ON THE CONVOLUTIVE MIXTURE SOURCE SEPARATION BY THE DECORRELATION APPROACH

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ABSTRACT

In this paper, we consider the problem of blind separation of causal minimum phase convolutive mixtures of two sources. We study in detail the so-called decorrelation approach. It consists in finding a causal minimum phase filter which, driven by the observations, produces decorrelated outputs. It is well established that this approach allows to separate the sources if the mixing filter is a non static FIR filter. We show that this result is no longer true in the IIR case. We establish that it exists infinitely many causal minimum phase filters producing decorrelated outputs and provide a parameterisation of these filters. This clearly shows that the decorrelation approach is, in practice, non robust. In order to overcome this drawback, we propose an alternative approach based on a linear prediction scheme, which, as the decorrelation approach, uses essentially the second order statistics of the observations.

1. INTRODUCTION

This paper deals with the problem of separating a convolutive mixture of two sources when there are two sensors. More precisely, let us suppose that two sources $s_1(n)$ and $s_2(n)$ are observed on two sensors after having been convolutively mixted by a two input/two output unknown linear system. The transfert function of this system will be noted

$$H(z) = \begin{pmatrix} H_{11}(z) & H_{12}(z) \\ H_{21}(z) & H_{22}(z) \end{pmatrix}$$

The observed signal $y_1(n)$ and $y_2(n)$ can then be written :

$$\begin{cases} y_1(n) = [H_{11}(z)]s_1(n) + [H_{12}(z)]s_2(n) \\ y_2(n) = [H_{21}(z)]s_1(n) + [H_{22}(z)]s_2(n) \end{cases}$$

Put $s(n) = (s_1(n), s_2(n))^T$. The source separation problem consists in recovering the contributions $[H_{i,j}(z)]s_j(n)$ of each source signal $s_j(n)$ on each sensor using only the observation of the signal $y(n) = (y_1(n), y_2(n))^T$ for $(i,j) \in \{1,2\}^2$ [1]. In other words, one wishes to reconstruct the signals that would be observed if the unknown system were driven separately by $[s_1(n), 0]^T$ and by $[0, s_2(n)]^T$. Therefore, we can replace without any noticeable restriction $s_1(n)$ by $[H_{11}(z)]s_1(n)$ and $s_2(n)$ by $[H_{22}(z)]s_2(n)$, equivalent to normalising the diagonal terms of $H(z)$ to 1. Therefore, the model of the observed signals is written :

$$\begin{cases} y_1(n) = s_1(n) + [H_{12}(z)]s_2(n) \\ y_2(n) = [H_{21}(z)]s_1(n) + s_2(n) \end{cases} \tag{1}$$

In the rest of this paper, the following hypothesis will be made :

- H1 : the filters $H_{12}(z)$ and $H_{21}(z)$ are causal
- H2 : the filter $H(z)$ is minimum phase i.e. its inverse is causal ; this can be written : $1 - H_{12}(z)H_{21}(z) \neq 0 \quad \forall z \quad | \quad |z| > 1$

Although rather restrictive, these hypothesis are realistic in certain situations and, therefore, often met (see e.g. [5, 4, 6]). In a number of works devoted to the model (1), it is proposed to identify a filter $G(z)$ of the form

$$G(z) = \begin{pmatrix} 1 & -G_{12}(z) \\ -G_{21}(z) & 1 \end{pmatrix}$$

such as its application on the observation $y(n)$ generates a signal $r(n) = [r_1(n), r_2(n)]^T = [G(z)]y(n)$ which components are independent. The form of $G(z)$ is justified by the fact that the separation is achieved (i.e. $r(n) = s(n)$) if and only if $G_{ij}(z) = H_{ij}(z) \quad i \neq j \quad (i,j) \in \{1,2\}^2$.

The statistical independence of r_1 and r_2 is of course difficult to express analytically. Therefore, the matrix $G(z)$ is often sought in order to cancel some cross-cumulants of the signals $r_1(n)$ and $r_2(n)$. The so-called decorrelation approach introduced in [4] consists in adapting $G(z)$ so as to decorrelate the signals $r_1(n)$ and $r_2(n)$. It is quite clear that it exists infinitely many filters $G(z)$ decorrelating $r_1(n)$ and $r_2(n)$ (see e.g. [7]). However, it has been proved recently by Lindgren and Broman [5] that if $H(z)$ is a non static (i.e. $H(z)$ is not reduced to a constant matrix) finite impulse response (FIR) filter satisfying the hypotheses H1 and H2 then, the filter $G(z)$ defined by $G_{ij}(z) = H_{ij}(z)$ is the *unique* FIR minimum phase causal filter decorrelating the two components of $[G(z)]y(n)$. Therefore, it seems that the decorrelation approach can be successful in the FIR case by restricting the filters $G(z)$ to a certain domain. This result is potentially attractive because the

This work has been supported by EDF/DER

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[1] without addtitional assumptions, it is impossible to reconstruct the source signals $s_j(n)$ themselves

decorrelation approach is based on the exclusive use of the second order statistics of the observation. Therefore, it may be used to separate convolutive mixtures of *Gaussian* signals.

In this paper, we first study the decorrelation approach under the hypotheses H1 and H2 in section 2. By contrast with the work of Lindgren and Broman, we consider the more general case where $H(z)$ is an infinite impulse response (IIR) filter. We establish that it exists infinitely many causal and minimum phase filters $G(z)$ decorrelating $r_1(n)$ and $r_2(n)$ which do not separate the sources s_1 and s_2. We moreoever provide a parameterisation of these filters. In the FIR case, we reconcile our results with [5] : if $H(z)$ is FIR, the unique minimum phase FIR separating filter is $G_{ij}(z) = H_{ij}(z)$.These results prove that the decorrelation approach is not robust because the FIR model is very often a simplified model. Moreover, in practice, the decorrelating filters are obtained by minimizing a certain non-quadratic cost function. Our results suggest that even if $H(z)$ is FIR, this cost function has a number of spurious local minima corresponding to FIR truncations of non separating IIR solutions of the decorrelation equations. In section 2, we propose an alternative approach based on a linear prediction scheme which is shown to be more efficient.

2. THE SOLUTIONS OF THE DECORRELATION EQUATIONS

By decorrelation equations we mean the set of relations expressing that the two components of $[G(z)]y(n)$ are uncorrelated. The separating solution corresponds - up to the term $(1 - H_{12}(z)H_{21}(z))^{-1}$ - to the inverse of the filter $H(z)$ which is supposed causal and of causal inverse (hypotheses H1 and H2). The solution sought is hence a causal filter with causal inverse as well. Therefore, as mentionned in section 1, it is natural to limit our research of the solutions of the decorrelation equations to the set of minimum phase causal filters $G(z)$.

In order to simplify the results, the spectra of the source signals $s_1(n)$ and $s_2(n)$ will be supposed rational. With this hypothesis, the concept of minimum causal phase factorisation of their spectral density can be easily defined. More precisely, there exist two unit variance white noises $\nu_1(n)$ and $\nu_2(n)$ and two minimum phase causal filters $f_1(z)$ and $f_2(z)$ for which $s_i(n) = [f_i(z)]\nu_i(n)$ for $i = 1, 2$. Moreover, $\nu_1(n)$ and $\nu_2(n)$ coincide with the normalised (in the sense that $E(\nu_i^2(n)) = 1$) innovation processes of s_1 and s_2 respectively. Now, let $G(z)$ be a minimum phase causal filter solution of the decorrelation equations i.e. such that the two components $r_1(n)$ and $r_2(n)$ of the signal $r(n) = [G(z)]y(n)$ are uncorrelated signals. Let $g_i(z)$ $i = 1, 2$ be the minimum phase causal factorisations of these signals and $\mu_i(n)$ their corresponding normalised innovation processes. The decorrelation of $r_1(n)$ and $r_2(n)$ is then equivalent to the one of the white noises $\mu_1(n)$ and $\mu_2(n)$. Furthermore, let us set $f(z) = \begin{bmatrix} f_1(z) & 0 \\ 0 & f_2(z) \end{bmatrix}$, $g(z) = \begin{bmatrix} g_1(z) & 0 \\ 0 & g_2(z) \end{bmatrix}$, $\mu(n) = [\mu_1(n), \mu_2(n)]^T$ and $\nu(n) = [\nu_1(n), \nu_2(n)]^T$ Then,

$$\begin{cases} r(n) = [g(z)]\,\mu(n) \\ s(n) = [f(z)]\,\nu(n) \end{cases}$$

Alternatively, $r(n) = [G(z)][H(z)]s(n)$.

The expression of the signals $r(n)$ and $s(n)$ according to their normalised innovation processes leads to :

$$\mu(n) = \Theta(z)\nu(n) \tag{2}$$

with

$$\Theta(z) = g(z)^{-1}G(z)H(z)f(z) \tag{3}$$

But the two components of $\mu(n)$ and $\nu(n)$ are both unit variance white noises. The spectral densities of the 2-variate sequences $\mu(n)$ and $\nu(n)$ thus both coincide with the 2×2 identity matrix I. Therefore, $\Theta(z)$ must satisfy :

$$\Theta(z)\Theta(z^{-1})^T = I$$

or equivalently $(\Theta(z))^{-1} = \Theta(z^{-1})^T$. On the other hand, $\Theta(z)$ and its inverse are products of four causal filters of causal inverses. They are therefore causal of causal inverses themselves. As we have seen that the inverse of $\Theta(z)$ verifies $\Theta(z)^{-1} = \Theta(z^{-1})^T$, it is causal if and only if $\Theta(z)$ is reduced to an orthogonal 2×2 constant matrix.

This matrix can be parametrised as :

$$\Theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \cos\theta \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

where $t = \tan\theta$. By using this parametrisation and simplifying the relation (3), the filter $G(z)$ can be expressed as :

$$\begin{cases} G_{12}(z) = \frac{f_2(z)H_{12}(z) - tf_1(z)}{f_2(z) - tf_1(z)H_{21}(z)} \\ G_{21}(z) = \frac{tf_2(z) + f_1(z)H_{21}(z)}{tf_2(z)H_{12}(z) + f_1(z)} \end{cases} \tag{4}$$

Therefore, we have shown that any minimum phase causal filter $G(z)$ solution of the decorrelation equations can be parametrised in the form (4). Conversely, it is clear that every minimum phase causal filter $G(z)$ given by (4) is a solution of the decorrelation equations. It remains to characterise the filters (4) which are minimum phase and causal. It can easily be checked that $\det(G(z)) = (1 + t^2)\det(H(z))$ so that, for any value of t, $\det(G(z)) \neq 0$ for $|z| > 1$ with the hypothesis H2. To conclude, the filter $G(z)$ is causal of inverse causal if and only if it is causal. In order to ensure the causality to $G(z)$ a necessary and sufficient condition is that the denominators of $G_{12}(z)$ and $G_{21}(z)$ have no zero outside the unit circle. This holds not only for the separating solution (which corresponds to $t = 0$) but for any t sufficiently small as well. Therefore, the decorrelation equations have infinitely many causal solutions. If we further suppose that $H(z)$ is FIR, we can impose the same constraint on $G(z)$ and a close look at (4) suggests that, in this case, the only possible solution is the separating one (i.e. $t = 0$). This last result confirms the one expressed by Lindgren and Broman [5]. However, the former analysis clearly shows that the separating character of the solutions of the decorrelation equations is fundamentally non-robust. This result and its possible consequences will be illustrated in the simulation section.

3. THE LINEAR PREDICTION APPROACH

The linear prediction approach originates from the work of Comon devoted to the identification of non monic MA models [2]. It has been later used in the more general context of source separation by Delfosse and Loubaton in [3] when the number of sensors is strictly greater than the number of sources. Here, we will adapt this result to the simpler case of two sources and two sensors.

First, let us remind some basic notions and definitions. The innovation process of the multivariate observation $y(n)$ is the signal defined by $i(n) = y(n) + \sum_{k=1}^{+\infty} A_k y(n-k) = [A(z)]y(n)$ where the filter $A(z) = I + \sum_{k\geq 1} A_k z^{-k}$ is defined by : $A(z) = \arg\min_B E\left[\left\|y(n) + \sum_{k=1}^{\infty} B_k y(n-k)\right\|^2\right]$.
$i(n)$ is a white noise (i.e. $E[i(n+k)i(k)^T] = 0$ for $n \neq 0$) whose instantaneous covariance matrix $D = E[i(n)i(n)^T]$ is not necessarily reduced to the identity matrix : indeed, the two components $i_1(n)$ and $i_2(n)$ of $i(n)$ are neither decorrelated nor unit-variance. The normalised innovation process of $y(n)$ is then any white noise $v(n)$ defined through $i(n)$ by $v(n) = L^{-1}i(n)$ where L satisfies[2] $D = LL^T$. Consequently, $E[v(n)v(n)^T] = I$. It is clear that the normalised innnovation process is defined up to an orthogonal matrix : if $v(n)$ is a normalised innovation process, so is $w(n) = Qv(n)$ where Q is an orthogonal matrix.

The principle of the linear prediction approach contents three steps :

Step 1. As we have supposed that the filter $H(z)$ is minimum phase, so is $H(z)f(z)$ where we recall that $f(z)$ is the diagonal filter matrix which elements are the minimum phase factorisation of the spectral densities of the sources $s_1(n)$ and $s_2(n)$. Under these conditions, the two-dimensional white noise $\nu(n) = (\nu_1(n), \nu_2(n))^T$ constructed from the normalised innovations of the source signals $s_1(n)$ and $s_2(n)$ coincides with *a* normalised innovation of the observation $y(n)$ (recall that the normalised innovation process is not uniquely defined in the multivariate case). $\nu(n)$ can thus be extracted, *up to an orthogonal matrix*, from $y(n)$ by a linear prediction algorithm : having evaluated the prediction error filter $A(z)$ of $y(n)$, the innovation $i(n)$ and its covariance matrix D are extracted. Finally, one generates $v(n) = L^{-1}i(n)$, where L is a particular square root of D.

Step 2. $v(n) = \Theta\nu(n)$ where Θ is an unknown constant orthogonal matrix. In order to extract $\nu(n)$, we just have to notice that $v(n)$ is an instantaneous mixture of the two statistically independent signals $\nu_1(n)$ and $\nu_2(n)$. $\nu_1(n)$ and $\nu_2(n)$ can be extracted by any classical separation algorithm of instantaneous mixture (see e.g. [1, 2]) if the source signals are non Gaussian.

Step 3. The last step consists in reconstructing the contribution of each source on each sensor, i.e. the signals $s_1(n)$ and $s_2(n)$. In order to do that, let us set

$$K(z) = \Theta^T L^{-1} A(z)$$

Therefore,

$$H(z)f(z) = K^{-1}(z)$$

and, consequently

$$K^{-1}(z) \begin{bmatrix} \nu_1(n) \\ 0 \end{bmatrix} = \begin{bmatrix} s_1(n) \\ * \end{bmatrix}$$

Therefore, $s_1(n)$ can be reconstructed from $K(z)$. $s_2(n)$ is reconstructed in the same way. It is worth noticing that the instantaneous separation algorithm applied to $v(n)$ generates $\nu(n)$ up to a permutation and a sign. We thus have to deal with this indeterminacy in the reconstruction of $s_1(n)$ and $s_2(n)$, possibly by adapting the formerly presented scheme.

Finally, the infinite order prediction error filter $A(z)$ can not be exactly computed because the exact second order statistics of $y(n)$ are not available. In practice, $A(z)$ is replaced by an empirical estimate $\hat{A}_N(z)$ of a finite order prediction error filter $A_N(z) = I + \sum_{k=1}^{N} A_{k,N} z^{-k}$, with N sufficiently big. The first two steps of the previously described procedure can be adapted without any serious methodological problem. For the last one, $K(z)$ has to be replaced by a FIR filter of length N $K_N(z) = \Theta_N^T L_N^{-1} A_N(z)$, where the $\hat{}$ means that all the quantities are empirical estimates and the index N signifies that Θ_N and L_N are evaluated on the basis of the empirical partial innovation $i_N(n) = [A_N(z)]y(n)$. For this last step to have a meaning, the filter $K_N(z)^{-1}$ has to be stable which is true if $A_N(z)^{-1}$ is stable. In practice, the coefficients of A_N are estimated by solving a Yule-Walker type equation based on an estimator $\hat{R}_N$ of the covariance matrix of the vector $Y(n) = (y(n)^T, \cdots, y(n-N)^T)^T$. It is well known that, as soon as $\hat{R}_N$ is chosen to be block Toeplitz and positive definite, the stability of $A_N(z)^{-1}$ is guaranteed. Practically, it is hard to impose simultaneously these two conditions to the estimator $\hat{R}_N$ but any reasonnable estimator generates a stable filter if the number of observations is big enough.

As a conclusion, the linear prediction approach seems to be a satisfactory alternative to the decorrelation technique. Its only disadvantage is that it implicitely supposes to approximate $H(z)$ and $f(z)$ by FIR filters. If the spectra of the sources $s_1(n)$ and $s_2(n)$ have zeros, this last step will be hard to accomplish and the efficiency of the procedure will decrease. However, the inversion of $H(z)$ can be intrinsequely easy, particularly by using a decorrelation technique if $H(z)$ is exactly FIR.

4. RESULTS AND SIMULATIONS

In the following simulations, the filter $H(z)$ will be taken such as $H_{12}(z) = \frac{1+0.5z^{-1}}{1-0.1z^{-1}}$ and $H_{21}(z) = \frac{1+0.2z^{-1}}{1-0.1z^{-1}}$ in the IIR case and $H_{12}(z) = 1 + 0.2\sqrt{2}z^{-1}$ and $H_{21}(z) = -1 + 0.2\sqrt{2}z^{-1}$ in the FIR case. We note that in the IIR case, the poles of $H_{12}(z)$ and of $H_{21}(z)$ are very far from the unit circle. In other words, our IIR filter $H(z)$ can be approximated with a very good accuracy by a low degree FIR filter. The observation sample size is set to $T = 10000$.

4.1. Decorrelation

In practice, the filters $G_{12}(z)$ and $G_{21}(z)$ are parametrised by FIR filters of fixed degree p, even in the IIR case. Let us call ic the vector whose coefficients are the first $(4p+1)$ empirical intercorrelation coefficients between $r_1(n)$ and $r_2(n)$. Then, the coefficients of $G_{12}(z)$ and $G_{21}(z)$ are determined so as to minimize the sum of the square of the coefficients of ic. For each fixed value of $G_{12}(z)$ (resp. $G_{21}(z)$), the corresponding cost function is quadratic in the coefficients of $G_{21}(z)$ (resp. $G_{12}(z)$). It is therefore possible to derive a very simple iterative relaxation algorithm converging to one of the local minima of the cost function (see e.g. [7]). It is clear that the initialisation of the algorithm is crucial and that its convergence toward the global minimum is not guaranteed.

First, the existence of spurious solutions expressed in (4) is illustrated in the IIR case. The Figure 1 shows the norm ρ of the vector ic for different filters $G(z)$ obtained by calculating the polynomial truncation of (4) for different values of t. The plot can be compared with Figure 2 which shows the evolution versus t of the restoration rate defined by $sep = sep_1 + sep_2$ and

[2] In our case, D is invertible

$sep_i = \frac{\sum_{n=1}^{T}(s_i(n)-r_i(n))^2}{\sum_{n=1}^{T} s_i(n)^2}$ with T representing the number of samples.

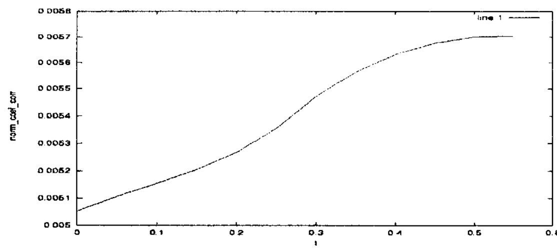

Figure 1: $\rho(t)$

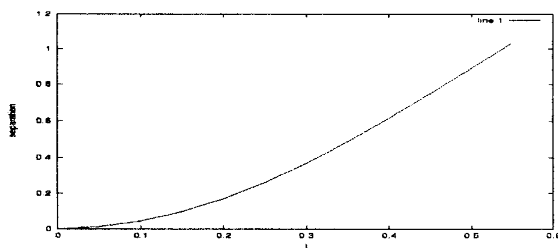

Figure 2: $sep(t)$

In these plots, it should be noticed that the value of ρ is nearly constant whereas the restoration rate increase noticeably. Moreover, at $t = 0$ and only at this value, the solution is the separating one. We again stress that our IIR filter $H(z)$ is "numerically" FIR, thus showing the robustness problems of the decorrelation approach.

4.2. Linear prediction

The following plot (Figure 3) compares the signal $s_1(n)$ $(*)$ and its estimate $r_1(n)$ $(+)$ on a duration of fifty samples in the IIR case. It is clear that the restoration rate is satisfactory. More details will

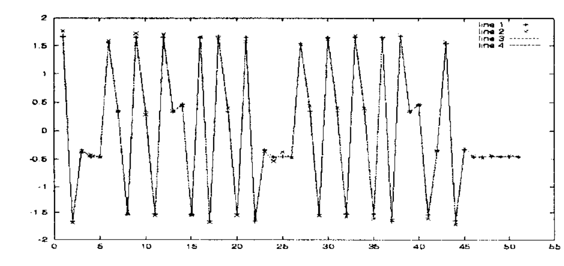

Figure 3: $s_1(n)$ and $\hat{s}_1(n)$

be found in the following table.

4.3. Comparison between these two methods

In the Table 1, comparative results between the two proposed approaches are shown. The case of $H(z)$ FIR is shown first, followed by the IIR case. For each case, two different initial values of $G(z)$ have been tested for the decorrelation method : the first one is closed to the optimal solution whereas the second one is farther.

	Decorrelation		Linear Prediction	
	Init. 1	Init. 2		
sep_1	0.0016169	0.92525	0.021393	FIR case
sep_2	0.0061503	2.9199	0.073438	
sep_1	0.15435	0.58399	0.0015590	IIR case
sep_2	0.15074	0.55316	0.0013727	

Table 1: Comparison

Even in the FIR mixture case, the decorrelation approach has some robustness problems : indeed, the results are very sensitive to the initialisation of $G(z)$ which seems to prove the existence of local minima. This is consistent with the IIR case where the decorrelation equations have an infinity of solutions. In practice, in the IIR case, the decorrelation algorithm can never produce a filter with good properties even if it is initialised closed to the optimal solution.

5. CONCLUSION

In this paper, we have considered the blind separation of a particular convolutive mixture. We have first analysed in some details the decorrelation approach. We have shown that it exists infinitely many non separating minimum phase causal solutions of the decorrelation equations and have provided a parameterisation of this set. We have deduced from these results that the decorrelation approach is, in practice, not robust. We have finally suggested to use a linear prediction based approach providing good results in the non Gaussian sources case.

6. REFERENCES

[1] J.F. Cardoso, B. Laheld, "Equivariant adaptive source separation", *IEEE Transactions on Signal Processing* vol. 44, n° 12, December 1996.

[2] P. Comon, "Analyse en composantes indépendantes et identification aveugle", *Revue Traitement du Signal*, vol. 7, n° 5, December 1990, pp. 435-450.

[3] N. Delfosse, Ph. Loubaton, "Adaptative blind identification of convolutive mixtures", *Proc. Int. Conf. Asilomar 95*,November 1995, pp. 341-345.

[4] S. Van Gerven, D. Van Compernolle, "Signal separation by symmetric adaptive decorrelation : stability, convergence and uniqueness",*IEEE Transactions on Signal Processing* vol 43, n° 7, July 1995, pp. 1602-1612.

[5] U. Lindgren, H. Broman, "Source separation using a criterion based on second order statistics", submitted to *IEEE Transactions on Signal Processing*.

[6] H.L. Nguyen Thi, Ch. Jutten, "Blind sources separation for convolutive mixtures", *Signal Processing*, vol. 45, pp. 209-229, 1995.

[7] E. Weinstein, M. Feder, A.V. Oppenheim, "Multi-channel signal separation by decorrelation", *IEEE Transactions on Speech Audio Processing*, vol 1, n° 4, October 1993.