Microwave measurements of the complex permittivity of construction materials using Fresnel reflection coefficients and reflection ellipsometry

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Abstract

In this paper, we compare two free-space reflection methods dedicated to the determination of the complex permittivity of materials at microwave frequencies. Original numerical methods, based on constant value lines charts and nonlinear least squares optimization, have been developed for each approach. Results are given in the case of homogeneous, single layer building slabs of finite and infinite thicknesses.

Introduction

We present a new free-space measurement approach and its associated experimental setup based on reflection measurements to determine the complex permittivity of building samples [1]. This setup can be used in two configurations, depending if the reflected power is measured as a function of the angle of incidence (Fresnel method) or versus the axial rotation angle of the receiving antenna (ellipsometry) [1-2]. The main difficulty associated with the Fresnel method is that, as the angle of incidence varies, the contribution of the line-of-sight wave path to the overall field received by the antenna changes as well. Ellipsometry consists in measuring the proper characteristics of the ellipse of polarization (excentricity, azimuth angle) of the reflected electric field as a function of the analyzing angle of the receiving antenna. The complex permittivity can then be determined from these elliptical characteristics. Specifical numerical approaches associated with the two experimental configurations and based on nonlinear-least-squares methods, have been developed in order to find the estimate of the complex permittivity corresponding at best to the data. In this paper, we compare estimates of the complex permittivity of common building materials provided by both methods.

Theoretical developments

A. The Fresnel method

In the case of a single layer sample (thickness), the Fresnel formulae relate the electric reflected field to the incident field for the two polarizations as follows [3] :

$$\tilde{r}(f,\theta_i) = \frac{1-\exp(-j\beta)}{1-\tilde{r}'\exp(-j2\beta)}\tilde{r}' \qquad (1)$$

where : $\beta = k_0 e\sqrt{\tilde{\varepsilon}_r - \sin^2\theta_i}$ is the complex propagation factor through the sample slab, $k_0 = 2\pi/\lambda_0$ is the free wave number of the radiation, λ_0 is the free space wavelength, $\tilde{\varepsilon}_r = \varepsilon_r' - j\varepsilon_r''$ is the relative complex permittivity of the dielectric sample, and $\tilde{r}'$ is the reflection coefficient of the air-medium interface.

Choosing $\tilde{r}' = \tilde{r}_p$ (respectively $\tilde{r}' = \tilde{r}_s$) in formula (1) where $\tilde{r}_p$ and $\tilde{r}_s$ are given by :

$$\tilde{r}_p = \frac{\sqrt{\tilde{\varepsilon}_r - \sin^2\theta_i} - \tilde{\varepsilon}_r \cos\theta_i}{\tilde{\varepsilon}_r \cos\theta_i + \sqrt{\tilde{\varepsilon}_r - \sin^2\theta_i}} \qquad (2a)$$

$$\tilde{r}_s = \frac{\cos\theta_i - \sqrt{\tilde{\varepsilon}_r - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\tilde{\varepsilon}_r - \sin^2\theta_i}} \qquad (2b)$$

yields the Fresnel reflection coefficients corresponding to the parallel (respectively perpendicular) polarization.

In the Fresnel method, the reflected power, for both polarizations, is recorded by the receiving antenna as a function of the angle of incidence θ_i. The corresponding Fresnel coefficients are deduced by substracting, from each measurement, the power detected by the antennas when they are placed in the line-of-sight configuration, at a distance equal to twice the distance between the antenna and the spot of reflection.

To estimate the complex permittivity from the measurements, we have developped a multi-step method which consists in locating a first estimate of $\tilde{\varepsilon}$, say $\tilde{\varepsilon}_0$, on the chart of constant $\theta_{Brewster}$ and constant $|\tilde{r}_p / \tilde{r}_s|^2$ value lines (see Fig. 1). Then, from an estimation of the uncertainties associated with the measurement of θ_B and $|\tilde{r}_p / \tilde{r}_s|^2$, a parallelepipedic uncertainty area centered on $\tilde{\varepsilon}_0$ is deduced. Several values of $\tilde{\varepsilon}$ are regularly sampling in this area ; among the curves of $|\tilde{r}_p|^2$ and $|\tilde{r}_s|^2$ as a function of θ_i associated to each $\tilde{\varepsilon}$, the best one approximating the measurement data (in a least square sense) is selected. Its corresponding parameter $\tilde{\varepsilon}_{opt}$ is adopted as the best estimated value for $\tilde{\varepsilon}$.

B. The reflection ellipsometry model

In this case, once the angle of incidence is fixed, the reflected power is recorded as a function $I_d = f(A)$ of the axial rotation angle A of the receiving antenna (called the analyzer) [4]. For simplicity, the position of the transmitting antenna is fixed to an angle $P = \pm\pi / 4$. In this case, the intensity of the electric field detected versus the analyzis angle A writes simply as :

$$\begin{aligned} I_d = I\,[1 - \cos(2A)\cos(2\psi_r) \\ \pm \sin(2A)\sin(2\psi_r)\cos(\Delta_r)] \end{aligned} \qquad (3)$$

where I is a constant intensity, and angles ψ_r and Δ_r denote the changes of amplitude and phase experienced upon reflection by the parallel and perpendicular components of the electric field. The determination of $\tilde{\varepsilon}$ requires the estimation of $\tilde{\rho}$ defined as the ration of the complex Fresnel reflection coefficients for the (p) and (s) polarizations :

$$\tilde{\rho} = \frac{\tilde{r}_p}{\tilde{r}_s} = \tan(\psi_r)\, e^{j\Delta_r} \qquad (4)$$

where Δ_r is the difference between the phases associated with $\tilde{r}_p$ and $\tilde{r}_s$ .For that purpose, two relevant parameters must be extracted from the measured curve $I_d = f(A)$, namely the angle α corresponding to the position A of the maximal intensity I_{max} of the curve, and the ratio (I_{min} / I_{max}) of the extremal values of I_d. These parameters allow to retreive the complex permittivity according to the following steps:

1) defining :

$$\tan\chi = \sqrt{\frac{I_{min}}{I_{max}}} \qquad (5)$$

the parameters ψ_r and Δ_r can be deduced from the angles α and χ according to the two fundamental relations of ellipsometry :

$$\tan 2\alpha = \tan 2\psi_r \cos\Delta_r \qquad (6)$$

$$\sin 2\chi = \sin 2\psi_r \sin\Delta_r \qquad (7)$$

2) knowledge of the angles ψ_r and Δ_r allows to compute $\tilde{\rho}$ according to relation (4)
3) $\tilde{\varepsilon}$ can be estimated analytically from $\tilde{\rho}$.

To determine the complex permittivity from the measurements, we have developped a multi-step method which consists in measuring the values of I_{max} and α to provide a first guess $\tilde{\varepsilon}_0 = (\varepsilon_0', \varepsilon_0'')$ of the complex permittivitty with a constant line chart (Fig. 2). Extracting the I_{min} value from the theoretical curve $I_d = f(A)$ associated with $\tilde{\varepsilon}_0$, couples of real and imaginary permittivities ($\varepsilon', \varepsilon''$) are regularly sampled in the uncertainty parallelepipedic area centered on $\tilde{\varepsilon}_0$ (Fig. 3). The best estimate of $\tilde{\varepsilon}$ is retained as the one corresponding to the curve fitting at best the measured data points, according to a least-square criterion.

Experimental setup and results

The experimental setup, which allows several methods of measuring the reflected waves, namely the Fresnel method or ellipsometry, is depicted on Fig. 4. A bistatic configuration is considered, which consists of transmitting and receiving horn antennas. We have adopted two experimental conditions, associated with the Fresnel and the ellipsometry methods, in which the antenna-reflection spot distance is 1 m at frequency 10 GHz or 0.6 m at frequency 8.5 GHz respectively, in order to allow the best trade-offs in the measurement process of the reflected waves.

Measurements have been made for usual single-layer construction materials with minimum size of 2 m x 1 m: a concrete wall ($e = 19.5\ cm$) and a fibreboard ($e = 10\ mm$ and $16\ mm$). The maximal uncertainties relative to the measured power I_d and the analyzer angle A or the angle of incidence θ_i are respectively estimated to $\pm 0.4\ dB$ and $\pm 0.5°$.

A. Concrete wall (thick sample)

The reflection coefficients, and their ratio, as measured by the Fresnel method, are plotted as a function of the angle of incidence on Fig. 5. Using the nonlinear least squares approach to fit either jointly the two reflection coefficients, or their ratio, yields estimated permittivities equal respectively to $\tilde{\varepsilon}_r = 3.55(\pm 0.29) - 1.19(\pm 0.13)j$ and $\tilde{\varepsilon}_r = 3.47(\pm 0.29) - 1.23(\pm 0.13)j$.

In the case of the ellipsometry method, with $\theta_i = 45°$, the measured data follow almost exactly a sinus curve, as shown on Fig. 5. The theoretical curve fitting at best the data corresponds to an estimated complex permittivity $\tilde{\varepsilon}_r = 3.49(\pm 0.53) - 1.94(\pm 0.53)j$.

B. Fibreboard (thin sample)

Because the thicknesses of the two different fibreboard samples are small compared to the wavelength ($\lambda_0 = 3.5\ cm$ at 8.5 GHz), absorbing materials have been placed behind the samples. In the case of the Fresnel method, the estimated permittivities are, for the 10 mm and 16 mm slabs, $\tilde{\varepsilon}_r = 2.77(\pm 0.26) - 0.01(\pm 0.18)j$ and $\tilde{\varepsilon}_r = 2.14(\pm 0.19) - 1.08(\pm 0.13)j$ respectively.

In the case of ellipsometry, the experimental curves, as shown on Fig. 6, yield to the following estimated permittivities for the 10 mm and 16 mm slab (with $\theta_i = 35°$ and $\theta_i = 45°$): $\tilde{\varepsilon}_r = 2.91(\pm 0.43) - 1.15(\pm 0.45)j$ and $\tilde{\varepsilon}_r = 2(\pm 0.39) - 1.7(\pm 0.31)j$ respectively.

As a conclusion, from the numerical point of view, the choice of the optimization method to determine the best fitting theoretical curve to the data is much more critical with the Fresnel approach. Concerning the results, the estimated real permittivities issued from both methods agree satisfactorily. The deviation between estimates of the imaginary permittivity is inherent to the free-space feature of these methods. The estimated uncertainties associated with ellipsometry appear sligthly greater than the Fresnel ones, but the Fresnel approach cannot be exploited on the whole range of angles of incidence, because of the Fresnel zone whose dimensions vary with θ_i.

As a conclusion, ellipsometry appears as a new promissing method that competes favorably with the Fresnel approach.

References

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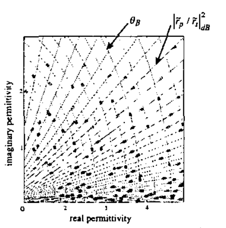

Fig. 1. Chart of constant θ_s and constant $|\tilde{r}_p / \tilde{r}_s|^2$ value lines at 10 GHz for an infinite thickness sample according to the Fresnel method ($\tilde{\varepsilon} = 3.5 - j$)

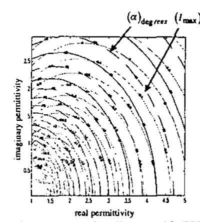

Fig. 2 Chart of constant value lines at 10 GHz - I_{max} (dB) and α - for an infinite thickness sample according to ellipsometry ($\theta_i = 45°$)

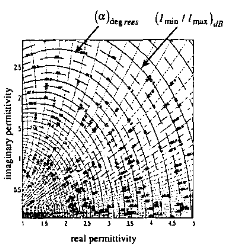

Fig. 3 Chart of constant value lines at 10 GHz - I_{min} / I_{max} (dB) and α - for an infinite thickness sample according to ellipsometry ($\theta_i = 45°$)

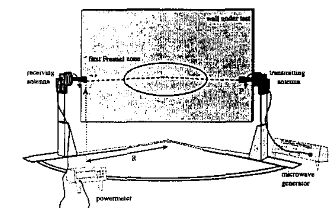

Fig. 4 Setup for the measurement of reflected waves

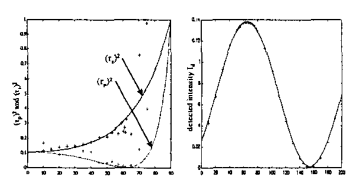

Fig. 5 Measurement data and fitting for concrete wall of infinite thickness using the Fresnel approach and ellipsometry ($\theta_i = 45°$)

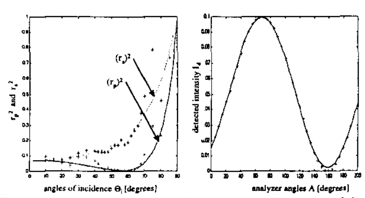

Fig. 6 Measurement data and fitting for a fibreboard slab (16 mm) using the Fresnel approach and ellipsometry ($\theta_i = 45°$)