Poincaré's Observation and the Origin of Tsallis Generalized Canonical Distributions

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In their paper [1], Plastino and Plastino have shown how Tsallis canonical distributions can be derived from the assumption that the system is in contact with a finite thermal bath: this result represented a major advance in the study of Tsallis statistics, since it replaces the infiniteness of the thermal bath - leading to the classical Boltzmann-Gibbs statistics - by the more physical finiteness assumption. In this paper, we show that an alternate approach, based on Poincaré's observation, can be used to obtain the same result. Poincaré's observation states that a $k-$variate Boltzmann-Gibbs distribution can be obtained as the marginal distribution of a uniform distribution on the sphere $S_n = \{x \in \mathbb{R}^n | \|x\| = \sqrt{n}\}$ as $n \to +\infty$.

Our contribution is based on the remark that Tsallis $q-$distributions (distributions that maximize Tsallis $q-$entropy under covariance constraint) can be obtained as marginals of a uniform distribution on the *finite* dimensional sphere S_n provided that, with $q > 1$ or $q \leq 0$: $n = k + 2q/(q-1)$. Thus the finiteness of the underlying sphere parallels the finiteness of the thermal bath, but contrarily to the approach in [1], we do not need here any power-law assumption. Moreover, we show how this approach can be extended to the case $q < 1$: the marginal distributions (orthogonal projections) need to be replaced by the gnomonic projections of the uniform distribution. One advantage of this approach is that it gives a physical meaning to the nonextensivity parameter q in terms of dimensions n and k. Another advantage is that it expresses any Tsallis canonical distributions as a very simple by-product - a projection - of the most natural possible distribution - the uniform distribution: this is a strong and new argument in favor of ubiquity of Tsallis statistics.

Moreover, we extend this result to more a general class of even moments constraints: if the set of constraints is $\{E[X_i^{2p_i}] = \theta_i, p_i \in \mathbb{N}\}$, then the Tsallis distributions with $q > 1$ are marginals of a uniform distribution on an hyperellipsoid $H_n = \{x \in \mathbb{R}^n | \sum_{i=1}^n x_i^{2p_i}/\theta_i = \sqrt{n}\}$. At last, we prove that Tsallis entropy is (up to a monotonous function) the only functional whose minimizers under a set of constraint are the marginals of a uniform distribution on a surface directly constructed using these constraints.

[1] A.R. Plastino, A. Plastino, *Phys. Lett.* **A193** (1994) 140-143