directional Water-Filling

- When achievable rate function $r(p)$ is arbitrary,

- **Solution:** constrained water-filling with generalized water levels: $v_i = \frac{\partial}{\partial p} r(p) \Big|_{p_i} = \sum_{j=i}^{N} (\lambda_j - \mu_j) - \eta_i$

- Arises in most known IC sum-capacity expressions

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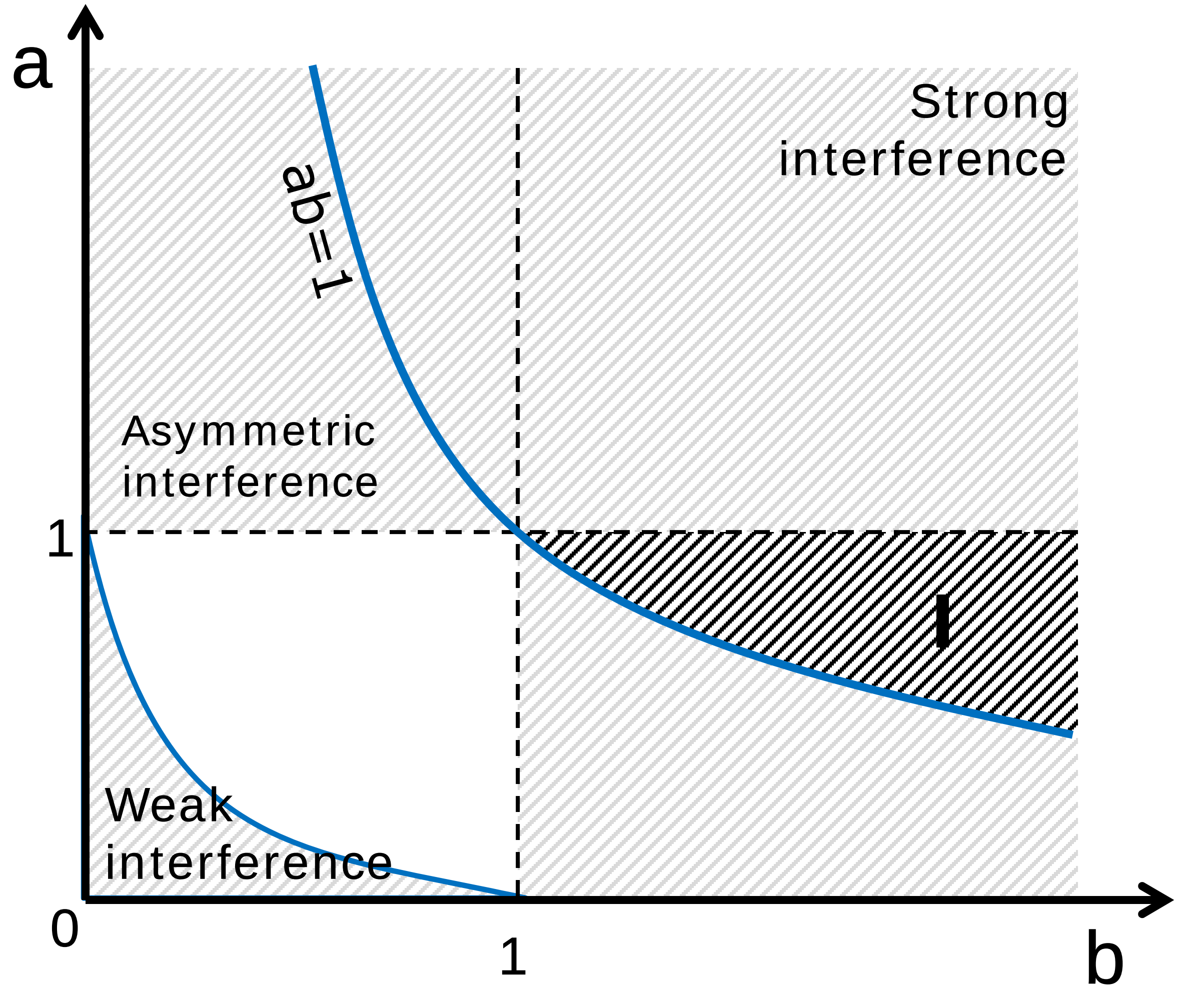
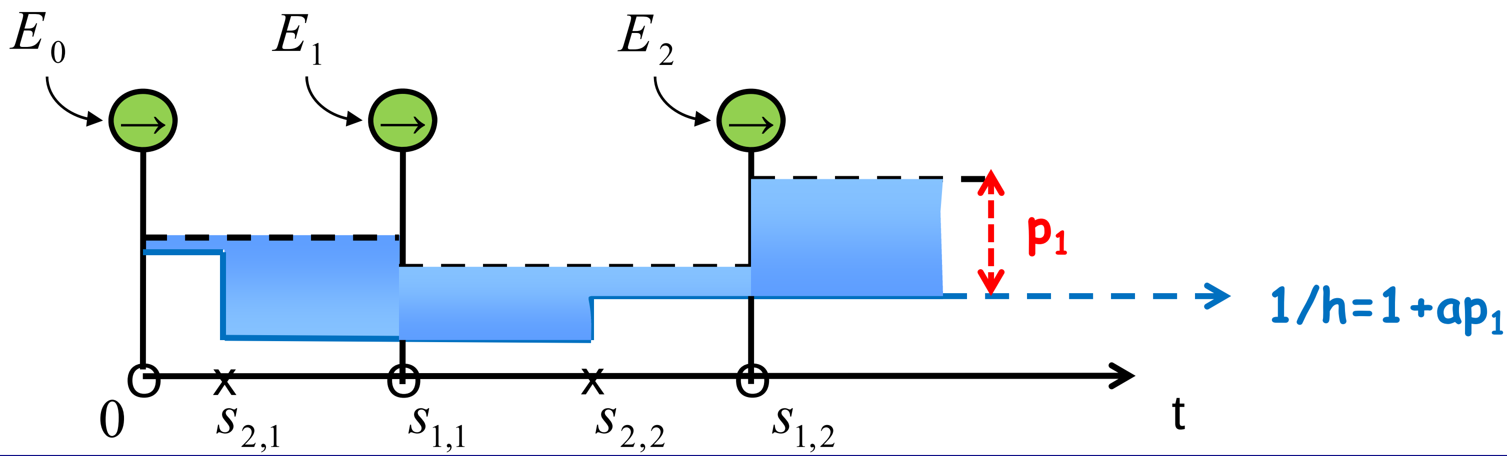
Sum-Rate for Asymmetric IC



Region I: ab>1 (a<1, b>1)

- $C_s^I = \frac{1}{2}\log\left(1+\frac{p_1}{1+ap_2}\right)+\frac{1}{2}\log\left(1+p_2\right) \qquad ab \geq 1$

- **User 1:** $C_s^I = \frac{1}{2}\log\left(1+hp_1\right)+C \qquad h=\frac{1}{1+ap_2}$
  - **Directional water-filling** with base levels as $\frac{1}{h}=1+ap_2$



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Region I: ab>1 (a<1, b>1)

- **User 2:**
  - KKT

$$\frac{d}{dp_2} C_s^I(p_2) - \sum_{j=k}^{N} \lambda_j - \sum_{j=k}^{N} \mu_j + \eta_j = 0$$

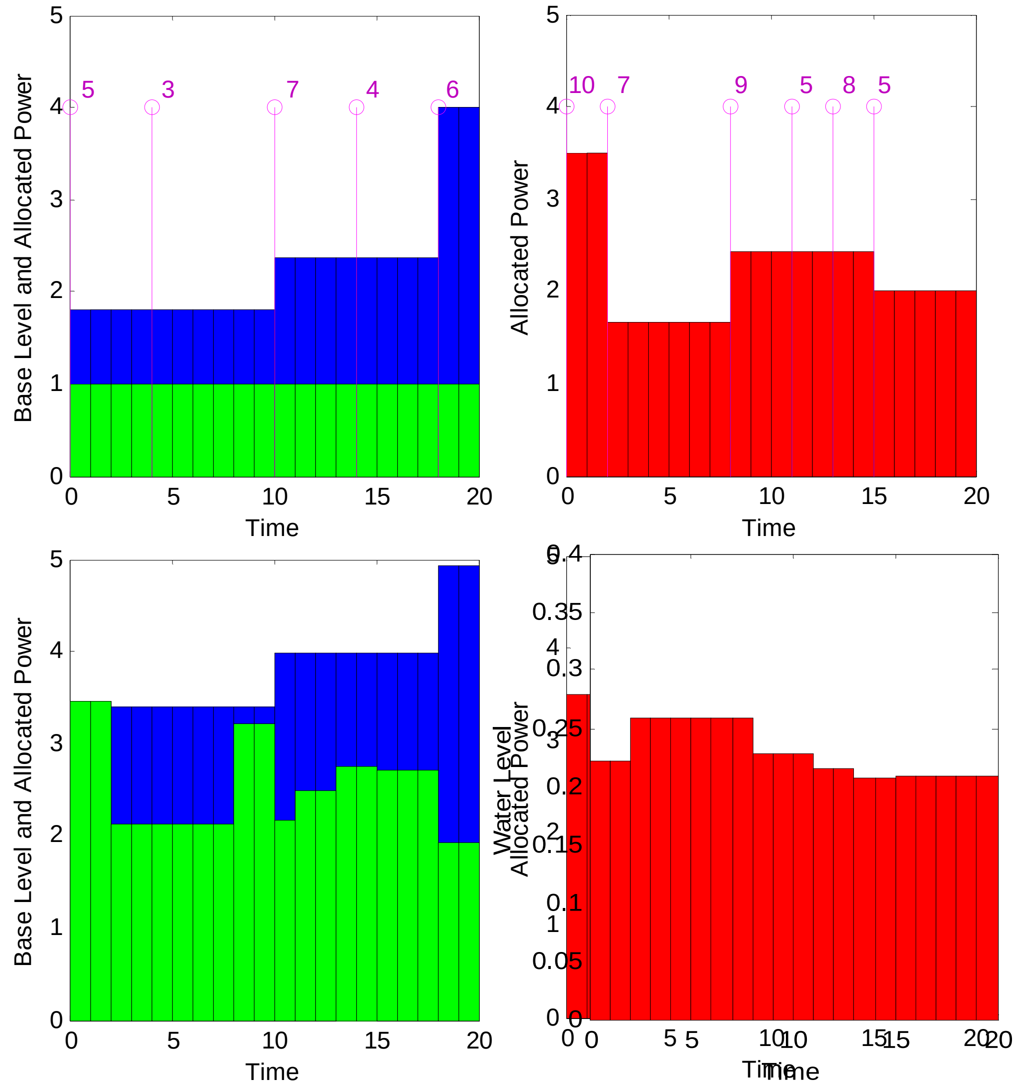
  - Complementary Slackness => directional water-filling type water level changes

  - **Generalized directional water-filling** with water levels:

$$\frac{d}{dp_2} C_s^I(p_2) = \frac{-ap_1}{2(1+p_1+ap_2)(1+ap_2)} + \frac{1}{2(1+p_2)}$$

Simulations

- Energy arrivals: magenta
- Base levels: green,
- Optimal p_1 significantly differs from the single user level.
- This in return affects the optimal p_2 found using generalized water-filling.



Conclusion

- New paradigm: Networking with energy harvesting nodes

- New design insights arising from new energy constraints

- In this presentation, we covered
  - Optimal scheduling policies for a single transmitter,
  - Directional water-filling for fading channels,
  - Extension to the Interference Channel.

Future Directions and Open Problems

- Information theoretic limits, optimal coding schemes for energy harvesters
- Energy harvesting relays, receivers,...
- Efficient online algorithms, simple practical implementations