Blind Equalization in the presence of jammers and unknown noise. Solutions based on second-order cyclostationary statistics [1,2].

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Abstract

This paper addresses the blind identification of a linear time invariant channel, using some second-order cyclostationnary statistics. In contrast to other contributions, the case where the second-order statistics of the noise and of the jammers are totally unknown is considered. It is shown that the channel can be identified consistently by adapting the so-called subspace method of Moulines *et al.*: this adaptation is valid for Fractionally Spaced systems and, more interestingly, for the general systems exhibiting Transmitter Induced Cyclostationnarity introduced by Tsatsanis and Giannakis. The new subspace method is based in both cases on a common tool, *i.e.,* a general spectral factorization algorithm. The identifiability conditions are specified and some simulation examples are given.

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1 Introduction

Under standard hypotheses (linear modulation, time-invariant channel), the complex envelope of the (noise-free) received signal in a digital communication context is[1] $x_a(t) = \sum_{k\in\mathbb{Z}} s(k)h_a(t-kT)$, where $\{s(n)\}$ is the sequence of symbols, T the symbol period, and $h_a(.)$ a composite time-limited causal mapping - the support of which is $[0, M_aT]$, say - accounting for shaping, the multi-path effects and the reception filter. It is easily checked that $x_a(t)$ can be rewritten as

$$x_a(t) = \sum_{k\in\mathbb{Z}} \tilde{s}(n)h_a(t-k\frac{T}{q}) \tag{1.1}$$

where $q \geq 1$ is any integer, and $\{\tilde{s}(n)\}$ is the so-called zero-padding sequence at the rate $\frac{q}{T}$, given by $\tilde{s}(qn) = s(n)$ and $\tilde{s}(nq+k) = 0$ for $k = 1\cdots q-1$. Hence, the expression of the sampled version of $x_a(t)$ at the rate $\frac{q}{T}$ is

$$x(n) = [h(z)].\tilde{s}(n) \tag{1.2}$$

where $h(z) = \sum_{k=0}^{qM_a} h_k z^{-k}$, and $h_k = h_a(k\frac{T}{q})$ (without any restriction, we have imposed qM_a to be an integer). Thus the sampled observation is the output of an unknown finite impulse response filter $h(z)$ of degree qM_a driven by the sequence $\{\tilde{s}(n)\}$, leading to Inter-Symbol-Interference (ISI). It is of interest to identify the unknown channel $h(z)$ from the second-order statistics of the observation so as to remove the ISI. Concerning this point, the model (1.2) calls for comments:

- If $q = 1$ (standard systems), $\tilde{s}(n) = s(n)$ and, in general, the second-order statistics of $\{x(n)\}$ do not allow the identification of $h(z)$;

- *A contrario*, the case $q > 1$, which corresponds to a Fractional Sampling (FS) system, deserves consideration. Noticing that $\{\tilde{s}(n)\}$ is cyclostationary at the cyclic frequencies $0, ..., \frac{q-1}{q}$, it was proved indeed that if $h(z)$ does not possess q zeros on a circle, separated by $\frac{2\pi}{q}$ radian angles, the *entire* second-order statistics of $\{x(n)\}$ enables the identification of $h(z)$. Various time-domain estimation algorithms of $h(z)$ based on the second order statistics of the observation have been proposed in [15] [14] [7]. These approaches can be extended to the case where the useful signal $x_a(t)$ is corrupted by an additive noise or / and interferences with known (up to a scalar factor) second order statistics.

The purpose of this paper is to develop a simple blind identification scheme, relying on *certain second-order statistics*, which remain consistent when $x_a(t)$ is corrupted by an additive noise or/and interference process $i_a(t)$, uncorrelated with $x_a(t)$, the second-order statistics of which are *unknown.*

[1]the subscript "a" stands for "analog".

In the case of an FS system with $q > 2$, it was briefly remarked by Giannakis in [8] that it is possible to identify $h(z)$ from the cyclo-statistics of the noisy observed signal[2] $x(n) + i(n)$ at the non-null cycles $1/q, \ldots, (q-1)/q$, provided that these cycles are not cycles of $\{i(n)\}$.

This principle of separating the contributions of the different cycles, then removing the corrupted statistics, is clever as far as the struggle against jammers is concerned. One should nevertheless note that this theoretical approach may prove useless in certain FS contexts, since the band-limited character of a communication channel makes most of the cyclo-statistics of interest numerically negligible, and the aforementioned method is often prone to degeneracies (see [1] [2] [3]). In order to deal with numerically significant cycles, one idea is to *impose* some second-order cyclic properties at the emitter: the so-called concept of Transmitter Induced Cyclostationarity (TIC) was introduced in [11] and has met with various extensions since (see e.g. [13] [10] [12]).

The principle of TIC is to transmit a sequence of pseudo-symbols $\{v(n)\}$ at a larger rate ($\frac{1}{T'}$) than that of the original symbol sequence[3]. The transmission is such that the noise-free analog signal, so received, can be written as

$$x_a(t) = \sum_{k \in \mathbb{Z}} v(n) h_a(t - kT'). \tag{1.3}$$

The sampled version at the rate $\frac{1}{T'}$ is then

$$x(n) = [h(z)].v(n). \tag{1.4}$$

This time, $h(z) = \sum_{k=0}^{\frac{T}{T'} M_a} h_k z^{-k}$, and $h_k = h_a(kT')$ where, as usual, $\frac{T}{T'} M_a$ is assumed to be an integer. This formulation shows that Equation (1.4) is a direct generalization of Equation (1.2) in which $v(n) = \tilde{s}(n)$ and $T' = T/q$. The reader may wish to consult the various contributions to appreciate the communication-oriented problems inherent to TIC systems.

Dealing with the model (1.4), we propose here to show that the subspace method of [5] can be adapted so as to identify $h(z)$ from some reliable statistics, namely, those free of any corruption. In Section 2, a general spectral factorization algorithm is presented. Section 3 applies this algorithm to blind identification; rather than developing a general method, we will focus on three particular cases: the FS case, the repetition coding case [11] and the modulation case [13]. Extensions to other contexts are possible. In each scenario, we will thoroughly depict the spectral factorization and make some remarks on the identifiability conditions. Simulation examples are subsequently given and analyzed. Section 4 summarizes the main points previously studied for various TIC systems.

[2] $i(n)$ is the sampled version of $i_a(t)$ at the rate $\frac{q}{T}$

[3] The one-to-one correspondence between $\{v(n)\}$ and $\{s(n)\}$ is obviously assumed.

2 A factorization algorithm

Let $S(z)$ be a $q \times 1$ rational function[4] of the form

$$S(z) = H(z) l^*(z^{-1}) \tag{2.5}$$

where $H(z) = \sum_{k=0}^{M} H_k z^{-k}$ is a $q \times 1$ degree M polynomial of the variable z^{-1}, $l(z)$ is a scalar-valued[5] causal rational transfer function, and $l^*(z)$ is obtained by conjugating the coefficients of $l(z)$. Of course, $S(z)$ has a Laurent expansion $S(z) = \sum_{k \in \mathbb{Z}} S_k z^{-k}$ converging around the unit circle. We focus on the problem of retrieving $H(z)$ from $S(z)$. As $H(z)$ is a polynomial, it is clear that $S_k = 0$ as soon as $k > M$, and we consequently consider

Problem 2.1 *Given $N \geq M$, under what conditions is it possible to recover $H(z)$ from the Laurent coefficients $\{S_k\}_{k=-N,\ldots,N}$ of $S(z)$? Exhibit a means of extracting $H(z)$.*

We assume the following hypotheses:

- **H1** $H(z) \neq 0$ for each z, including ∞,
- **H2** $l_0 = l(\infty)$ is nonzero.

Problem (2.1) could be solved by developing a linear prediction-like method. However, for simplicity, we shall rather generalize the noise subspace approach of [7].

We first need to recall an important result presented in [5]. Let $\mathcal{B}_N$ be the set of all $q(N+1)$-dimensional row vectors $G = [G_0, \ldots, G_N]$ satisfying the linear relation $G(z)H(z) = 0$, where $G(z) = \sum_{k=0}^{N} G_k z^{-k}$. $\mathcal{B}_N$ is of course a linear subspace of $\mathbb{C}^{q(N+1)}$. Let Π_N be the orthogonal projector onto $\mathcal{B}_N$. For every $q \times 1$ polynomial $A(z) = \sum_{k=0}^{M} A_k z^{-k}$ of degree M, we denote by $\mathcal{T}_N(A)$ the $q(N+1) \times (N+M+1)$ Sylvester matrix defined as

$$\mathcal{T}_N(A) = \begin{bmatrix} A_0 & A_1 & \ldots & A_M & 0 & \ldots & 0 \\ 0 & A_0 & A_1 & \ldots & A_M & \ddots & \vdots \\ \vdots & \ddots & \ddots & & & \ddots & 0 \\ 0 & \ldots & 0 & A_0 & A_1 & \ldots & A_M \end{bmatrix} \tag{2.6}$$

Then, we have the following result:

Theorem 2.1 *Let $F(z)$ be a $q \times 1$ polynomial of degree M. If* **H1** *is true, then, for $N \geq M$, the linear equation*

$$\Pi_N \mathcal{T}_N(F) = 0 \tag{2.7}$$

holds if and only if $F(z)$ coincides with $H(z)$ up to a scalar factor.

[4] In the sequel, capital letters stand for matrices or vectors.

[5] the case when $l(z)$ is vector-valued can also be treated this way

In other words, $H(z)$ can be identified from the subspace $\mathcal{B}_N$ by solving a linear system. This follows from properties of minimal polynomial bases of a rational subspace; see [4] for details. It now remains to show that $\mathcal{B}_N$, and hence Π_N, can under **H2** be extracted from the Laurent coefficients $\{S_k\}_{-N,\cdots,N}$ of $S(z)$. It is clear that $G(z) = \sum_{k=0}^{N} G_k z^{-k}$, of dimension $1 \times q$, satisfies $G(z)H(z) = 0$ if and only if $G(z)S(z) = 0$. Denoting by $\mathcal{S}_N$ the $q(N+1) \times (2N+1)$ matrix defined as

$$\mathcal{S}_N = \begin{bmatrix} S_0 & \dots & S_N & & \mathbf{0} \\ \vdots & \ddots & \vdots & \ddots & \\ S_{-N} & \dots & S_0 & \dots & S_N \end{bmatrix},$$

$G(z)S(z) = 0$ implies in particular that $G\mathcal{S}_N = 0$, which is equivalent to the condition $[G(z)S(z)]_- = 0$, where the notation $[.]_-$ stands for the causal truncation of the function inside the brackets. Conversely, $[G(z)S(z)]_- = 0$ also means that $[k(z)l^*(z^{-1})]_- = 0$, where $k(z)$ is the scalar-valued polynomial $G(z)H(z)$. Assuming now that **H2** holds, we deduce immediately that $k(z) = 0$. Hence the space $\mathcal{B}_N$ coincides with the left kernel of matrix $\mathcal{S}_N$ associated with $S(z)$.

Problem (2.1) is solved: we have found a method for recovering $H(z)$, which is valid as soon as **H1** and **H2** hold. We now recast some blind second-order problems into this framework.

3 Application to blind identification

Generally speaking, the correlation coefficient at lag τ and at cycle α of a second-order process $\{y(n)\}$ is defined as (see [16] and the references therein) :

$$R_y^{(\alpha)}(\tau) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} E[y(n+\tau)y(n)^*]e^{-i2\pi\alpha n}, \tag{3.8}$$

and the corresponding cyclo-spectrum as $S_y^{(\alpha)}(e^{i\omega}) = \sum_\tau R_y^{(\alpha)}(\tau)e^{-i\omega\tau}$.

According to model (1.4), it is easy to prove that the expression of the cyclo-spectrum of $\{x(n)\}$ at cycle α is

$$S_x^{(\alpha)}(e^{i\omega}) = h(e^{i\omega})h(e^{i(\omega-2\pi\alpha)})^* S_v^{(\alpha)}(e^{i\omega}). \tag{3.9}$$

The expression (3.9) remains valid in a noisy context, as soon as the noise and the jammers are decorrelated from the signal of interest and do not admit α as a cycle.

Although it is possible to develop further the general framework depicted thus far, such would result in cumbersome relations. We shall thus concentrate henceforth on the three cases raised in the introduction.

3.1 The FS case.

It is assumed here that $q > 2$, so that at least two non-null cycles can be exploited. The contribution of the jammers $\{i(n)\}$ is assumed not to exhibit cyclostationarity at the shifts $\frac{1}{q}, ..., \frac{q-1}{q}$, and we recall that $\{v(n) = \tilde{s}(n)\}$ and $T' = T/q$. We propose to develop a scheme in the case where the input symbols are white; the method can be extended directly when the symbols are colored. Under this assumption, $S_v^{(\alpha)}(e^{i\omega}) = \frac{1}{q}$, for all $\alpha = \frac{1}{q}, ..., \frac{q-1}{q}$. Consider now the following vector:

$$S(e^{i\omega}) = q[S_x^{(1/q)}(e^{i\omega})^*, ..., S_x^{((q-1)/q)}(e^{i\omega})^*]^T$$

Recalling (3.9), and defining

$$H(z) = \left[h(ze^{\frac{i2\pi}{q}}), ..., h(ze^{\frac{i2\pi(q-1)}{q}})\right]^T$$

and $l(z) = h(z)$, the factorization (2.5) holds. The results of the previous section then apply: provided $h(z)$ does not possess $q-1$ zeros on a circle, equally separated by $\frac{2\pi}{q}$ radians, one can identify $h(z)$ from the cyclo-statistics of the observation at the non-null cycles. In practice, the Laurent coefficients of $S(z)$ are unknown, but they can be replaced by consistent estimates in the procedure sketched in Section 2 (the equation (2.7) should be solved in the least squares sense). The proposed estimate of the channel is of course consistent.

Let us now consider the practical aspects of the above-mentioned approach. So as to enlighten the identifiability condition, one may consider increasing the oversampling rate: indeed, the more cycles, the less stringent the identifiability condition. But

- the order M increases with q,
- since the analog filter $h_a(t)$ is band-limited, a larger q reduces the bandwidth of $h(z)$; in other words, according to Equation (3.9), there are only two non-null numerically relevant cycles: $\frac{1}{q}, \frac{q-1}{q}$.

A good practical choice is $q = 3$. The question is: what is the relevance of removing the zero cycle? When the problem is well-conditioned (this occurs for a large excess bandwidth and for short impulse responses), the color of the noise does not impact significantly the performance of the full method (all cycles considered, see [7]), even if consistency is lost. By contrast, consistency is crucial in ill-conditionned problems: in this case the full method is very sensitive to the perturbations brought by the color of the noise, and fails to provide a good estimate. Excluding the zero cycle is then recommended; of course, the bad conditioning compels one to use a large analysis window.

Simulation results: $q = 3$. We consider a GSM channel: the shaping is a raised-cosine with 85% excess-bandwidth; the symbol period is $T = 3.7\mu s$ and the carrier frequency is $f_0 = 1.087654GHz$. A three path realization is studied. The characteristics of the channel are given in Tables 1 and 2. The digital channel $h(z)$ is consequently a degree $M = 8$ polynomial. The symbols are BPSK. The colored noise is the output of $r(z) = \frac{1}{\sqrt{3}}(1 + z^{-2} + z^{-4})$ driven by a white Gaussian noise independent of the symbols. The averaged square Euclidean distance between the estimate and the true channel (MSE) is estimated from 200 Monte-Carlo trials. In Figure (1), the SNR is fixed to $9.2dB$ whereas the duration of observation varies from $N = 500$ to $N = 1700$ symbol periods. As expected, the standard subspace method of [7] reaches a limit due to its inconsistency; the consistency of our method is observed. In Figure (2), $N = 1000$ and the SNR goes from $4dB$ to $16dB$. As expected, removing the zero cycle is all the less pertinent as the SNR increases.

3.2 The repetition coding and interleaving scheme [11].

One now transmits the sequence $\{v(n)\}$ at the rate $\frac{1}{T'} = \frac{2}{T}$, which may be read as consecutive blocks of the type $[s(nL), s(nL+1), ..., s(nL+L-1)\|s(nL), s(nL+1), ..., s(nL+L-1)]$. It is easy to prove that $\{v(n)\}$ admits $(\frac{2k+1}{2L})_{k=0,...,L-1}$ as non-zero cyclic frequencies.

Let us moreover assume that $\{s(n)\}$ is white. Setting $\mu_k = 1/(L(1 - e^{-i2\pi(2k+1)/2L}))$, a simple computation gives

$$S_v^{(2k+1/2L)}(z) = \mu_k(z + z^{-1}).$$

Consider now the following vector:

$$\tilde{S}(e^{i\omega}) = [\frac{1}{\mu_{k_1}^*} S_x^{((2k_1+1)/2L)}(e^{i\omega})^*, ..., \frac{1}{\mu_{k_N}^*} S_x^{(2k_N-1/2L)}(e^{i\omega})^*]^T$$

for any collection of N distinct $k_i \in \{1, ..., L\}$. This yields $\tilde{S}(z) = z^{-1}H(z)l^*(z^{-1})$, where

$$H(z) = \left[h(ze^{i2\pi(2k_1+1)/2L}), ..., h(ze^{i2\pi(2k_N+1)/2L})\right]^T$$

and $l(z) = (1 + z^{-2})h(z)$. The results of Section (2) still hold if one considers $S(z) = z\tilde{S}(z)$. Noticing that L is a design parameter and can then be chosen arbitrarily big, two crucial remarks follow:

- *many cycles in the factorization:* let us exploit all the non-null cycles, so that there are $L-1$ entries in $H(z)$. On the one hand, the identifiability condition is that $h(z)$ does not possess $L-1$ zeros on a circle, equally separated by $\frac{2\pi}{L}$. On the other hand, $h(z)$ has M zeros. If one imposes $L > M$, the previous points are contradictory, hence showing that the identifiability condition is automatically fulfilled (see [11]).

- *two well-chosen cycles:* take any two cycles $\frac{2k_1+1}{2L}$ and $\frac{2k_2+1}{2L}$ so that $k_2 - k_1$ and L are coprime (this is always possible since L can be chosen arbitrarily large). According to a structural result of [13] (see also [10]), the identification of $h(z)$ is possible, without any restriction on $h(z)$, as soon as $L > M$.

Therefore, the identification in a repetition context is robust as regards the unknown channel.

3.3 The modulation case.

In this model, $T' = T$, and the sequence $\{v(n) = f(n)s(n)\}$ is transmitted, where $f(n)$ is a deterministic (almost) periodic sequence. This scheme has been proposed independently in [13] and [10]. For sake of simplicity, we restrict the study to i.i.d. sequences of symbols[6]. Under this condition, it is easy to prove that $S_v^{(\alpha)}(z) = \lambda_\alpha$ for some α and λ_α depending on the development of $f(n)$ as a Fourier series. Suppose α_1 and α_2 are non-null cycles. Consider the function[7]

$$S(e^{i\omega}) = [\frac{1}{\lambda_{\alpha_1}^*} S_x^{(\alpha_1)}(e^{i\omega})^*, \frac{1}{\lambda_{\alpha_2}^*} S_x^{(\alpha_2)}(e^{i\omega})^*]^T.$$

If $H(z) = \left[h(ze^{i2\pi\alpha_1}), h(ze^{i2\pi\alpha_2})\right]^T$ and $l(z) = h(z)$, we have $S(z) = H(z)l^*(z^{-1})$.

As in the previous section, the identifiability condition vanishes in the following two cases (see [13] [10]):

- $\alpha_2 - \alpha_1$ is irrational, or
- $\alpha_2 - \alpha_1 = \frac{k}{p}$, k and p coprime, is such that $p > M$, M being as usual the degree of $h(z)$.

As compared to the repetition scheme, the modulation of the symbols also brings a robust way of identifying the channel. The advantage of modulation over repetition is that the channel order is halved, thus yielding faster algorithms, with a lower computational burden.

<u>Simulation results:</u> The pulse is a raised-cosine with 20% excess bandwidth; $T = 3.7\mu s$ and $f_0 = 1.087654GHz$. We averaged over 10 realizations of channels resulting from 5 multi-paths. The maximum delay is set to $2T$, so that the degree is $M = 4$. The impulse responses are in Table 3. The deterministic sequence is $f(n) = \frac{1}{\sqrt{1+\gamma^2}}(1 + \gamma e^{i2\pi\alpha n})$, with $\gamma = 0.5$ and $\alpha = \frac{51}{360}$. The symbols are white BPSK sequences. In the factorization algorithm, we chose the two cyclo-frequencies α and $-\alpha$. In Figure (3), the MSE is given as the number of observed samples increases. The SNR is set to $10dB$; the noise is either white Gaussian, or is the output of $r(z) = \frac{1}{\sqrt{3}}(1 + z^{-2} + z^{-4})$ driven by a white Gaussian noise. Notice the consistency of the estimate when the noise has an unknown color. In Figure (4), the observation lasts 600 symbol periods. The SNR varies from 0 to $18dB$.

The results thus obtained are satisfactory; however, the "best" choice of a sequence $f(n)$ is currently under investigation.

4 Conclusion.

When cyclostationarity is induced at the transmitter, we have shown that some second-order cyclo-spectra provide a way of identifying the unknown channel. In contrast to the conventional approches, the consistency of the pro-

[6]For a generalization to any distribution, see [13].

[7]More cycles may be taken into account.

posed method is achieved when the observation is corrupted by interference, the second-order structures of which are unknown. We propose in Table 4 a sum-up of the main points developed in the paper.

References

[1] Z. Ding, “Characteristics of band-limited channels unidentifiable from second-order statistics”, *IEEE Sig. Proc. Letters*, pages 150-152, May 1996.

[2] A.J. van der Veen, ”Resolution limits of blind multi-user channels identification schemes: the band-limited case”, *Proc. ICASSP*, pages 2722-2725, 1996.

[3] Ph. Ciblat and Ph. Loubaton, “Égalisation aveugle au second ordre : le cas de signaux à bande limitée”, *actes du GRETSI*, Grenoble, France, Septembre 1997.

[4] T. Kailath, “Linear Systems” , *Prentice-Hall, 1978.*

[5] K. Abed Meraim, P. Loubaton and E. Moulines, ”A subspace algorithm for certain blind identification problems”, *IEEE Trans. on Info. Theory*, vol. 43, no 2, pages 499-511, March 1997.

[6] K. Abed-Meraim, J.F. Cardoso, A. Gorokhov, P. Loubaton and E.Moulines, “On subspace methods for blind identification of single-input/multi-output FIR systems”, *IEEE Trans. on Sig. Proc.*, vol. 45 no. 1, pages 42-56, January 1997.

[7] E. Moulines, P. Duhamel, J.F. Cardoso, S. Mayrargue, ”Subspace methods for the blind identification of multichannel FIR filters”, *IEEE Trans. on Sig. Proc.*, vol. 43, pages 516-526, February 1995.

[8] G. B. Giannakis, ”A linear cyclic correlation approach for blind identification of FIR channels”, *Proc. Asilomar Conf.*, pages 420-424, 1994.

[9] G. B. Giannakis, “Filterbanks for blind channel identification and equalization”, *Proc. Wavelet, Subband, and Block Transforms in Communications Symposium,* New Jersey Inst. of Tech., March 1997.

[10] E. Serpedin and G. B. Giannakis, “Blind channel identification and equalization with modulation induced cyclostationarity”, *Proc. 31st Conf. on Info. Sciences and Systems*, The Johns Hopkins Univ., Baltimore, March 1997.

	delays	attenuation
1^{st} path	$0.0000T$	1
2^{nd} path	$0.2084T$	1
3^{rd} path	$0.8880T$	1

Table 1: *FS: realisation of the paths*

$h_1(n)$	$h_2(n)$	$h_3(n)$
0.0415 + 0.0095i	0.1393 - 0.2058i	0.5407 - 0.1906i
0.1499 - 0.0171i	0.2099 - 0.2797i	0.3819 - 0.0896i
0.1794 - 0.0971i	0.4252 - 0.2715i	0.1096 - 0.0188i

Table 2: *FS: polyphase components of the channel*

[11] M. K. Tsatsanis and G. B. Giannakis, "Coding induced cyclostationarity for blind channel equalization", *Proc. 29th Conf. on Info. Sciences and systems*, pages 685-690, The John Hopkins Univ., Baltimore, March 22-24 1995; also in *IEEE Trans. on Sig. Proc.*, 1997 (to appear).

[12] G. B. Giannakis, "Filterbanks for blind channel identification and equalization", *IEEE Sig. Proc. Letters*, June 1997.

[13] A. Chevreuil and P. Loubaton, " Modulation and second-order cyclo-stationarity: structured subspace method and identifiability", *IEEE Sig. Proc. Letters*, July 1997.

[14] D. Slock, "Blind fractionnally spaced equalizers, perfect reconstruction filter banks and multichannel linear prediction", *Proc. ICASSP*, vol. 4, pages 585-588, 1994.

[15] L. Tong, G. Xu and T.Kailath, "A new approach to blind identification and equalization of multi-path channels, *Proc. Asilomar conf.*, pages 856-860, 1991.

[16] M.J. Genossar, H. Lev-Ari and T. Kailath, "Consistent estimation of the cyclic-autocorrelation", *IEEE Trans. on Sig. Proc.*, vol. 42, no 3, pages 595-603, March 1994.

$h(0)$	$h(1)$	$h(2)$	$h(3)$	$h(4)$
0.0766 - 0.0472i	0.1727 + 0.2774i	-0.5916 + 0.2153i	-0.2405 + 0.6518i	-0.0645 - 0.0394i
-0.0300 + 0.0204i	0.1597 - 0.0300i	0.4166 + 0.7988i	-0.1903 + 0.3093i	-0.1673 + 0.0215i
0.4175 - 0.1257i	0.1802 + 0.3764i	-0.6259 + 0.3107i	-0.3777 + 0.0284i	0.0128 + 0.0591i
0.8130 + 0.0577i	-0.2574 - 0.1393i	-0.0456 - 0.2748i	0.0356 + 0.3854i	-0.0951 - 0.1151i
-0.0402 + 0.0787i	0.2298 - 0.1537i	0.2845 - 0.0574i	-0.4862 + 0.7622i	0.0265 - 0.1149i
0.2502 + 0.0668i	0.8357 + 0.4010i	-0.1944 + 0.0851i	-0.1363 + 0.0901i	0.0324 - 0.0279i
0.3149 - 0.0940i	-0.1115 - 0.1508i	0.1901 - 0.8604i	-0.0521 - 0.2384i	-0.0681 + 0.1261i
-0.0383 - 0.0137i	0.4540 + 0.0190i	0.1122 - 0.6541i	0.3695 - 0.4206i	-0.1635 + 0.1052i
0.1348 - 0.1371i	0.6038 - 0.0735i	-0.5815 + 0.0650i	-0.4808 - 0.0372i	0.1239 + 0.0512i
-0.0251 - 0.0294i	0.5445 + 0.46879	0.1939 + 0.4547i	-0.4566 + 0.1545i	0.0738 + 0.0024i

Table 3: *Modulation: 10 channel realizations.*

	FS at $3/T$	repetition	modulation
support of $h_a(t)$	M_aT	M_aT	M_aT
degree of $h(z)$ to identify	$M = 3M_a$	$M = 2M_a$	$M = M_a$
number of cycles used	$p = 2$	$2 \leq p < L$	$p \geq 2$
identifiability condition	$h(z_0) = 0 \Rightarrow h(z_0 e^{i2\pi/3}) \neq 0$	none	none
control of cycles	no	yes	yes
estimation in jammers	consistent	consistent	consistent
generalization to colored sources	yes	yes	yes
over-estimation of the degree	no	yes	yes

Table 4: *TIC and blind second-order identification.*

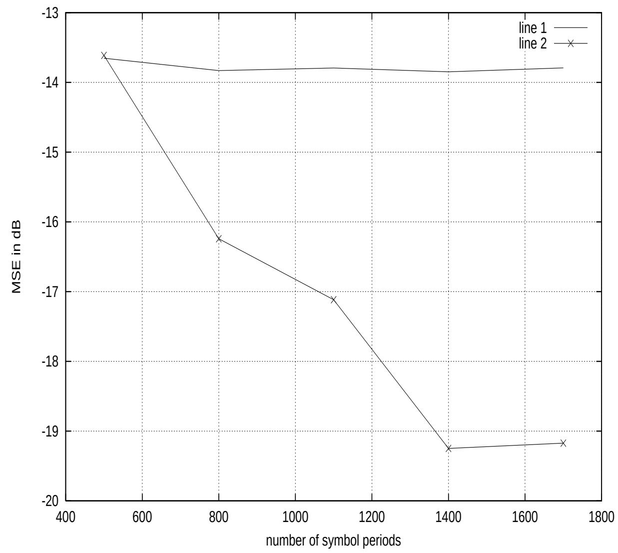

Figure 1: *FS system. Line 1: standard method. Line 2:* 0 *cycle excluded* ; $SNR = 9.2dB$

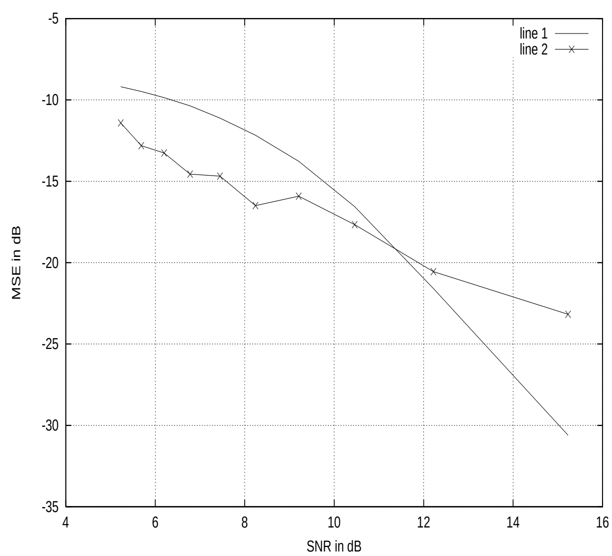

Figure 2: *FS system. Line 1: standard method. Line 2:* 0 *cycle excluded* ; $N = 1000$

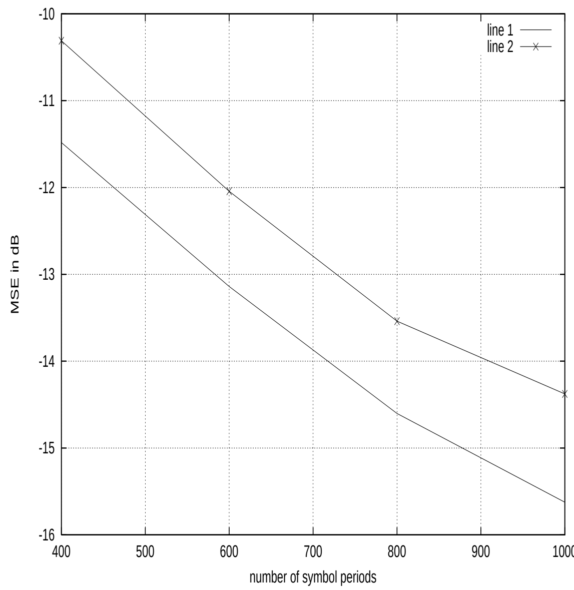

Figure 3: *Modulation. Line 1: white noise. Line2: colored noise* ; $SNR = 10dB$

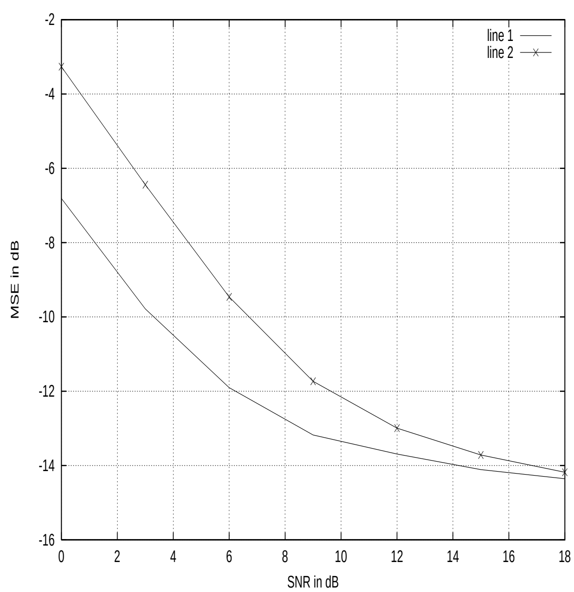

Figure 4: *Modulation. Line 1: white noise. Line 2: colored noise* ; $N = 600$ samples.