Performance Analysis over Slow Fading Channels of a Half-Duplex Single-Relay Protocol: Decode or Quantize and Forward

Nassar Ksairi[(1)], Philippe Ciblat[(2)], Pascal Bianchi[(2)], Walid Hachem[(2)]

Abstract

In this work, a static relaying protocol, called *Decode or Quantize and Forward* (DoQF), is introduced for half duplex single-relay networks, and its performance is studied in the context of communications over slow fading wireless channels. The proposed protocol is inspired by the so-called *Compress-and-Forward* (CF) but only needs statistical Channel State Information at the Transmitter (CSIT). First, we analyze the behavior of the outage probability P_o of the proposed protocol as the SNR ρ tends to infinity. In this case, we prove that $\rho^2 P_o$ converges to a constant ξ. We refer to this constant as the *outage probability gain* and we derive its closed-form expression for a general class of wireless channels that includes Rayleigh and Rice. We furthermore prove that the DoQF protocol has the best achievable outage gain in the wide class of half-duplex static relaying protocols and we minimize ξ w.r.t the power allocation to the source and the relay and the durations of the slots. Next, we focus on Rayleigh channels to derive the *Diversity-Multiplexing Tradeoff* (DMT) of the DoQF. Our results show that the DoQF achieves the 2 by 1 MISO DMT upper-bound for multiplexing gains $r < 0.25$.

[(1)] Supélec, Plateau de Moulon 91192 Gif-sur-Yvette Cedex, France (nassar.ksairi@supelec.fr). Phone: +33 1 69 85 14 54, Fax: +33 1 69 85 14 69.

[(2)] CNRS / Telecom ParisTech (ENST), 46 rue Barrault 75634 Paris Cedex 13, France (philippe.ciblat@enst.fr,bianchi@telecom-paristech.fr,walid.hachem@enst.fr). Phone: +33 1 45 81 83 60, Fax: +33 1 45 81 71 44.

I. Introduction

Relaying has become a widely accepted means of cooperation in wireless networks. In this paper, we focus on networks composed of one source, one destination and one relay that operates under the half-duplex constraint *i.e.*, the relay can either receive or transmit, but not both at the same time. The relay thus listens to the source signal during a certain amount of time (the first slot) and is allowed to transmit towards the destination during the rest of the time (the second slot).

A wide range of relaying protocols have been proposed so far. Most of these protocols belong to one of the following families of relaying schemes: Amplify and Forward (AF) [1], [2], [3], Decode and Forward (DF) [4], [5], [6], [7] and Compress and Forward (CF) [4], [8], [9], [10], [11]. The first classical family of relaying protocols is formed by Amplify and Forward (AF) protocols for which the relay retransmits a scaled version of its received signal. A second well known family of protocols is formed by the Decode and Forward (DF) approaches. In this case, the relay listens to the source during the first slot of transmission and tries to decode the source message. If it succeeds, the relay forwards the (re-coded) source message during the second slot. In this context, Azarian *et al.* [7] proposed a *dynamic* version of the DF (DDF, Dynamic Decode and Forward) in which the slots durations are supposed to be adaptive as a function of the (random) state of the source-relay channel. Although the DDF is attractive from a theoretical point of view, an implementation of the DDF requires the use of coders-decoders with adaptive length. To the best of our knowledge, the design of such codes for the DDF is still in its early stages [12], [13], [14]. We now go back to the *static* protocols for which the relay listening time is constant and thus regardless of the channels realization. One of the most widespread *static* DF protocols is the so-called *non orthogonal* DF [4] (as opposed to the *orthogonal* DF [5]).

By "non orthogonal" it is meant that the source and the relay are simultaneously transmitting during the second slot. The non orthogonal DF will be simply designated as DF in the rest of this paper. Finally, another classical family of relaying protocols is the Compress and Forward (CF) [4], [8], [9], [10], [11]. In the standard version of the CF [8], the relay uses a Wyner-Ziv encoder [15] to produce a source encoded version of its received signal and forwards it assuming that the destination disposes of a side information (the signal received on the source-destination link). Moreover, the relay is assumed to have perfect knowledge of the the relay-destination and source-destination channel gains. In order to overcome the Wyner-Ziv encoder and/or the perfect CSIT assumption, a few strategies inspired by the CF scheme have also been proposed in the literature. We cite for example [4], [11] where the strong assumption of perfect knowledge by the relay of the source-destination and the relay-destination channels is replaced by a quantized feedback link from the destination to the relay. In [11], the case of no CSIT at the relay is also treated and the performance degrades dramatically. In [10], vector quantization is performed by carefully choosing the relay data rate in order to have reliable link between relay and destination and then applies a *Successive Interference Canceller* (SIC) at the destination side. Perfect CSIT is thus needed.

We recall the DMT [20] of any relaying scheme with a single relay is upper-bounded by the DMT of a 2×1 MISO system given by $d_{\text{MISO}}(r) = 2(1-r)^+$. In [7], it is shown the DDF achieves the MISO upper-bound for $r < 0.5$. As for the DF, it is known from [21] that it does not achieve the MISO bound for any r. Concerning the CF, it is MISO-achieving provided that Wyner-Ziv coding and perfect CSIT are assumed. In [10], it is proven that replacing Wyner-Ziv encoder with a standard vector quantization leads to a significant degradation of the DMT. In [2], new protocols corresponding to a hybrid AF and CF approach that does not need CSIT are proposed, but no DMT is provided to assess the merit of this approach. In the recent work [22], [23], a static protocol called "quantize-map-and-forward" is proven to achieve the MISO upper-bound of the DMT for any multiplexing gain. However, no practical coding-decoding architecture has been proposed yet to implement it. Therefore developing new *static* powerful protocol (without instantenous CSIT) whose the performance are close to the MISO upper-bound of the DMT is still worthy.

In our contribution, we consider the context where the instantaneous realizations of the source-destination and relay-destination channels are completely unknown by the relay. We only assume that the average powers of the channels are available. In this context, *we propose a new relaying technique which we shall refer to as the* Decode or Quantize and Forward *(DoQF), and we analyze its performance over slow fading wireless channels through the DMT and the outage gain. We especially show that the DoQF is DMT-optimal for multiplexing gains less than* 0.25 *and that its outage gain coincides with the lower-bound on outage gains of the wide class of half-duplex static protocols.* The DoQF can be considered either as an *augmented* DF scheme or as a non-standard *degraded* CF scheme without the need of perfect CSIT. Indeed, in DoQF protocol, the relay first tries to decode the source message based on the signal received during the first slot. If the latter step is successful, then similarly to the classical DF scheme, the relay retransmits a coded version of this message during the second slot based on an independent codebook. If the relay is not able to decode the message, it does not remain inactive, but it quantizes the received signal vector using a well chosen distortion value as done in [10], [11], but unlike these two works, the design parameters in our work are obtained assuming statistical CSIT. Moreover, the relay in [10], [11] always quantizes and never decodes and so only relies on CF whereas we combine the DF and the CF approaches.

The paper is organized as follows: the performance metrics and general notations are drawn in Section II. A detailed description of the new DoQF protocol is provided in Section III. The outage performance analysis and minimization at high SNR for a constant transmission rate R is addressed in Section IV. Section V is devoted to the DMT of DoQF. Numerical results are drawn in Section VI. Finally, Section VII is devoted to the conclusions. Due to page limitation, the proofs of all the theorems are omitted and are available on the following webpage [1].

II. Performance metrics and Notations

The source wants to transmit R nats per channel use [2]. The outage probability $P_o(\rho)$ is the probability that the number of transmitted nats exceeds the mutual information associated with the whole channel.Deriving $P_o(\rho)$ for all possible values of the SNR ρ is a difficult problem, but $P_o(\rho)$ can be well approximated in the high SNR regime. Indeed, $\rho^2 P_o(\rho)$ usually converges to a non-zero constant ξ as ρ tends to infinity. This constant is referred to as the *outage gain* [16], [17], [18], [19] and is a relevant performance metric for the design of relaying protocols.

The derivation of the outage gain assumes that the rate R is a constant w.r.t. the SNR ρ. One could as well take benefit of an increasing SNR to increase the transmission rate. When the rate $R = R(\rho)$ depends on the SNR, a relevant performance metric is the Diversity-Multiplexing Tradeoff (DMT) introduced in [20]. We remind that a relaying protocol achieves *multiplexing gain* r and *diversity gain* $d(r)$ if $R(\rho)$ and $P_o(\rho)$ satisfy:

$$\lim_{\rho\to\infty} \frac{R(\rho)}{\log \rho} = r \qquad\qquad \lim_{\rho\to\infty} \frac{\log P_o(\rho)}{\log \rho} = -d(r)\ . \tag{1}$$

[1] http://perso.telecom-paristech.fr/∼ciblat/publications.html#jnl

[2] for the sake of simplicity in the derivations part, the rate is evaluated via the natural logarithm instead of the base-2 logarithm; therefore, we introduce the ”nats” and not the ”bits”

Here, $d(r)$ will be referred to as the DMT of the relaying protocol.

Node 0 will coincide with the source, node 1 with the relay and node 2 with the destination. We denote by H_{ij} the complex random variable representing the wireless channel between node i and node j. Coefficients H_{ij} are independent and perfectly known at the receiving node j but unknown at each other node of the network. We define $G_{ij} = |H_{ij}|^2$, and we write as usual $f(\rho) \doteq \rho^d$ if $\lim_{\rho\to\infty} \frac{\log f(\rho)}{\log(\rho)} = d$. Notations $\dot{>}$, $\dot{<}$ are similarly defined. Finally, $(x)^+ = \max(0, x)$.

III. The Proposed DoQF Protocol

A. Description of the Protocol

The source needs to send information at a rate of R nats per channel use. The source has at its disposal a frame of length T and a dictionary of $\lfloor e^{RT} \rfloor$ Gaussian independent vectors with independent $\mathcal{CN}(0,1)$ elements each. We partition the word X_0 selected by the source as $X_0 = \left[X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}\right]^{\mathrm{T}}$ where the length of X_{00} and X_{01} is t_0T and t_1T respectively with $t_1 = 1 - t_0$. Here $t_0 < 1$ is a fixed parameter. The source transmits the vector $\sqrt{\alpha_0\rho}X_0 = \left[\sqrt{\alpha_0\rho}X_{00}^{\mathrm{T}}, \sqrt{\alpha_0\rho}X_{01}^{\mathrm{T}}\right]^{\mathrm{T}}$, where ρT represents the total energy spent by both the source and the relay. Note that $E_0 = \alpha_0\rho T$ is the source share of the total energy. Denote by E_1 the *average* energy spent by the relay. The energy E_1 should be selected such that the following (long-term) power constraint is respected

$$E_0 + E_1 \leq \rho T \; . \tag{2}$$

The relay listens to the source message for a duration of t_0T channel uses (slot 0). At the end of this slot, the signal of size t_0T received by the relay writes

$$Y_{10} = \sqrt{\alpha_0\rho}H_{01}X_{00} + V_{10} \; , \tag{3}$$

where each component of vector V_{10} is a unit variance Additive White Gaussian Noise (AWGN). Figure 1 represents the transmit and receive signals for each node.

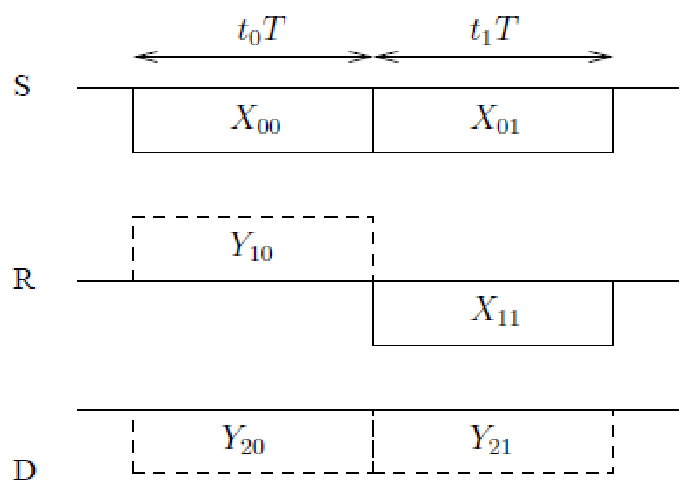

Figure 1. Transmit/Receive signals for source (S), relay (R) and destination (D)

We now consider separately the case when the relay manages to decode the source message and the case when it does not.

- **Case when the relay decodes the source message**

We can check from (3) that the relay is able to decode the source message if the event

$$\mathcal{E} = \{\omega : t_0 \log(1 + \alpha_0 \rho G_{01}(\omega)) > R\} \tag{4}$$

is realized. If this is the case, the relay transmits during the remainder of the frame (slot 1) the corresponding codeword of length t_1T from its own codebook. The relay codebook is composed of $\lfloor e^{RT} \rfloor$ Gaussian independent vectors with independent $\mathcal{CN}(0,1)$ elements each. The relay selects the codeword X_{11} and transmits $\sqrt{\alpha_1\rho}X_{11}$, which means that $\alpha_1\rho T$ is the relay share of the total energy. Finally, during the slots 0 and 1, the destination receives the signal

$$[Y_{20}^{\mathrm{T}}, Y_{21}^{\mathrm{T}}]^{\mathrm{T}} = \mathbf{H}_{\mathcal{E}}[X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}, X_{11}^{\mathrm{T}}]^{\mathrm{T}} + [V_{20}^{\mathrm{T}}, V_{21}^{\mathrm{T}}]^{\mathrm{T}} \;, \tag{5}$$

where

$$\mathbf{H}_{\mathcal{E}} = \begin{bmatrix} \sqrt{\alpha_0\rho}H_{02}\mathbf{I}_{t_0T} & 0 & 0 \\ 0 & \sqrt{\alpha_0\rho}H_{02}\mathbf{I}_{t_1T} & \sqrt{\alpha_1\rho}H_{12}\mathbf{I}_{t_1T} \end{bmatrix} .$$

Items of V_{20} (resp. V_{21}) are unit variance AWGN at the destination during slot 0 (resp. slot 1).

- **Case when the relay does not decode the source message (event $\overline{\mathcal{E}}$ is realized)**

The relay quantizes in this case the received signal during slot 0 and transmits a coded version of the quantized vector during slot 1 using the following steps.

a) Quantization: Denote by $\tilde{Y}_{10}$ the quantized version of the received vector Y_{10}. Vector $\tilde{Y}_{10}$ is constructed as follows. Clearly, all t_0T components of vector Y_{10} are independent and $\mathcal{CN}(0, \alpha_0\rho G_{01} + 1)$ distributed. Denote by $\Delta^2(\rho)$ the desired squared-error distortion per vector component:

$$\mathbb{E}|\tilde{Y}_{10}(i) - Y_{10}(i)|^2 \leq \Delta^2(\rho) \;.$$

The Rate Distortion Theorem for Gaussian sources [24] tells us that there exists a $(\left\lfloor e^{Q(\rho)t_0T} \right\rfloor, t_0T)$-rate distortion code (for some $Q(\rho) > 0$) which is achievable for distortion $\Delta^2(\rho)$ provided that

$$Q(\rho) > \log\left(\frac{\alpha_0\rho G_{01} + 1}{\Delta^2(\rho)}\right) \;. \tag{6}$$

Such a code can be constructed by properly selecting the quantized vector $\tilde{Y}_{10}$ among a quantizer-codebook formed by $\left\lfloor e^{Q(\rho)t_0T} \right\rfloor$ independent random vectors with distribution $\mathcal{CN}(0, (\alpha_0\rho G_{01} + 1 - \Delta^2(\rho))\mathbf{I}_{t_0T})$. Vector $\tilde{Y}_{10}$ is selected from this codebook in such a way that sequences Y_{10}

and $\tilde{Y}_{10}$ are jointly typical w.r.t. the joint distribution $p_{(Y,\tilde{Y})}$ given by

$$Y = \tilde{Y} + \Delta(\rho)Z \;, \tag{7}$$

where $\tilde{Y}$ and Z are independent random variables with respective distributions $\mathcal{CN}(0, \alpha_0\rho G_{01} + 1 - \Delta^2(\rho))$ and $\mathcal{CN}(0,1)$. Condition (6) ensures that such a vector $\tilde{Y}_{10}$ exists with high probability as $T \to \infty$. Parameter $Q(\rho)$ can be interpreted as the number of nats used to quantize one component of the received vector Y_{10}. It must be chosen such that (6) is satisfied. As the rhs of (6) depends on the channel gain G_{01}, it looks impossible at first glance to construct a fixed quantizer which is successful for any channel state. Nevertheless, recall that we are considering the case where event $\mathcal{E}$ is *not* realized *i.e.,* $t_0 \log(1 + \alpha_0\rho G_{01}) < R$. It is thus sufficient to define $Q(\rho) = \log\left(\frac{K}{\Delta^2(\rho)}\right)$, where K is any constant such that $K \geq e^{\frac{R}{t_0}}$. We choose $K = e^{\frac{R}{t_0}}$.

Remark: Condition (6) implies that inequality $\alpha_0\rho G_{01}+1 > \Delta^2(\rho)$ should hold. The quantization step is thus possible provided that the following event is realized

$$\mathcal{S} = \left\{\omega : \alpha_0\rho G_{01}(\omega) + 1 > \Delta^2(\rho)\right\} . \tag{8}$$

Event $\overline{\mathcal{S}}$ happens with negligible probability provided that $\Delta^2(\rho)$ is chosen properly.

b) Forwarding the Relay Message: During the second slot of length $t_1 T$, the relay must forward the index of the quantized vector among the possible $\left\lfloor e^{Q(\rho)t_0 T}\right\rfloor$ ones. To that end, it uses a Gaussian codebook with rate $Q(\rho)t_0/t_1$. If we denote by X_{11} the corresponding codeword, the signal transmitted by the relay can be written as $\sqrt{\phi(\rho)}X_{11}$, where $\phi(\rho)$ is the power of the relay. Function $\phi(\rho)$ should be selected such that the power constraint given by (2) is respected.

c) Processing at Destination: In case the relay has quantized the source message (event $\mathcal{S}$ defined by (8) is realized), the destination proceeds as follows. It first tries to recover the relay message

X_{11} received during slot 1 and uses it to help decode the source message. The signal of length $t_1 T$ received by the destination during the second slot can be written as

$$Y_{21} = \sqrt{\phi(\rho)} H_{12} X_{11} + \sqrt{\alpha_0 \rho} H_{02} X_{01} + V_{21} \; . \tag{9}$$

Note that (9) can be seen as a Multiple Access Channel (MAC). In order to recover X_{11} (and consequently $\tilde{Y}_{10}$) from (9), the destination interprets the source contribution as noise. It succeeds in recovering $\tilde{Y}_{10}$ if the event

$$\mathcal{F} = \left\{ \omega : t_1 \log \left(1 + \frac{\phi(\rho) G_{12}(\omega)}{\alpha_0 \rho G_{02}(\omega) + 1} \right) > Q(\rho) t_0 \right\} \tag{10}$$

is realized. We distinguish between three possible cases.

Events $\mathcal{S}$ and $\mathcal{F}$ are realized: In this case, the contribution of X_{11} in (9) can be canceled, and the resulting signal can be written as $Y_{21}' = \sqrt{\alpha_0 \rho} H_{02} X_{01} + V_{21}$. Moreover, it is a straightforward result of (7) that the conditional distribution $p_{\tilde{Y}|Y}$ is Gaussian with mean $\mathbb{E}\left[\tilde{Y}|Y\right] = \frac{1+\alpha_0\rho G_{01} - \Delta^2(\rho)}{1+\alpha_0 \rho G_{01}} Y$ and variance $\mathrm{var}\left(\tilde{Y}|Y\right) = \frac{\Delta^2(\rho)\left(1+\alpha_0\rho G_{01}-\Delta^2(\rho)\right)}{1+\alpha_0\rho G_{01}}$. We thus write

$$\tilde{Y}_{10} = \frac{1 + \alpha_0 \rho G_{01} - \Delta^2(\rho)}{1 + \alpha_0 \rho G_{01}} Y_{10} + \sqrt{\frac{\Delta^2(\rho)\left(1 + \alpha_0 \rho G_{01} - \Delta^2(\rho)\right)}{1 + \alpha_0 \rho G_{01}}} \tilde{Z} \; , \tag{11}$$

where vector $\tilde{Z}$ is AWGN independent of Y_{10} such that each of its components $\tilde{Z}(i)$ satisfies $\tilde{Z}(i) \sim \mathcal{CN}(0,1)$. Plugging $Y_{10} = \sqrt{\alpha_0 \rho} H_{01} X_{00} + V_{10}$ into (11), it follows that

$$\tilde{Y}_{10} = \sqrt{\gamma(G_{01}, \rho) \alpha_0 \rho} H_{01} X_{00} + \tilde{V}_{10} \; ,$$

where $\gamma(G_{01}, \rho) = \frac{\left(1+\alpha_0\rho G_{01} - \Delta^2(\rho)\right)^2}{(1+\alpha_0\rho G_{01})^2}$ and where vector $\tilde{V}_{10}$ is AWGN whose components satisfy $\tilde{V}_{10}(i) \sim \mathcal{CN}\left(0, \gamma(G_{01}, \rho) + \Delta^2(\rho) \sqrt{\gamma(G_{01}, \rho)}\right)$. In order to decode the source message, the overall received signal can be reconstructed as $Y_2 = \left[Y_{20}^{\mathrm{T}}, \tilde{Y}_{10}^{\mathrm{T}}, (Y_{21}')^{\mathrm{T}}\right]^{\mathrm{T}}$ given by

$$Y_2 = \mathbf{H}_{\mathcal{F}}[X_{00}^{\mathrm{T}}, X_{01}^{\mathrm{T}}]^{\mathrm{T}} + \check{V}_{10}, \tag{12}$$

where

$$\mathbf{H}_{\mathcal{F}} = \begin{bmatrix} \sqrt{\alpha_0\rho}H_{02}\mathbf{I}_{t_0T} & 0 \\ \sqrt{\gamma(G_{01},\rho)\alpha_0\rho}H_{01}\mathbf{I}_{t_0T} & 0 \\ 0 & \sqrt{\alpha_0\rho}H_{02}\mathbf{I}_{t_1T} \end{bmatrix},$$

and where $\check{V}_{10} = \left[V_{20}^T, \tilde{V}_{10}^T, V_{21}^T\right]^T$ is a zero-mean Gaussian noise with covariance matrix

$$\mathbb{E}[\check{V}_{10}\check{V}_{10}^*] = \begin{bmatrix} \mathbf{I}_{t_0T} & 0 & 0 \\ 0 & \sqrt{\gamma(G_{01},\rho) + \Delta^2(\rho)\sqrt{\gamma(G_{01},\rho)}}\mathbf{I}_{t_0T} & 0 \\ 0 & 0 & \mathbf{I}_{t_1T} \end{bmatrix}.$$

Events $\mathcal{S}$ and $\overline{\mathcal{F}}$ are realized: The destination will only be able to use Y_{20}, the signal received during slot 0. Note that in such a case, we get $Y_{20} = \sqrt{\alpha_0\rho}H_{02}X_{00} + V_{20}$.

Event $\overline{\mathcal{S}}$ is realized: In this case, the relay does not quantize the source message. This is like the case of a non cooperative transmission.

Finally, the outage probability of the DoQF protocol writes

$$P_o(\rho) = P_{o,1}(\rho) + P_{o,2}(\rho) + P_{o,3}(\rho) + P_{o,4}(\rho)\,, \tag{13}$$

where

- $P_{o,1}(\rho)$ is the probability that the destination is in outage *and* that the event $\mathcal{E}$ is realized:

$$P_{o,1}(\rho) = \Pr[t_0\log(1+\alpha_0\rho G_{02}) + t_1\log(1+\alpha_0\rho G_{02} + \alpha_1\rho G_{12}) \le R](1 - \Pr\left[\mathcal{E}\right])\,; \tag{14}$$

- $P_{o,2}(\rho)$ is the probability that the destination is in outage and that events $\overline{\mathcal{E}}$, $\mathcal{F}$, $\mathcal{S}$ are realized:

$$P_{o,2}(\rho) = \Pr\left[t_1 \log(1+\alpha_0\rho G_{02}) + t_0 \log\left(1+\alpha_0\rho G_{02} + \frac{\gamma(G_{01},\rho)\alpha_0\rho G_{01}}{\gamma(G_{01},\rho)+\Delta^2(\rho)\sqrt{\gamma(G_{01},\rho)}}\right) \right.$$
$$\left. \leq R,\ \overline{\mathcal{E}}, \mathcal{F}, \mathcal{S}\right] ; \tag{15}$$

- $P_{o,3}(\rho)$ is the probability that the destination is in outage and that events $\overline{\mathcal{E}}$, $\overline{\mathcal{F}}$, $\mathcal{S}$ are realized:

$$P_{o,3}(\rho) = \Pr[t_0 \log(1+\alpha_0\rho G_{02}) \leq R, \overline{\mathcal{E}}, \overline{\mathcal{F}}, \mathcal{S}] ; \tag{16}$$

- $P_{o,4}(\rho)$ is the probability that the destination is in outage and that events $\overline{\mathcal{E}}$, $\overline{\mathcal{S}}$ are realized:

$$P_{o,4}(\rho) = \Pr\left[\log(1+\alpha_0\rho G_{02}) \leq R, \overline{\mathcal{E}}, \overline{\mathcal{S}}\right] . \tag{17}$$

In Figure 2, the data processing steps at the destination node are summarized.

B. *On the selection of parameters* $t_0, t_1, \alpha_0, \alpha_1, \phi(\rho), \Delta^2(\rho)$

Parameters t_0, t_1, α_0, α_1, $\phi(\rho)$ should be selected such that constraint (2) is respected *i.e.*, such that $E_0+E_1 \leq \rho T$. Let us derive E_0 and E_1. The source transmits the signal $[\sqrt{\alpha_0\rho}X_{00}, \sqrt{\alpha_0\rho}X_{01}]$ spending the energy $E_0 = \alpha_0\rho T$. If event $\mathcal{E}$ is realized, then the relay transmits the signal $\sqrt{\alpha_1\rho}X_{11}$ and spends $\alpha_1\rho t_1 T$ Joules. If events $\overline{\mathcal{E}}$ and $\mathcal{S}$ are realized, the relay transmits $\sqrt{\phi(\rho)}X_{11}$ spending $\phi(\rho)t_1 T$ Joules. As for the case where event $\overline{\mathcal{S}}$ is realized, the relay remains inactive spending no energy. The average energy spent by the relay is thus $E_1 = \alpha_1\rho t_1 T\left(1-\Pr\left[\overline{\mathcal{E}}\right]\right) + \phi(\rho)t_1 T\Pr\left[\overline{\mathcal{E}}, \mathcal{S}\right]$. Putting all pieces together, the power constraint given by (2) writes

$$\alpha_0\rho + \alpha_1\rho t_1\left(1-\Pr\left[\overline{\mathcal{E}}\right]\right) + \phi(\rho)t_1\Pr\left[\overline{\mathcal{E}}, \mathcal{S}\right] \leq \rho\, . \tag{18}$$

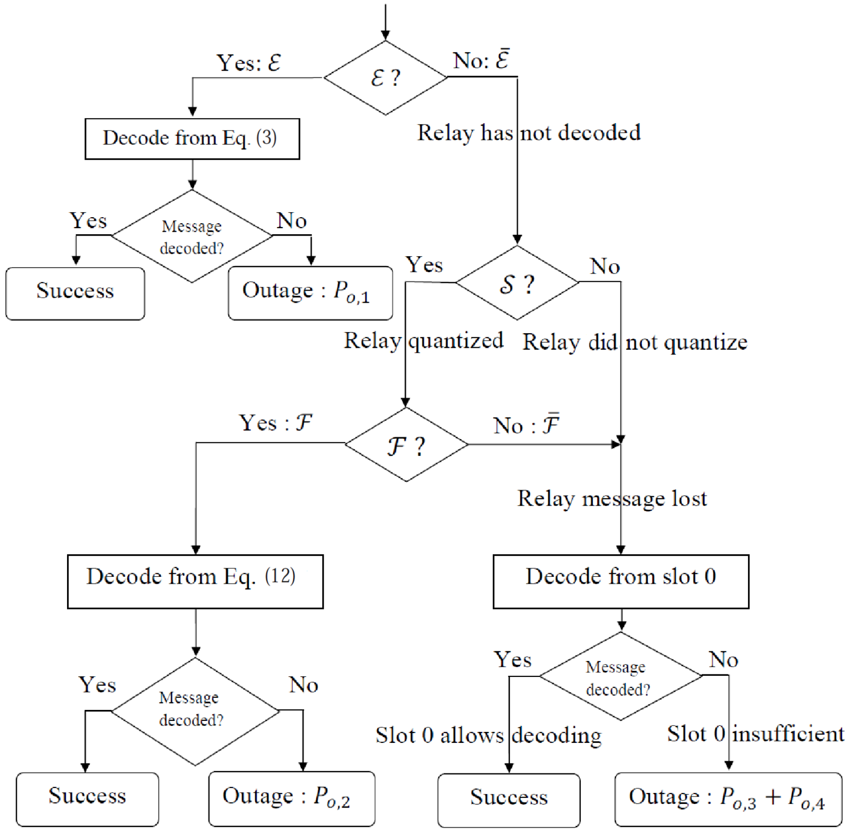

Figure 2. Data processing at the destination

The selection of t_0, t_1, α_0, α_1, $\phi(\rho)$ such that (18) is respected is addressed (along with the selection of $\Delta^2(\rho)$) in Sections IV and V. The rest of the paper is devoted to the study of the performance of the DoQF using two performance metrics: The outage gain and the DMT.

IV. Outage Probability Analysis of the DoQF Protocol

A. Notations and Channel Assumptions

Recall that H_{ij} is the random variable that represents the wireless channel between nodes i and j of the network ($i, j \in \{0, 1, 2\}$), and that $G_{ij} = |H_{ij}|^2$ designates the power gain of this channel. In this section, all variables G_{ij} are assumed to have densities $f_{G_{ij}}(x)$ which are right continuous at zero. This assumption is satisfied in particular by the so-called Rayleigh and Rice channels. Note that except for this mild assumption, we do not make any assumption on the channels probability distributions. We denote by c_{ij} the limit $c_{ij} = f_{G_{ij}}(0^+)$ and we assume that all these limits are positive and available to the resource allocation unit. For instance, in the Rayleigh case, H_{ij} is complex circular Gaussian with zero mean and variance σ_{ij}^2. In this case, G_{ij} has the exponential distribution $f_{G_{ij}}(x) = \sigma_{ij}^{-2} \exp(-x/\sigma_{ij}^2)\mathbf{1}\{x \geq 0\}$, and in particular $c_{ij} = \sigma_{ij}^{-2}$. Here, for any subset $\mathcal{A}$ of $\mathbb{R}$, we denote by $\mathbf{1}\{\mathcal{A}\}$ the indicator function of the set $\mathcal{A}$.

B. Lower Bound on the Outage Gain of Static Half-Duplex Protocols

Before deriving the outage gain of the DoQF protocol, we first derive a bound on the outage performance of the wide class of half-duplex static relaying protocols. This class is indexed using parameters t_0, α_0, α_1. For each value of these parameters, the class is denoted by $\mathcal{P}_{\text{HD}}(t_0, \alpha_0, \alpha_1)$ and is defined as the set of all half-duplex static relaying protocols which satisfy:

- The source has at its disposal a dictionary of $\lfloor e^{RT} \rfloor$ codewords. Each codeword $X_0 = \left[X_{00}^{\text{T}}, X_{01}^{\text{T}}\right]^{\text{T}}$ is a vector of length T channel uses.
- The source transmit power $\frac{1}{T}\sum_{i=1}^{T} E\left[|X_0(i)|^2\right]$ satisfies the following high SNR constraint

$$\lim_{\rho\to\infty} \frac{\frac{1}{T}\sum_{i=1}^{T} \mathbb{E}\left[|X_0(i)|^2\right]}{\rho} \leq \alpha_0 \ . \tag{19}$$

- The relay listens to the source signal during the first $t_0 T$ channel uses out of the T channel uses which is the duration of the whole transmission. The relay has at its disposal a dictionary of codewords X_{11} of length $(1-t_0)T$ channel uses each.
- During the last $(1-t_0)T$ channel uses, the relay average transmit power satisfies

$$\lim_{\rho\to\infty} \frac{\frac{1}{(1-t_0)T}\sum_{i=1}^{(1-t_0)T} \mathbb{E}\left[|X_{11}(i)|^2\right]}{\rho} \leq \alpha_1 \ . \tag{20}$$

The above definition does not impose any particular codewords distribution neither any constraints on the powers for finite values of the SNR ρ. Constraints (19) and (20) restrict only the way the average transmit powers behave in *the high SNR regime*.

Theorem 1. *For any static half-duplex relaying protocol from the class* $\mathcal{P}_{HD}(t_0, \alpha_0, \alpha_1)$*, the outage gain* $\xi = \lim_{\rho\to\infty} \rho^2 P_o(\rho)$ *is lower-bounded by* $\xi_{\text{CS-HD}}$*, where*

$$\xi_{\mathrm{CS-HD}} = \frac{c_{02}c_{01}}{\alpha_0^2}\left(\frac{1}{2} + \frac{\exp(2R)}{4t_0 - 2} - \frac{t_0 \exp(R/t_0)}{2t_0 - 1}\right) + \frac{c_{02}c_{12}}{\alpha_0\alpha_1}\left(\frac{1}{2} + \frac{\exp(2R)}{4t_1 - 2} - \frac{t_1 \exp(R/t_1)}{2t_1 - 1}\right) . \tag{21}$$

The above lower-bound has been derived using the Cut-Set (CS) bound for Half-Duplex (HD) relay channels. This explains the use of the subscript (CS-HD) to designate this bound.

We now derive and compare the outage gain of the DoQF protocol with the above lower-bound.

C. Outage Gain of the DoQF Protocol

Theorem 2. *Assume that the quantization squared-error* $\Delta^2(\rho)$ *and the relay power* $\phi(\rho)$ *satisfy*

$$\lim_{\rho} \phi(\rho) = +\infty, \tag{22}$$

$$\lim_{\rho} \frac{\phi(\rho)}{\rho^2} = 0, \tag{23}$$

$$\lim_{\rho} \Delta^2(\rho) = 0, \tag{24}$$

$$\lim_{\rho} \left(\phi(\rho)^{t_1}\, \Delta^2(\rho)^{t_0}\right) = +\infty. \tag{25}$$

The outage gain ξ_{DoQF} *associated with the proposed DoQF protocol coincides with the lower-bound given by* (21)*, i.e.,* $\xi_{\mathrm{DoQF}} = \xi_{\mathrm{CS-HD}}$.

Theorem 2 states that the DoQF is outage-gain-optimal in the wide class of half-duplex static relaying protocols. Moreover, due to (22)-(25), we can choose $\phi(\rho) = \alpha_1\rho$ (provided that $\rho^{-\frac{t_1}{t_0}} \dot{<} \Delta^2(\rho) \dot{<} 1$). It is thus optimal from an outage gain perspective to let the relay transmit at a constant power regardless of whether the source message has been decoded or not.

D. Power and Time Optimization

We derive t_0, t_1, α_0, α_1 minimizing ξ_{DoQF} subject to constraint (18). Let us examine (18) when the SNR ρ tends to infinity. We first divide the two sides of this power constraint by ρ, which leads to $\alpha_0 + \alpha_1 t_1 \left(1 - \text{Pr}\left[\overline{\mathcal{E}}\right]\right) + \frac{\phi(\rho)}{\rho} t_1 \text{Pr}\left[\overline{\mathcal{E}}, \mathcal{S}\right] \leq 1$, where $\text{Pr}[\overline{\mathcal{E}}] = \text{Pr}\left[t_0 \log(1 + \alpha_0 \rho G_{01}) \leq R\right]$. It is useful to write the term $\frac{\phi(\rho)}{\rho} t_1 \text{Pr}\left[\overline{\mathcal{E}}, \mathcal{S}\right]$ in the lhs of the above inequality as $t_1 \frac{\phi(\rho)}{\rho^2} \rho \text{Pr}\left[\overline{\mathcal{E}}, \mathcal{S}\right]$. Recall that due to (23), $\lim_{\rho\to\infty} \frac{\phi(\rho)}{\rho^2} = 0$. Furthermore, it is straightforward to check that $\rho \text{Pr}\left[\overline{\mathcal{E}}, \mathcal{S}\right]$ is upper-bounded for any $\rho \in \mathbb{R}_+$. Indeed, $\lim_{\rho\to\infty} \rho \text{Pr}\left[\overline{\mathcal{E}}, \mathcal{S}\right]$ is a constant. Putting all pieces together, the power constraint at high SNR writes as $\alpha_0 + t_1\alpha_1 \leq 1$. Note that this constraint is not convex in α_0, α_1, t_1. It will be convenient to replace it with a convex constraint by making the change of variables $\beta_0 = \alpha_0$ and $\beta_1 = \alpha_1 t_1$. The power constraint thus becomes

$$\beta_0 + \beta_1 \leq 1 \, . \tag{26}$$

It can be shown [19] that $(t_1, \beta_0, \beta_1) \mapsto \xi_{\text{DoQF}}$ is convex on $(0,1) \times (0,\infty)^2$. Furthermore, the minimization of $\xi_{\text{DoQF}}(t_1, \beta_0, \beta_1)$ given constraint (26) reduces to minimizing ξ_{DoQF} on the line segment of $\mathbb{R}_+^2$ defined by $\beta_0 + \beta_1 = 1$. Function $\xi_{\text{DoQF}}(t_1, \beta_0, 1 - \beta_0)$ defined on $(0,1)^2$ is convex as it coincides with the restriction of $\xi_{\text{DoQF}}(t_1, \beta_0, \beta_1)$ to a line segment. So $\xi_{\text{DoQF}}(t_1, \beta_0, 1 - \beta_0)$ goes to infinity on the frontier of $(0,1)^2$. Therefore, the minimum is in the interior of $(0,1)^2$, and can be obtained by a descent method [25]. The optimization problem is convex which simplifies greatly the algorithm complexity. The simplest way is to proceed into two steps: we first evaluate the cost function on a 2D (*coarse* step) grid in $(0,1)^2$ to find rough estimation of this optimal power and time distribution. Then a *fine* step can be implemented through a gradient-descent algorithm initialized with the *coarse* estimates. Notice that the optimal distribution has to be updated only when the channel statistics (and not the channel realization) are varying. As the channel statistics have usually a large coherence time, the distribution update has to be done only seldom and so does not consume a lot of energy and time.

V. DMT Analysis of the DoQF Protocol

In this section, wireless channels are assumed to be Rayleigh distributed and the transmission rate is assumed to be a function of the SNR ρ satisfying $R = R(\rho) \doteq r \log \rho$ (see (1)).

A. On the Selection of $\Delta^2(\rho)$ and $\phi(\rho)$ from a DMT Perspective

In Section IV, parameters $\Delta^2(\rho)$ and $\phi(\rho)$ were chosen from an outage gain perspective such that (22)-(25) are satisfied. In the current section, we are interested in choices of $\Delta^2(\rho)$ and $\phi(\rho)$ that are relevant from a DMT perspective. In the sequel, we assume

$$\Delta^2(\rho) \doteq \rho^{\delta} , \tag{27}$$

where parameter δ will be fixed later. The power $\phi(\rho)$ should be chosen without violating constraint (18). We recall that the term Pr $\left[\overline{\mathcal{E}}, \mathcal{S}\right]$ in (18) is given by Pr $\left[\overline{\mathcal{E}}, \mathcal{S}\right] = \Pr\left[t_0 \log(1 + \alpha_0 \rho G_{01}) \le R(\rho), 1 + \alpha_0 \rho G_{01} > \Delta^2(\rho)\right]$. It is straightforward to show that Pr $\left[\overline{\mathcal{E}}, \mathcal{S}\right] \doteq \rho^{-(1-r/t_0)^+}$ (provided that $\delta \le 1 - \left(1 - \frac{r}{t_0}\right)^+$). The (asymptotic) power constraint can be thus written as

$$\phi(\rho) \dot{\le} \rho^{1+(1-r/t_0)^+} . \tag{28}$$

If we choose $\delta < 0$, then $\Delta^2(\rho)$ and $\phi(\rho)$ given by (27) and (28) also satisfy constraints (22)-(25). However, this does not necessarily yield the best DMT performance of the protocol.

B. DMT of the DoQF protocol

Denote by $d(t_0, \delta, r)$ the DMT of DoQF for fixed values of t_0 and δ:

$$d(t_0, \delta, r) = -\lim_{\rho\to\infty} \frac{\log P_o(\rho)}{\log \rho} , \tag{29}$$

where $P_o(\rho)$ is the outage probability of the protocol. We define the final DMT of DoQF as

$$d^*_{\mathrm{DoQF}}(r) = \sup_{t_0,\delta} d(t_0, \delta, r) , \tag{30}$$

Theorem 3. *Assume that the relay power and quantization squared-error distortion satisfy $\phi(\rho) \doteq \rho^{1+(1-r/t_0)^+}$ and $\Delta^2(\rho) \doteq \rho^\delta$, respectively. The DMT of the DoQF is given by*

$$d^*_{\mathrm{DoQF}}(r) = \begin{cases} 2(1-r)^+ & \text{for } r \le \frac{1}{4} \\ 2 - \frac{r}{1-v^*(r)} & \text{for } \frac{1}{4} < r \le \frac{2(\sqrt{5}-1)}{9-\sqrt{5}} \\ 2 - \frac{2}{3-\sqrt{5}} r & \text{for } \frac{2(\sqrt{5}-1)}{9-\sqrt{5}} < r \le \frac{\sqrt{5}-1}{\sqrt{5}+1} \\ (2-r)(1-r) & \text{for } r > \frac{\sqrt{5}-1}{\sqrt{5}+1} \end{cases} , \tag{31}$$

where $v^*(r)$ is the unique solution in $\left[\frac{1}{2}, \frac{2}{\sqrt{5}+1}\right]$ to the following equation.

$$2(1+r)v^3 - (4+5r)v^2 + 2(1+4r)v - 4r = 0 . \tag{32}$$

The MISO upper-bound is thus reached by the DoQF for $r < 0.25$, but the DMT of the protocol deviates from the MISO bound for $r > 0.25$. Note that we allowed t_0 and δ to depend on the multiplexing gain r. This additional degree of freedom will not change the fact that the

DoQF protocol is static. Indeed, parameters t_0 and δ do not depend on any channel coefficients.

VI. Numerical Illustrations and Simulations

Simulations have been carried out assuming that channels are Rayleigh distributed *i.e.*, $H_{ij} \sim \mathcal{CN}(0, \sigma_{ij}^2)$. Variance σ_{ij}^2 is a function of the distance d_{ij} between nodes i and j following a path loss model with exponent equal to 3: $\sigma_{ij}^2 = Cd_{ij}^{-3}$, where the constant C is chosen in such a way that $\sigma_{02}^2 = 1$. The data rate is fixed to 2 bits per channel use.

In Figure 3, outage probability performance with equal duration time slots and equal amplitudes for both the DF and the DoQF is compared to the performance after time and power optimization for different values of the SNR ρ. Both the simulated outage probability $P_o(\rho)$ and the approximated outage probability $\frac{\xi_{DoQF}}{\rho^2}$ are plotted in this figure. The relay is assumed to lie at two thirds of the source-destination distance on the source-destination line segment. Substantial gains are observed between the DF and the DoQF, and between optimized and non optimized protocols. Note that minimizing the outage gain continues to reduce the outage probability of the protocol even for moderate values of the SNR.

Figure 4 represents the outage gains for the DoQF and the DF versus the position $d_{0,1}$ of the relay. Note from the figure that the farther the relay from the source is, the better DoQF compared to DF works. This fact can be explained as follows: If the relay is close to the destination, it will be more often in outage and the Quantization step will thus operate more often.

In Figure 5, we plot the ratios of the outage gains with equal times and equal powers to the optimized outage gains as a function of the position $d_{0,1}$ of the relay. Note from this figure that optimizing the slots durations and the power allocation yields larger performance gains for both the DF and the DoQF when the relay is too close or too far from the source.

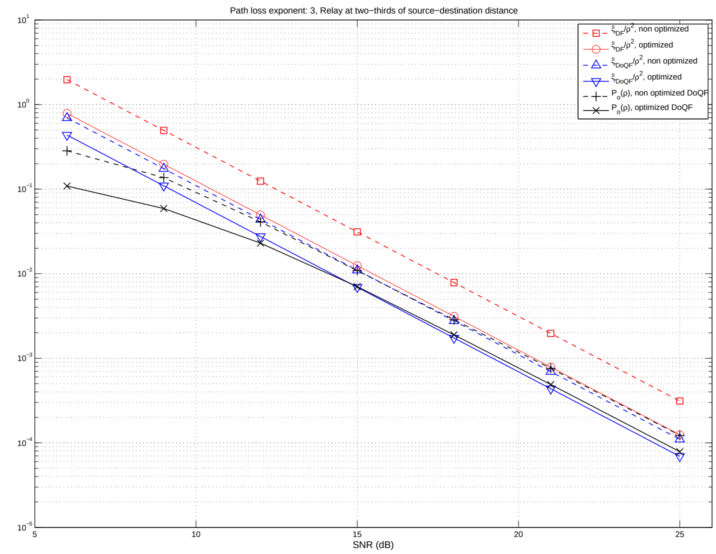

Figure 3. Outage performance of the DF and DoQF protocols

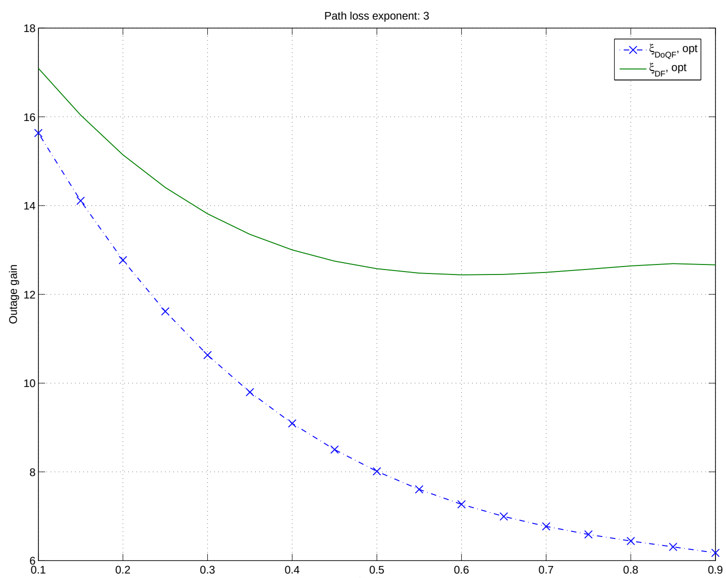

Figure 4. Outage gain of DF and DoQF versus relay position

In Figure 6, we plot the DMT of the DoQF, orthogonal DF, (non orthogonal) DF, non orthogonal AF (NAF), DDF, CF (with and without Wyner-Ziv coding [10]) and the MISO

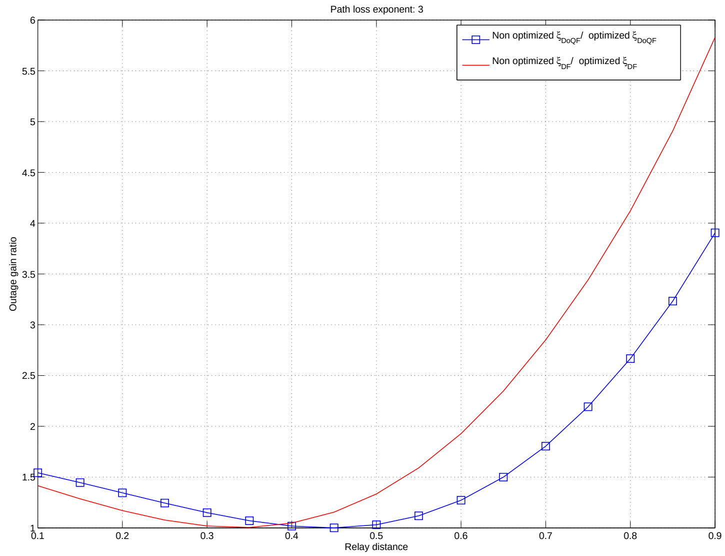

Figure 5. Outage gain ratio of DF and DoQF versus relay position

upper-bound. The DoQF outperforms the other static protocols that are *not* based on perfect CSIT. In contrast, the DDF protocol is still better than the DoQF but its dynamic approach leads to several implementation difficulties. The CF protocol with Wyner-Ziv coding (which needs perfect CSIT at the relay node) is DMT-optimal while its non Wyner-Ziv variant without CSIT [11] never achieves the MISO upper-bound and unfortunately offers poor performance.

In Figure 7, the optimal sizes of slot 0 for the DF (as computed in [21]) and the DoQF are plotted. We remark that, when r is small enough, slots 0 and 1 have the same length. When r increases, the duration of relay listening increases also. As a consequence, the duration for the quantization step thus decreases and the DoQF becomes closer to the DF as seen on the DMT.

VII. Conclusions

A static relaying protocol (DoQF) has been introduced for half-duplex single-relay scenarios. The proposed DoQF involves practical coding-decoding strategies at both the relay and the

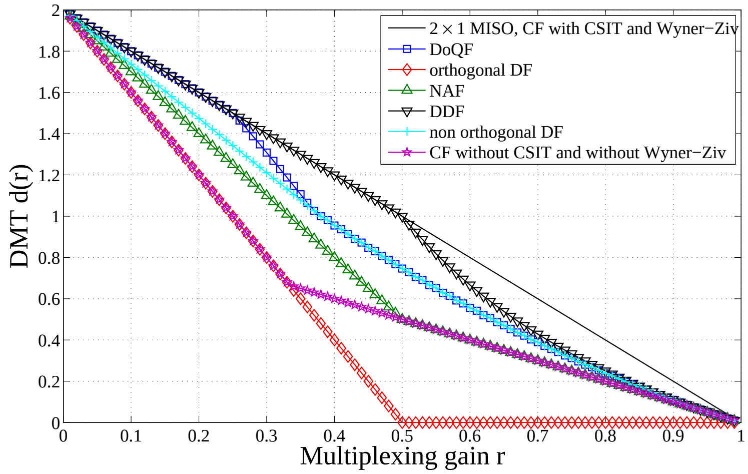

Figure 6. DMT of the DoQF and other protocols

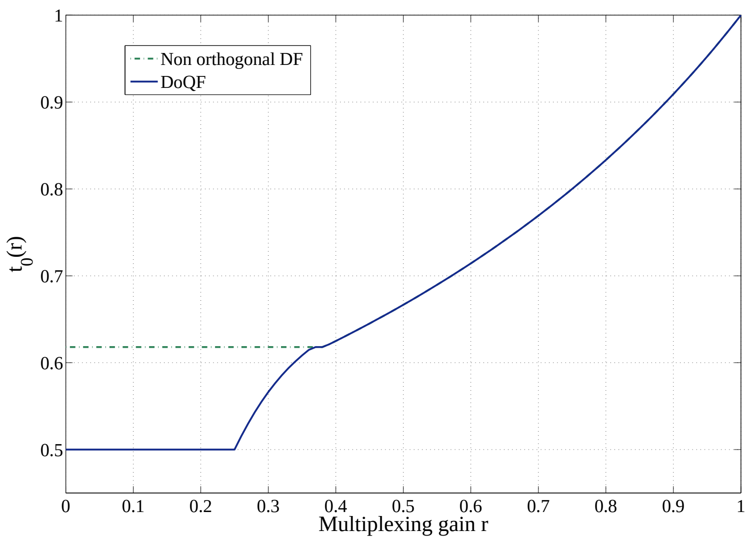

Figure 7. Optimal t_0 for DF and DoQF

destination. The performance of this protocol has been studied in the context of communications over slow fading wireless channels using two relevant performance metrics: The outage gain and the diversity multiplexing tradeoff (DMT). The DoQF protocol has been shown to be optimal in terms of outage gain in the wide class of half-duplex static relaying protocols. The proposed protocol has been finally shown to achieve the DMT of MISO for multiplexing gains $r \leq 0.25$.

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APPENDIX A

PROOF OF THEOREM 1

The capacity of any static relaying protocol is limited by the cut-set upper-bound. In this appendix, we derive the outage gain associated with the cut-set capacity. We prove next that this outage gain is equal to $\xi_{\text{CS-HD}}$ given by (21).

The cut-set upper-bound on the capacity of any half-duplex single-relay protocol from the class $\mathcal{P}_{\text{HD}}(t_0, \alpha_0, \alpha_1)$, with a listening time equal to $t_0 T$ and a cooperation time equal to $(1-t_0)T = t_1 T$, is given by

$$C_{\text{CS-HD}} = \lim_{T\to\infty} \frac{1}{T} \max_{p(X_{00},X_{01},X_{11})} \min\Big\{ I(X_{00};Y_{10},Y_{20}) + I(X_{01};Y_{21}|X_{11}), \\ I(X_{00};Y_{20}) + I(X_{01},X_{11};Y_{21})\Big\} , \tag{33}$$

where the maximization in (33) is with respect to all the joint distributions of X_{00}, X_{01} and X_{11} that satisfy the power constraints (19) and (20). It can be shown that the maximum in (33) is achieved when vectors X_{00}, X_{01} and X_{11} are zero-mean i.i.d Gaussian with covariance matrices that satisfy constraints (19) and (20). The cut-set upper-bound can thus be written as

$$\begin{aligned} C_{\text{CS-HD}} = \min\Big\{ & t_0 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{01} + \mathbb{E}\left[|X_0(i)|^2\right] G_{02}\right) + t_1 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{02}\right) , \\ & t_0 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{02}\right) + t_1 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{02} + \mathbb{E}\left[|X_{11}(i)|^2\right] G_{12}\right) \Big\} \\ = \min & \{C_{\text{SIMO}}, C_{\text{MISO}}\} , \end{aligned} \tag{34}$$

where C_{SIMO} and C_{MISO} are defined in order to simplify the presentation of the proof as follows:

$$C_{\text{SIMO}} = t_0 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{01} + \mathbb{E}\left[|X_0(i)|^2\right] G_{02}\right) + t_1 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{02}\right)$$

$$C_{\text{MISO}} = t_0 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{02}\right) + t_1 \log\left(1 + \mathbb{E}\left[|X_0(i)|^2\right] G_{02} + \mathbb{E}\left[|X_{11}(i)|^2\right] G_{12}\right) \ .$$

We now prove that the limit $\lim_{\rho\to\infty} \rho^2 \Pr[C_{\mathrm{CS-HD}} \leq R]$ exists and that it is equal to $\xi_{\mathrm{CS-HD}}$ given by (21). For that sake, note that the following holds:

$$\begin{aligned}
\Pr[C_{\text{CS-HD}} \leq R] =& 1 - \Pr\left[C_{\text{CS-HD}} > R\right] \\
=& 1 - \Pr[C_{\text{SIMO}} > R, C_{\text{MISO}} > R] \\
\geq& 1 - \Pr\left[C_{\text{SIMO}} > R\right] \times \Pr\left[C_{\text{MISO}} > R\right] \\
=& 1 - \left(1 - \Pr\left[C_{\text{SIMO}} \leq R\right]\right) \times \left(1 - \Pr\left[C_{\text{MISO}} \leq R\right]\right) \ .
\end{aligned}$$

Now define

$$P_{o,\text{SIMO}} = \Pr\left[C_{\text{SIMO}} \leq R\right]$$

$$P_{o,\text{MISO}} = \Pr[C_{\text{MISO}} \leq R] \ .$$

Using these new notations, we conclude that the following lower-bound on $\Pr[C_{\text{CS-HD}} \leq R]$ holds:

$$\Pr[C_{\text{CS-HD}} \leq R] \geq P_{o,\text{SIMO}} + P_{o,\text{MISO}} - P_{o,\text{SIMO}} P_{o,\text{MISO}} \ . \tag{35}$$

In the same way, it is straightforward to show that $\Pr[C_{\text{CS-HD}} \leq R]$ can be upper-bounded as follows.

$$\Pr[C_{\text{CS-HD}} \leq R] \leq P_{o,\text{SIMO}} + P_{o,\text{MISO}} + P_{o,\text{SIMO}} P_{o,\text{MISO}} \ . \tag{36}$$

Now, we can use the same arguments and tools employed in Subsection B to prove that

$$\lim_{\rho\to\infty} \rho^2 P_{o,\text{SIMO}} = \frac{c_{02}c_{01}}{\alpha_0^2} \int_{\mathbb{R}_+^2} \mathbf{1}\left\{t_1 \log(1+u) + t_0 \log\left(1+u+v\right) \leq R\right\} dudv \tag{37}$$

$$\lim_{\rho\to\infty} \rho^2 P_{o,\text{MISO}} = \frac{c_{02}c_{12}}{\alpha_0\alpha_1} \int_{\mathbb{R}_+^2} \mathbf{1}\{t_0 \log(1+u) + t_1 \log(1+u+v) \leq R\} dudv \tag{38}$$

$$\lim_{\rho\to\infty} \rho^2 P_{o,\text{SIMO}} P_{o,\text{MISO}} = 0\ . \tag{39}$$

Note that the integrals in the rhs of (37) and (38) coincide with the two integrals in the rhs of (48). We can thus write

$$\lim_{\rho\to\infty} \rho^2 P_{o,\text{SIMO}} = \frac{c_{02}c_{01}}{\alpha_0^2} \left(\frac{1}{2} + \frac{\exp(2R)}{4t_0 - 2} - \frac{t_0 \exp(R/t_0)}{2t_0 - 1} \right) \tag{40}$$

$$\lim_{\rho\to\infty} \rho^2 P_{o,\text{MISO}} = \frac{c_{02}c_{12}}{\alpha_{01}\alpha_{02}} \left(\frac{1}{2} + \frac{\exp(2R)}{4t_1 - 2} - \frac{t_1 \exp(R/t_1)}{2t_1 - 1} \right)\ . \tag{41}$$

Combining (35), (36), (39), (40) and (41) we conclude that

$$\lim_{\rho\to\infty} \rho^2 \Pr[C_{\text{CS-HD}} \leq RT] = \xi_{\text{CS}-\text{HD}}\ ,$$

where $\xi_{\text{CS-HD}}$ is the lower-bound defined by (21). Note that since $C_{\text{CS-HD}}$ is an upper-bound on the capacity of any static half-duplex relaying protocol belonging to the class $\mathcal{P}_{\text{HD}}(t_0, \alpha_0, \alpha_1)$, then $\xi_{\text{CS-HD}}$ which satisfies $\xi_{CS-HD} = \lim_{\rho\to\infty} \rho^2 \Pr[C_{\text{CS-HD}} \leq RT]$ is a lower-bound on the outage gain of any protocol from the class $\mathcal{P}_{\text{HD}}(t_0, \alpha_0, \alpha_1)$. This completes the proof of Theorem 1.

Appendix B

Proof of Theorem 2

Recall the definition of $P_o(\rho)$ given by (13) as the outage probability associated with the DoQF protocol. In order to prove Theorem 2, we need to show that $\rho^2 P_o(\rho)$ converges as

$\rho \to \infty$ and to derive the outage gain ξ_{DoQF} given by $\xi_{\text{DoQF}} = \lim_{\rho\to\infty} \rho^2 P_o(\rho)$. According to (13), $P_o(\rho) = P_{o,1}(\rho) + P_{o,2}(\rho) + P_{o,3}(\rho) + P_{o,4}(\rho)$, where $P_{o,1}(\rho)$, $P_{o,2}(\rho)$, $P_{o,3}(\rho)$ and $P_{o,4}(\rho)$ are defined by (14), (15), (16) and (17) respectively. Therefore, we need to first compute the limits $\lim_{\rho\to\infty} \rho^2 P_{o,1}(\rho)$, $\lim_{\rho\to\infty} \rho^2 P_{o,2}(\rho)$, $\lim_{\rho\to\infty} \rho^2 P_{o,3}(\rho)$ and $\lim_{\rho\to\infty} \rho^2 P_{o,4}(\rho)$ in order to obtain the outage gain ξ_{DoQF}. It has been proved in [19] that

$$\lim_{\rho\to\infty} \rho^2 P_{o,1}(\rho) = \frac{c_{02}c_{12}}{\alpha_0\alpha_1} \int_{\mathbb{R}_+^2} \mathbf{1}\{t_0 \log(1+u) + t_1 \log(1+u+v) \le R\} du dv \, , \tag{42}$$

where c_{01} and c_{12} has been defined in Subsection IV-A as $c_{01} = f_{G_{01}}(0+)$ and $c_{12} = f_{G_{12}}(0+)$ respectively. The steps of the proof that (42) holds are very similar to the steps given below for the derivation of $\lim_{\rho\to\infty} \rho^2 P_{o,2}(\rho)$. Refer to the definition of $P_{o,2}(\rho)$ given by (15) as

$$\begin{aligned} P_{o,2}(\rho) = \Pr\Bigg[& t_1 \log(1+\alpha_0\rho G_{02}) + t_0 \log\left(1 + \alpha_0\rho G_{02} + \frac{\gamma(G_{01},\rho)\alpha_0\rho G_{01}}{\gamma(G_{01},\rho) + \Delta^2(\rho)\sqrt{\gamma(G_{01},\rho)}}\right) < R, \\ & \overline{\mathcal{E}}, \mathcal{F}, \mathcal{S} \Bigg] \, , \end{aligned} \tag{43}$$

where $\gamma(G_{01},\rho) = \frac{\left(1+\alpha_0\rho G_{01} - \Delta^2(\rho)\right)^2}{(1+\alpha_0\rho G_{01})^2}$. Plugging the definitions of events $\mathcal{E}$, $\mathcal{S}$ and $\mathcal{F}$ given respectively by (4), (8) and (10) into (43) leads to

$$\begin{aligned} P_{o,2}(\rho) = \int_{\mathbb{R}_+^3} & \mathbf{1}\left\{ t_1 \log(1+\alpha_0\rho x) + t_0 \log\left(1 + \alpha_0\rho x + \frac{\gamma(y,\rho)\alpha_0\rho y}{\gamma(y,\rho) + \Delta^2(\rho)\sqrt{\gamma(y,\rho)}}\right) \le R \right\} \\ & \times \mathbf{1}\left\{ t_0 \log(1+\alpha_0\rho y) \le R \right\} \mathbf{1}\left\{ 1 + \alpha_0\rho y > \Delta^2(\rho) \right\} \\ & \times \mathbf{1}\left\{ t_1 \log\left(1 + \frac{\phi(\rho) z}{1+\alpha_0\rho x}\right) > t_0 Q(\rho) \right\} f_{G_{02}}(x) f_{G_{01}}(y) f_{G_{12}}(z) dx dy dz \, , \end{aligned}$$

By making the change of variables $u = \alpha_0 \rho x$ and $v = \alpha_0 \rho y$ we obtain

$$\rho^2 P_{o,2}(\rho) = \frac{1}{\alpha_0^2} \int_{\mathbb{R}_+^3} \mathbf{1} \left\{ t_1 \log(1+u) + t_0 \log\left(1 + u + \frac{\gamma(v,\rho)v}{\gamma(v,\rho) + \Delta^2(\rho)\sqrt{\gamma(v,\rho)}}\right) \leq R \right\}$$
$$\times \mathbf{1}\left\{t_0 \log(1+v) \leq R\right\} \mathbf{1}\left\{1 + v > \Delta^2(\rho)\right\}$$
$$\times \mathbf{1}\left\{ t_1 \log\left(1 + \frac{\phi(\rho)z}{1+u}\right) > t_0 Q(\rho)\right\} f_{G_{02}}\left(\frac{u}{\alpha_0\rho}\right) f_{G_{01}}\left(\frac{v}{\alpha_0\rho}\right) f_{G_{12}}(z) du dv dz \,. \tag{44}$$

Since $Q(\rho) = \log\left(K/\Delta^2(\rho)\right)$, it is possible and useful to write the last indicator as follows.

$$\mathbf{1}\left\{ t_1 \log\left(1 + \frac{\phi(\rho)z}{1+u}\right) > t_0 Q(\rho)\right\} = \mathbf{1}\left\{z > (1+u)\theta(\rho)\right\} \,, \tag{45}$$

where

$$\theta(\rho) = \frac{K^{\frac{t_0}{t_1}}}{\phi(\rho)\left(\Delta^2(\rho)\right)^{\frac{t_0}{t_1}}} - \frac{1}{\phi(\rho)} \,. \tag{46}$$

Define the function $\Phi(u,v,z,\rho)$ as the integrand in the rhs of (44) and let $\mathcal{C}$ be the compact subset of $\mathbb{R}_+^2$ defined as $\mathcal{C} = \left\{ (u,v) \in \mathbb{R}_+^2, t_1 \log(1+u) + t_0 \log\left(1 + u + \frac{\gamma(v,\rho)v}{\gamma(v,\rho)+\Delta^2(\rho)\sqrt{\gamma(v,\rho)}}\right) \leq R, t_0 \log(1+v) \leq R \right\}$. As $f_{G_{02}}$ and $f_{G_{01}}$ are right continuous at zero, then the function $(u,v) \mapsto f_{G_{02}}\left(\frac{u}{\alpha_0\rho}\right) f_{G_{01}}\left(\frac{v}{\alpha_0\rho}\right)$ is bounded on $\mathcal{C}$ for ρ large enough *i.e.*, there exist $\rho_0 > 0$ and $M > 0$ such that $\forall \rho \geq \rho_0$, $f_{G_{02}}\left(\frac{u}{\alpha_0\rho}\right) f_{G_{01}}\left(\frac{v}{\alpha_0\rho}\right) \leq M$. It is straightforward to verify that the following inequalities hold for all $\rho \geq \rho_0$:

$$\begin{aligned}\Phi(u,v,z,\rho) \leq & M \times \mathbf{1}\left\{t_1 \log(1+u) + t_0 \log\left(1+u+\frac{\gamma(v,\rho)v}{\gamma(v,\rho)+\Delta^2(\rho)\sqrt{\gamma(v,\rho)}}\right) \leq R\right\} \\ & \times \mathbf{1}\left\{t_0 \log(1+v) \leq R\right\} \mathbf{1}\left\{1+v > \Delta^2(\rho)\right\} \\ & \times \mathbf{1}\left\{z > (1+u)\theta(\rho)\right\} f_{G_{12}}(z) \\ \leq & M \times \mathbf{1}\left\{\log(1+u) \leq R\right\} \times \mathbf{1}\left\{t_0 \log(1+v) \leq R\right\} f_{G_{12}}(z)\,.\end{aligned}$$

The rhs of the latter inequality is an integrable function on $\mathbb{R}_+^3$ and it does not depend on ρ. Therefore, we can apply Lebesgue's Dominated Convergence Theorem (DCT) in order to compute $\lim_{\rho\to\infty} \rho^2 P_{o,2}(\rho)$ in (44). Note first that $\lim_{\rho\to\infty} \Delta^2(\rho) = 0$, $\lim_{\rho\to\infty} \frac{\gamma(v,\rho)}{\gamma(v,\rho)+\Delta^2(\rho)\sqrt{\gamma(v,\rho)}} = 1$ and $\lim_{\rho\to\infty} \theta(\rho) = 0$ due to assumptions (22)- (25). After some algebra, we can easily show that the following result holds.

$$\lim_{\rho\to\infty} \rho^2 P_{o,2}(\rho) = \frac{c_{02}c_{01}}{\alpha_0^2} \int_{\mathbb{R}_+^2} \mathbf{1}\left\{t_1 \log(1+u) + t_0 \log\left(1+u+v\right) \leq R\right\} dudv\,. \tag{47}$$

We now prove that $\lim_{\rho\to\infty} \rho^2 P_{o,3}(\rho) = 0$. First, recall that $P_{o,3}(\rho) = \Pr[t_0 \log(1+\alpha_0\rho G_{02}) < R, \overline{\mathcal{E}}, \overline{\mathcal{F}}, \mathcal{S}]$. Plugging the definition of events $\mathcal{E}$, $\mathcal{S}$ and $\mathcal{F}$ from (4), (8) and (10) respectively into the latter equation leads to

$$\begin{aligned}P_{o,3}(\rho) = \int_{\mathbb{R}_+^3} & \mathbf{1}\left\{t_0 \log(1+\alpha_0\rho x) \leq R\right\} \mathbf{1}\left\{t_0 \log(1+\alpha_0\rho y) \leq R\right\} \mathbf{1}\left\{1+\alpha_0\rho y > \Delta^2(\rho)\right\} \\ & \times \mathbf{1}\left\{t_1 \log\left(1+\frac{\phi(\rho)z}{1+\alpha_0\rho x}\right) \leq t_0 Q(\rho)\right\} f_{G_{02}}(x) f_{G_{01}}(y) f_{G_{12}}(z) dxdydz\,,\end{aligned}$$

Defining $u = \alpha_0 \rho x$ and $v = \alpha_0 \rho y$, we get

$$P_{o,3}(\rho) = \frac{1}{\alpha_0^2 \rho^2} \int_{\mathbb{R}_+^3} \mathbf{1}\left\{t_0 \log(1+u) \le R\right\} \mathbf{1}\left\{t_0 \log(1+v) \le R\right\} \mathbf{1}\left\{1+v > \Delta^2(\rho)\right\}$$
$$\times \mathbf{1}\left\{t_1 \log\left(1 + \frac{\phi(\rho) z}{1+u}\right) \le t_0 Q(\rho)\right\} f_{G_{02}}\left(\frac{u}{\alpha_0 \rho}\right) f_{G_{01}}\left(\frac{v}{\alpha_0 \rho}\right) f_{G_{12}}(z) du dv dz \, .$$

As we did in (45), we write the last indicator as follows.

$$\mathbf{1}\left\{t_1 \log\left(1 + \frac{\phi(\rho) z}{1+u}\right) \le t_0 Q(\rho)\right\} = \mathbf{1}\left\{z \le (1+u)\theta(\rho)\right\} \; ,$$

where $\theta(\rho)$ is defined by (46). In analogy with the approach we used to compute $\lim_{\rho\to\infty} \rho^2 P_{o,2}(\rho)$, we define $\mathcal{C}_1$ as the compact subset of $\mathbb{R}_+^3$ satisfying $\mathcal{C}_1 = \big\{(u,v,z) \in \mathbb{R}_+^3,\ t_0 \log(1+u) \le R,\ t_0 \log(1+v) \le R,\ z \le (1+u)\theta(\rho)\big\}$. Next, we use the fact that $f_{G_{02}}$, $f_{G_{01}}$ and $f_{G_{12}}$ are right continuous at zero, along with $\lim_{\rho\to\infty} \theta(\rho) = 0$, to show that the function $(u,v,z) \mapsto f_{G_{02}}\left(\frac{u}{\alpha_0\rho}\right) f_{G_{01}}\left(\frac{v}{\alpha_0\rho}\right) f_{G_{12}}(z)$ is bounded on $\mathcal{C}_1$ for ρ large enough *i.e.*, there exist $\rho_1 > 0$ and $M_1 > 0$ such that $\forall \rho \ge \rho_1$, $f_{G_{02}}\left(\frac{u}{\alpha_0\rho}\right) f_{G_{01}}\left(\frac{v}{\alpha_0\rho}\right) f_{G_{12}}(z) \le M_1$. It follows that the following inequalities hold for all $\rho \ge \rho_1$:

$$\rho^2 P_{o,3}(\rho) \le \frac{M_1}{\alpha_0^2} \int_{\mathbb{R}_+^2} \mathbf{1}\left\{1+u \le e^{\frac{R}{t_0}}\right\} \mathbf{1}\left\{z \le (1+u)\theta(\rho)\right\} du dz$$
$$\le \frac{M_1}{\alpha_0^2} \int_{\mathbb{R}_+} \mathbf{1}\left\{z \le e^{\frac{R}{t_0}} \theta(\rho)\right\} dz \le \frac{M_1}{\alpha_0^2} \int_0^{e^{\frac{R}{t_0}}\theta(\rho)} dz = \frac{M_1}{\alpha_0^2} e^{\frac{R}{t_0}} \theta(\rho) \, .$$

Now since $\lim_{\rho\to\infty} \theta(\rho) = 0$ due to assumptions (22)-(25), it follows that $\lim_{\rho\to\infty} \rho^2 P_{o,3}(\rho) = 0$. We can prove in the same way and without difficulty that $\lim_{\rho\to\infty} \rho^2 P_{o,4}(\rho) = 0$.

Putting all pieces together,

$$\begin{aligned}\lim_{\rho\to\infty}\rho^2 P_o = &\lim_{\rho\to\infty}\rho^2 P_{o,1}(\rho) + \lim_{\rho\to\infty}\rho^2 P_{o,2}(\rho) + \lim_{\rho\to\infty}\rho^2 P_{o,3}(\rho) + \lim_{\rho\to\infty}\rho^2 P_{o,4}(\rho)\\ =&\frac{c_{02}c_{12}}{\alpha_0\alpha_1}\int_{\mathbb{R}_+^2}\mathbf{1}\{t_0\log(1+u)+t_1\log(1+u+v)\le R\}dudv\\ +&\frac{c_{02}c_{01}}{\alpha_0^2}\int_{\mathbb{R}_+^2}\mathbf{1}\left\{t_1\log(1+u)+t_0\log\left(1+u+v\right)\le R\right\}dudv\,. \end{aligned} \tag{48}$$

The remaining task is to prove that the rhs of (48) is equal to the rhs of (21). This can be done by making the change of variables $x=\log(1+u)$ and $y=\log\left(1+\frac{v}{1+u}\right)$ in (48). The details of the proof can be found in [19]. The proof of Theorem 2 is thus complete.

Appendix C
Proof of Theorem 3

The outage probability associated with the DoQF protocol was given by (13) as

$$P_o(\rho) = P_{o,1}(\rho)+P_{o,2}(\rho)+P_{o,3}(\rho)+P_{o,4}(\rho)\,, \tag{49}$$

where probabilities $P_{o,1}(\rho)$, $P_{o,2}(\rho)$, $P_{o,3}(\rho)$ and $P_{o,4}(\rho)$ are respectively defined by (14), (15), (16) and (17). Inserting (49) into the definition of the DMT $d(t_0,\delta,r)$ given by (29) leads to

$$\begin{aligned}d(t_0,\delta,r) =& -\lim_{\rho\to\infty}\frac{\log\left(P_{o,1}(\rho)+P_{o,2}(\rho)+P_{o,3}(\rho)+P_{o,4}(\rho)\right)}{\log\rho}\\ =&\min\left\{d_1(t_0,r),d_2(t_0,\delta,r),d_3(t_0,\delta,r),d_4(t_0,\delta,r)\right\}\,, \end{aligned} \tag{50}$$

where

$$d_i(t_0,\delta,r) = -\lim_{\rho\to\infty}\frac{\log P_{o,i}(\rho)}{\log\rho}\,, \tag{51}$$

for $i = 1, 2, 3, 4$. Note that $d_1(t_0, r)$ is the only term in (50) that does not depend on parameter δ. The derivation of the DMT associated with the DoQF protocol will be thus done as follows:

1) Compute the terms $d_1(t_0, r)$, $d_2(t_0, \delta, r)$, $d_3(t_0, \delta, r)$ and $d_4(t_0, \delta, r)$ for fixed values of t_0 and δ as given by (51). This is done in this Subsection.
2) Compute $t^*_{0,\mathrm{DoQF}}(r)$ and $\delta^*_{0,\mathrm{DoQF}}(r)$ minimizing $d(t_0, \delta, r)$ defined from (50) as the minimum of $d_1(t_0, r)$, $d_2(t_0, \delta, r)$, $d_3(t_0, \delta, r)$ and $d_4(t_0, \delta, r)$.
3) The final DMT of the protocol can be finally obtained by calculating $d(t^*_{0,\mathrm{DoQF}}(r), \delta^*_{0,\mathrm{DoQF}}(r), r)$.

Derivation of the term $d_1(t_0, r)$, *i.e.*, event $\mathcal{E}$ is realized:

Recall the definition given by (14) of $P_{o,1}(\rho)$ as the probability that the destination is in outage and that the event $\mathcal{E}$ is realized. It is clear from (4) and (14) that $P_{o,1}(\rho)$ is a function of parameter t_0. This is why the DMT term $d_1(t_0, r)$ associated with $P_{o,1}(\rho)$ is also a function of this parameter. Following the steps used in Appendix D-A, one can show that the following result holds.

$$d_1(t_0, r) = \begin{cases} 2(1-r)^+ & \text{for } t_0 \leq 0.5 \\ 2 - \frac{r}{1-t_0} & \text{for } t_0 > 0.5 \text{ and } r < 1 - t_0 \\ \frac{(1-r)^+}{t_0} & \text{for } t_0 > 0.5 \text{ and } r \geq 1 - t_0 \end{cases} \tag{52}$$

Derivation of the term $d_2(t_0, \delta, r)$, *i.e.*, events $\overline{\mathcal{E}}$, $\mathcal{S}$ and $\mathcal{F}$ are realized:

Note from (10) and (15) that $P_{o,2}(\rho)$ is a function of parameters t_0 and δ. This is why the DMT $d_2(t_0, \delta, r)$ associated with $P_{o,2}(\rho)$ is function of t_0 and δ.

First, consider the case $t_0 \geq 0.5$.

If parameter δ is chosen such that $0 < \delta \leq 1 - \left(1 - \frac{r}{t_0}\right)^+$, then $d_2(t_0, \delta, r)$ can be written as

$$d_2(t_0, \delta, r) = \begin{cases} (1-r)^+ + \max\left\{\left(1 - \frac{r}{t_0}\right)^+, 1 - r - \delta\right\}, & \frac{r}{t_1} - \left(1 - \frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta \leq 1 - r \\ \frac{r}{t_1} - \left(1 - \frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta + \max\left\{\frac{1-2r}{t_0} + \frac{t_1}{t_0}\left(1 - \frac{r}{t_0}\right)^+, \left(1 - \frac{r}{t_0}\right)^+\right\}, & \frac{r}{t_1} - \left(1 - \frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta > 1 - r \end{cases} \tag{53}$$

As for the choice $\delta > 1 - \left(1 - \frac{r}{t_0}\right)^+$, we show in Appendix D-A that event $\overline{\mathcal{E}}\&\mathcal{S}$ cannot be realized in this case for any channel state provided that ρ is sufficiently large. Therefore, there exists $\rho_0 > 0$ such that $\forall \rho \geq \rho_0,\ P_{o,2}(\rho) = 0$. The corresponding DMT $d_2(t_0, \delta, r)$ will have no effect on the final DMT of the protocol. The value $d_2(t_0, \delta, r) = 2(1-r)^+$ is conveniently chosen in this case:

$$d_2(t_0,\delta,r) = 2(1-r)^+ \ \text{ for } \delta > 1-\left(1-\frac{r}{t_0}\right)^+ . \tag{54}$$

The proof of (53) and (54) is provided in Appendix D-A. We can show using the same arguments of the latter appendix that

$$d_2(t_0,\delta,r) = 2(1-r)^+ \ , \ \text{ for } \delta \leq 0 \ . \tag{55}$$

Similarly, we can obtain the expression (56) for $d_2(t_0,\delta,r)$ in the case $t_0 < 0.5$.

$$d_2(t_0,\delta,r) = \begin{cases} \left(1-\frac{r}{t_0}\right)^+ + \max\left\{(1-r)^+, \frac{1-r}{t_1} - \frac{t_0}{t_1}\left(1-\frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta\right\}, & \text{for } t_0 < 0.5 \text{ and } 2t_0t_1 \leq r \\ \left(1-\frac{r}{t_0}\right)^+ + \frac{r}{t_1} - \left(1-\frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta, & \text{for } t_0 < 0.5 \text{ and } 2t_0t_1 > r \end{cases} \tag{56}$$

Derivation of the term $d_3(t_0,\delta,r)$, *i.e.*, events $\overline{\mathcal{E}}$, $\mathcal{S}$ and $\overline{\mathcal{F}}$ are realized:

By referring to (10) and (16), it becomes clear that $P_{o,3}(\rho)$ is a function of parameters t_0 and δ. This explains the fact that $d_3(t_0,\delta,r)$ also depends on these two parameters.

The expression given below of $d_3(t_0,\delta,r)$ can be derived using the approach used in Appendix D-A.

$$d_3(t_0,\delta,r) = \begin{cases} 2\left(1-\frac{r}{t_0}\right)^+ + \left(2\left(1-\frac{r}{t_0}\right)^+ + \frac{t_0}{t_1}\delta - \frac{r}{t_1}\right)^+ & \text{for } \delta \leq 1-\left(1-\frac{r}{t_0}\right)^+ \\ 2(1-r)^+ & \text{for } \delta > 1-\left(1-\frac{r}{t_0}\right)^+ \end{cases} . \tag{57}$$

Recall that in the case $\delta > 1-\left(1-\frac{r}{t_0}\right)^+$, event $\overline{\mathcal{E}}\&\mathcal{S}$ cannot be realized, as we saw earlier, for any channel realization provided that ρ is sufficiently large. In this case $P_{o,3}(\rho) = 0$ and the corresponding DMT $d_3(t_0,\delta,r)$ will have no effect on the final DMT of the protocol. This is why the value $d_3(t_0,\delta,r)=2(1-r)^+$ was conveniently chosen in (57) in this case.

Derivation of the term $d_4(t_0,\delta,r)$, *i.e.*, events $\overline{\mathcal{E}}$ and $\overline{\mathcal{S}}$ are realized:

This is the case when the relay does not quantize even if it has not succeeded in decoding the source message. This happens when $\alpha_0\rho G_{01}+1<\Delta^2(\rho)$ which means that the relay stays inactive. Recall the definition of $P_{o,4}(\rho)$ as the probability that the destination is in outage and that events $\overline{\mathcal{E}}$ and $\overline{\mathcal{S}}$ are realized. It is straightforward to verify that

$$d_4(t_0,\delta,r)=\begin{cases}(1-r)^+ + \max\left\{\left(1-\frac{r}{t_0}\right)^+,(1-\delta)^+\right\} & \text{for } \delta>0\\ 2(1-r)^+ & \text{for } \delta\le 0\end{cases}. \quad (58)$$

Note that in the case $\delta\le 0$, the condition $\alpha_0\rho G_{01}+1>\Delta^2(\rho)$ is always satisfied for sufficiently large values of ρ for all channel realizations since $\Delta^2(\rho)\doteq\rho^\delta\le 1$. Therefore, there exists in this case $\rho_0>0$ such that $\forall\rho\ge\rho_0$, event $\overline{\mathcal{S}}$ is never realized and $P_{o,4}(\rho)=0$. The corresponding DMT $d_4(t_0,\delta,r)$ will have therefore no effect on the final DMT of the protocol, and as usual we can assign it conveniently the value $d_4(t_0,\delta,r)=2(1-r)^+$ as done in (58).

Derivation of the final DMT of the DoQF protocol:

At this point, the DMT terms $d_1(t_0,r)$, $d_2(t_0,\delta,r)$, $d_3(t_0,\delta,r)$ and $d_4(t_0,\delta,r)$ associated with all the possible cases encountered by the destination have been derived. the DMT $d(t_0,\delta,r)$ associated with the DoQF protocol for fixed values of t_0 and δ can now be obtained from (29) as the minimum of the above DMT terms. No closed-form expression of $d(t_0,\delta,r)$ is given in this paper. However, Theorem 3 does provide the closed-form expression of $d^*_{\text{DoQF}}(r)$ obtained by solving the optimization problem $d^*_{\text{DoQF}}(r)=\sup_{\delta,t_0} d(t_0,\delta,r)$. We derive $d^*_{\text{DoQF}}(r)$ as follows.

Before proceeding with the proof, we define $t^*_{0,\mathrm{DoQF}}(r)$ and $\delta^*_{\mathrm{DoQF}}(r)$ as the argument of the supremum in $d^*_{\mathrm{DoQF}}(r) = \sup_{\delta,t_0} d(t_0,\delta,r)$.

We will first compute $d^*_{\mathrm{DoQF}}(r)$ in the case $r \le 0.25$, and then in the case $r > 0.25$.

The case $r \le 0.25$

Let us plug $t_0 = 0.5$ and $\delta = 0$ into (52), (53), (57) and (58) to obtain

$$d_1(t_0,r) = d_2(t_0,\delta,r) = d_4(t_0,\delta,r) = 2(1-r)^+ \,, \tag{59}$$

$$d_3(t_0,\delta,r) = 2(1-2r)^+ + \big(2(1-2r)^+ - 2r\big)^+ = 2 - 8r \,. \tag{60}$$

Note that $d_3(t_0,\delta,r)$ is the only term that may be different from $2(1-r)^+$. However, one can verify by referring to (60) that $d_3(t_0,\delta,r) \ge 2(1-r)^+ \;\Leftrightarrow\; r \le 0.25$. We conclude that, for $r \le 0.25$, $d(0.5,0,r) = 2(1-r)^+$. We have thus proved that the MISO upper-bound is achieved by the DoQF for $r \le 0.25$ by choosing $t^*_{0,\mathrm{DoQF}}(r) = 0.5$ and $\delta^*_{\mathrm{DoQF}}(r) = 0$.

The case $r > 0.25$

The first step of the proof in this case is to reduce the size of the set of possible values of $t^*_{0,\mathrm{DoQF}}(r)$ and $\delta^*_{\mathrm{DoQF}}(r)$. For that sake, we first recall that the DMT of (non-orthogonal) DF in the general multiple-relay case has been derived in [21]. Denote by $P_{o,\mathrm{DF}}(\rho)$ the outage probability associated with the DF protocol. The DMT of DF for fixed values of t_0 can thus be defined as

$$d(t_0,r) = -\lim_{\rho\to\infty} \frac{\log P_{o,\mathrm{DF}}(\rho)}{\log \rho} \,, \tag{61}$$

and the final DMT of the protocol as $d^*_{\mathrm{DF}}(r) = \sup_{t_0} d(t_0, r)$. The closed-form expression of $d^*_{\mathrm{DF}}(r)$ in the case of a single relay is reproduced here by

$$d^*_{\mathrm{DF}}(r) = \begin{cases} 2 - \frac{2}{3-\sqrt{5}} r & \text{for } 0 \leq r \leq \frac{\sqrt{5}-1}{\sqrt{5}+1} \\ (2-r)(1-r) & \text{for } \frac{\sqrt{5}-1}{\sqrt{5}+1} < r \leq 1 \; . \end{cases} \tag{62}$$

Moreover, the optimal value of t_0, as function of r, that allows to achieve this DMT is given by

$$t^*_{0,\mathrm{DF}}(r) = \begin{cases} \frac{2}{\sqrt{5}+1} & \text{for } 0 \leq r \leq \frac{\sqrt{5}-1}{\sqrt{5}+1} \\ \frac{1}{2-r} & \text{for } \frac{\sqrt{5}-1}{\sqrt{5}+1} < r \leq 1 \; . \end{cases} \tag{63}$$

given the above results, we will prove in particular that the following three lemmas hold.

Lemma 1. *For any* $r \in [0,1]$, $d^*_{\mathrm{DoQF}}(r) \geq d^*_{\mathrm{DF}}(r)$.

In other words, Lemma 1 states that the DMT achieved by the DoQF protocol cannot be worse than the DMT achieved by the DF. The proof of Lemma 1 is given in Appendix D-B.

Lemma 2. *For any* $r \in [0,1]$, *the following inequalities hold true:* $\max\{0.5, r\} \leq t^*_{0,\mathrm{DoQF}}(r) \leq t^*_{0,\mathrm{DF}}(r)$.

Here, $t^*_{0,\mathrm{DF}}(r)$ is the value of t_0 defined by (63) which allows to achieve the DMT of the DF protocol. The proof of Lemma 2 is given in Appendix D-C.

Lemma 3. *Assume that* $r > 0.25$. *The following holds true:* $0 < \delta^*_{\mathrm{DoQF}}(r) < 1 - \left(1 - \frac{r}{t^*_{0,\mathrm{DoQF}}(r)}\right)^+$.

The proof of Lemma 3 is given in Appendix D-D.

These three lemmas will considerably simplify the derivation of $d^*_{\mathrm{DoQF}}(r)$. Indeed, with the help of Lemma 2 and Lemma 3, we will derive the DMT of the DoQF firstly in the case when $0.25 < r \leq \frac{2(\sqrt{5}-1)}{9-\sqrt{5}}$, and secondly in the case when $\frac{2(\sqrt{5}-1)}{9-\sqrt{5}} < r \leq 1$.

- $0.25 < r \leq \frac{2(\sqrt{5}-1)}{9-\sqrt{5}}$.

  We begin with the simplification of the DMT terms $d_1\left(t^*_{0,\text{DoQF}}(r), r\right)$, $d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$, $d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$ and $d_4\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$. The final DMT $d^*_{\text{DoQF}}(r)$ can then be deduced as the minimum of the above terms. Consider first the derivation of $d_1\left(t^*_{0,\text{DoQF}}(r), r\right)$. Since Lemma 2 states that $t^*_{0,\text{DoQF}}(r) \leq t^*_{0,\text{DF}}(r) = \frac{2}{\sqrt{5}+1}$, it follows from (52) that

$$d_1\left(t^*_{0,\text{DoQF}}(r), r\right) = 2 - \frac{r}{1 - t^*_{0,\text{DoQF}}(r)} \,. \tag{64}$$

We now proceed to the simplification of the expression of $d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$. Thanks to Lemma 2 and Lemma 3, we will prove that

$$d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = (1-r)^+ + \max\left\{1 - \frac{r}{t^*_{0,\text{DoQF}}(r)}, 1 - r - \delta^*_{\text{DoQF}}(r)\right\} \,. \tag{65}$$

For that sake, refer to (53) and note that proving (65) is equivalent to proving that

$$\frac{r}{1 - t^*_{0,\text{DoQF}}(r)} - \left(1 - \frac{r}{t^*_{0,\text{DoQF}}(r)}\right)^+ - \frac{t^*_{0,\text{DoQF}}(r)}{1 - t^*_{0,\text{DoQF}}(r)}\delta^*_{\text{DoQF}}(r) \leq 1 - r \,. \tag{66}$$

In order to show that (66) holds, we suppose to the contrary that $\frac{r}{1-t^*_{0,\text{DoQF}}(r)} - \left(1 - \frac{r}{t^*_{0,\text{DoQF}}(r)}\right)^+ - \frac{t^*_{0,\text{DoQF}}(r)}{1-t^*_{0,\text{DoQF}}(r)}\delta^*_{\text{DoQF}}(r) > 1 - r$. Since $\delta^*_{\text{DoQF}}(r) > 0$ according to Lemma 3, the latter assumption leads to

$$r > \frac{2t^*_{0,\text{DoQF}}(r)\left(1 - t^*_{0,\text{DoQF}}(r)\right)}{1 + t^*_{0,\text{DoQF}}(r)\left(1 - t^*_{0,\text{DoQF}}(r)\right)} \,. \tag{67}$$

Moreover, it is straightforward to show that

$$\min_{0.5 \leq t \leq \frac{2}{\sqrt{5}+1}} \frac{2t\,(1-t)}{1 + t\,(1-t)} > \frac{2(\sqrt{5}-1)}{9-\sqrt{5}} \,, \tag{68}$$

where the restriction to $0.5 \leq t \leq t^*_{0,\text{DF}}(r) = \frac{2}{\sqrt{5}+1}$ is due to Lemma 2. Now, we can combine (67) and (68) in order to get $r > \frac{2(\sqrt{5}-1)}{9-\sqrt{5}}$, which contradicts the fact that $r \leq \frac{2(\sqrt{5}-1)}{9-\sqrt{5}}$. We conclude that expression (65) holds true.

We can further simplify the expression (65) by proving that $1-r-\delta^*_{\text{DoQF}}(r) \geq 1-\frac{r}{t^*_{0,\text{DoQF}}(r)}$. The proof of this point uses the same arguments as above and is thus omitted. The term $d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$ can finally be written as

$$d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = 2(1-r)^+ - \delta^*_{\text{DoQF}}(r)\ . \tag{69}$$

As for $d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$ given by (57), it simplifies to

$$d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = 4 + \frac{t^*_{0,\text{DoQF}}(r)}{1-t^*_{0,\text{DoQF}}(r)}\delta^*_{\text{DoQF}}(r) - \left(4 + \frac{t^*_{0,\text{DoQF}}(r)}{1-t^*_{0,\text{DoQF}}(r)}\right)\frac{r}{t^*_{0,\text{ DoQF}}(r)} \tag{70}$$

The remaining task is to simplify the expression (58) which defines $d_4\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$. For that sake, we can resort to Lemma 1 to prove that

$$d_4\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = (1-r)^+ + (1-\delta^*_{\text{DoQF}}(r))\ .$$

It follows that $d_4\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) \geq d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$ and that it can thus be dropped from the derivation of the final DMT of the DoQF. Now that the DMT terms $d_1\left(t^*_{0,\text{DoQF}}(r), r\right)$, $d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$ and $d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right)$ have been expressed as functions of $t^*_{0,\text{DoQF}}(r)$ and $t^*_{0,\text{DoQF}}(r)$, we can proceed to the determination of $t^*_{0,\text{DoQF}}(r)$, $\delta^*_{\text{DoQF}}(r)$, and consequently $d^*_{\text{DoQF}}(r)$.

– Determination of $\delta^*_{\text{DoQF}}(r)$:

  Assume that $t^*_{0,\text{DoQF}}(r)$ has been already determined. It is straightforward to verify that $d_2(t, \delta, r)$ given by (69) is decreasing w.r.t δ, and that $d_3(t, \delta, r)$ given by (70) is increasing w.r.t δ on $\mathbb{R}^+$. Furthermore, $d_2(t,0,r) > d_3(t,0,r)$. We conclude that

$$d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) \,.$$

Therefore, $\delta^*_{\text{DoQF}}(r)$ can be given as a function of $t^*_{0,\text{DoQF}}(r)$ as follows

$$\delta^*_{\text{DoQF}}(r) = \left(4 - 3t^*_{0,\text{DoQF}}(r)\right)\frac{r}{t^*_{0,\text{DoQF}}(r)} - (2+2r)\left(1 - t^*_{0,\text{DoQF}}(r)\right) \,, \tag{71}$$

which leads to

$$\begin{aligned} &d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = \\ &2 - 2r + (2+2r)\left(1 - t^*_{0,\text{DoQF}}(r)\right) - \left(4 - 3t^*_{0,\text{DoQF}}(r)\right)\frac{r}{t^*_{0,\text{DoQF}}(r)} \,. \end{aligned} \tag{72}$$

– Determination of $t^*_{0,\text{DoQF}}(r)$:

We can show in the same way that $t^*_{0,\text{DoQF}}(r)$ can be obtained by writing

$$d_1\left(t^*_{0,\text{DoQF}}(r), r\right) = d_2\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) \,. \tag{73}$$

Plugging the expression of $\delta^*_{\text{DoQF}}(r)$ from (71) and the expression of $d_2\Big(t^*_{0,\text{DoQF}}(r),$ $\delta^*_{\text{DoQF}}(r),\, r\Big)$ from (72) into (73) leads to equation (32) given in Theorem 3 as

$$2(1+r)t^*_{0,\text{DoQF}}(r)^3 - (4+5r)t^*_{0,\text{DoQF}}(r)^2 + 2(1+4r)t^*_{0,\text{DoQF}}(r) - 4r = 0\,.$$

It can be shown after some algebra that the above equation admits a unique solution $v^*(r)$ on $\left[0.5, \frac{2}{\sqrt{5}+1}\right]$ provided that $r \leq \frac{2(\sqrt{5}-1)}{9-\sqrt{5}}$. This explains why the distinction $r \leq \frac{2(\sqrt{5}-1)}{9-\sqrt{5}}$ and $r > \frac{2(\sqrt{5}-1)}{9-\sqrt{5}}$ appears in Theorem 3. Once the solution $v^*(r)$ to the above equation has been computed, then $d^*_{\text{DoQF}}(r)$, $t^*_{0,\text{DoQF}}(r)$ and $\delta^*_{\text{DoQF}}(r)$ can be easily obtained.

- $\frac{2(\sqrt{5}-1)}{9-\sqrt{5}} < r \leq 1$.

  In this case, we need to prove that $d^*_{\text{DoQF}}(r) = d^*_{\text{DF}}(r)$. To that end, we can show that $d^*_{\text{DoQF}}(r) > d^*_{\text{DF}}(r)$ leads to a contradiction. The proof of this point is based on Lemmas 1, 2 and 3 and is omitted due to lack of space.

The proof of Theorem 3 is thus completed.

APPENDIX D

DERIVATION OF $d_2(t_0, \delta, r)$ AND PROOFS OF LEMMAS 1, 2, AND 3

A. *Derivation of $d_2(t_0, \delta, r)$ (for $t_0 \geq 0.5$ and $\delta > 0$)*

First, recall the definition of $d_2(t_0, \delta, r)$ as $d_2(t_0, \delta, r) = -\lim_{\rho\to\infty} \frac{\log(P_{o,2}(\rho))}{\log\rho}$, where the probability $P_{o,2}(\rho)$ is defined by (15) as

$$P_{o,2}(\rho) = \Pr\left[t_1 \log(1 + \alpha_0\rho G_{02}) + t_0 \log\left(1 + \alpha_0\rho G_{02} + \frac{\gamma(G_{01},\rho)\alpha_0\rho G_{01}}{\gamma(G_{01},\rho) + \Delta^2(\rho)\sqrt{\gamma(G_{01},\rho))}}\right) \leq R(\rho), \right.$$
$$\left. \overline{\mathcal{E}}, \mathcal{F}, \mathcal{S}\right], \tag{74}$$

where $\gamma(G_{01},\rho) = \frac{\left(1+\alpha_0\rho G_{01}-\Delta^2(\rho)\right)^2}{(1+\alpha_0\rho G_{01})^2}$, and where events $\mathcal{E}$, $\mathcal{S}$ and $\mathcal{F}$ are defined by (4), (8) and (10) respectively. Note that $\gamma(G_{01},\rho)$ is positive since event $\mathcal{S}$ *i.e.*, $1 + \alpha_0\rho G_{01} \geq \Delta^2(\rho)$, is realized. Furthermore, we can check that the following result holds.

$$\frac{\gamma(G_{01},\rho)}{\gamma(G_{01},\rho) + \Delta^2(\rho)\sqrt{\gamma(G_{01},\rho)}} \doteq \frac{1}{1+\Delta^2(\rho)} \doteq \rho^{-(\delta)^+}\,. \tag{75}$$

In the following, we assume that $R(\rho) = r\log\rho$ in accordance with (1), and we define as in [20] the *exponential order* associated with channel H_{ij} as $a_{ij} = -\frac{\log G_{ij}}{\log\rho}$. We can easily verify that a_{ij} is a *Gumbel* distributed random variable with the probability density function $f_{a_{ij}}(a) = \log\rho\, e^a e^{-e^{-a\log\rho}}$. By plugging $G_{01} = \rho^{-a_{01}}$ into (4), the probability of the event $\overline{\mathcal{E}}$ *i.e.*, $t_0 \log(1 + \alpha_0 \rho G_{01}) > R(\rho)$, can be written as

$$\Pr[\mathcal{E}] \doteq \Pr\left[(1 - a_{01})^+ \leq \frac{r}{t_0}\right] . \tag{76}$$

Similarly, we can verify that the probability of event $\mathcal{F}$ *i.e.*, $t_1 \log\left(1 + \frac{\phi(\rho)G_{12}}{\alpha_0\rho G_{02}+1}\right) > Q(\rho)t_0$, satisfies

$$\Pr[\mathcal{F}] \doteq \Pr\left[\left(1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1 - a_{02})^+\right)^+ \leq \frac{r}{t_1} - \frac{t_0}{t_1}\delta\right] , \tag{77}$$

and that the probability of $\mathcal{S}$ satisfies

$$\Pr[\mathcal{S}] \doteq \Pr[\delta \leq (1 - a_{01})^+] . \tag{78}$$

By plugging $R(\rho) = r\log\rho$, $G_{01} = \rho^{-a_{01}}$, $G_{02} = \rho^{-a_{02}}$, $G_{12} = \rho^{-a_{12}}$, (75), (76), (77) and (78) into (74), the following high SNR result holds for $\delta > 0$.

$$\begin{aligned} P_{o,2}(\rho) \doteq \Pr\Bigg[& t_1(1 - a_{02})^+ + t_0(1 - \min(a_{02}, a_{01} + \delta))^+ < r \, , \; (1 - a_{01})^+ < \frac{r}{t_0} \, , \\ & \left(1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1 - a_{02})^+\right)^+ > \frac{r}{t_1} - \frac{t_0}{t_1}\delta \, , \; \delta \leq (1 - a_{01})^+ \Bigg], \end{aligned} \tag{79}$$

or, equivalently,

$$P_{o,2}(\rho) \doteq \int_{\mathcal{O}} f_{a_{01}}(a_{01}) f_{a_{02}}(a_{02}) f_{a_{12}}(a_{12}) da_{01} da_{02} da_{12} \, , \tag{80}$$

where $f_{a_{ij}}(.)$ is the probability density function of a_{ij} and

$$\mathcal{O} = \left\{ (a_{01}, a_{02}, a_{12}) \in \mathbb{R}^3 \mid t_1(1-a_{02})^+ + t_0(1-\min(a_{02}, a_{01}+\delta))^+ < r \, , \, (1-a_{01})^+ < \frac{r}{t_0} \, , \right.$$
$$\left. \left(1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1-a_{02})^+ \right)^+ > \frac{r}{t_1} - \frac{t_0}{t_1}\delta \, , \, \delta \leq (1-a_{01})^+ \right\} . \tag{81}$$

Plugging the expression of $f_{a_{ij}}(.)$ given earlier into (80), $P_{o,2}(\rho)$ can be written as

$$P_{o,2}(\rho) \doteq \int_{\mathcal{O}} (\log \rho)^3 \rho^{-(a_{01}+a_{02}+a_{12})} e^{-\rho^{-a_{01}}} e^{-\rho^{-a_{02}}} e^{-\rho^{-a_{12}}} da_{01} da_{02} da_{12} \, .$$

It can be shown (refer to [20]) that the term $(\log \rho)^3$ can be dropped from the latter equation without losing its exactness. Moreover, integration in the same equation can be restricted to positive values of a_{01}, a_{02} and a_{12}. Define $\mathcal{O}_+ = \mathcal{O} \cap \mathbb{R}^3_+$. The probability $P_{o,2}(\rho)$ thus satisfies

$$P_{o,2}(\rho) \doteq \int_{\mathcal{O}_+} \rho^{-(a_{01}+a_{02}+a_{12})} da_{01} da_{02} da_{12} \, , \tag{82}$$

and the DMT $d_2(t_0, \delta, r)$ associated with $P_{o,2}(\rho)$ can now be written [20] as

$$d_2(t_0, \delta, r) = \inf_{(a_{01}, a_{02}, a_{12}) \in \mathcal{O}^+} (a_{01} + a_{02} + a_{12}) \, . \tag{83}$$

In this appendix, the derivation of $d_2(t_0, \delta, r)$ will be done only in the case characterized by $t_0 \geq 0.5$ and $\delta > 0$. The derivation in the case $\delta \leq 0$ or $t_0 < 0.5$ follows the same approach.

Consider first the case $0 < \delta \leq 1 - \left(1 - \frac{r}{t_0}\right)^+$. The infimum in (83) can be computed by partitioning $\mathcal{O}_+$ into subsets according to whether a_{01}, a_{02} are smaller or larger than 1.

- $\boldsymbol{a_{01} > 1}$. In this case, $(1-a_{01})^+ = 0$ and the fourth inequality in (81) reduces to $\delta \leq 0$. This result contradicts our assumption that $\delta > 0$. There is therefore no triples $(a_{01}, a_{02}, a_{12}) \in \mathcal{O}^+$ such that $a_{01} > 1$.

- $\boldsymbol{a_{01} \leq 1, a_{02} > 1}$. Since the third inequality in the definition of $\mathcal{O}$ given by (81) contains the term $\left(1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1 - a_{02})^+\right)^+$, then we should consider two categories of triples (a_{01}, a_{02}, a_{12}):

  - $1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1 - a_{02})^+ < 0$.

    For triples $(a_{01}, a_{02}, a_{12}) \in \mathcal{O}^+$ under this category, the third inequality in (81) can be reduced to $\delta > \frac{r}{t_0}$, which contradicts the second and the fourth inequalities in (81). This category can be therefore dropped out.

  - $1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1 - a_{02})^+ \geq 0$.

    Recall the first inequality in (81) ***i.e.,*** $t_1(1-a_{02})^+ + t_0(1-\min(a_{02}, a_{01}+\delta))^+ < r$. Since $\delta \leq (1 - a_{01})^+$ due to the fourth inequality in (81), then $a_{01} + \delta \leq a_{01} + (1 - a_{01})^+ = 1 \leq a_{02}$. The first inequality in (81) reduces thus to $a_{01} \geq \left(1 - \frac{r}{t_0}\right)^+$. We conclude that

$$\inf_{a_{01} \leq 1, a_{02} > 1} (a_{01} + a_{02} + a_{12}) = 1 + \left(1 - \frac{r}{t_0}\right)^+ . \tag{84}$$

    One can verify after some simple algebra that $\inf_{a_{01} \leq 1, a_{02} > 1}(a_{01} + a_{02} + a_{12}) = 1 + \left(1 - \frac{r}{t_0}\right)^+$ is always larger than $d_1(t_0, r)$ given by (52). Therefore, the term $\inf_{a_{01} \leq 1, a_{02} > 1}(a_{01} + a_{02} + a_{12})$ never coincides with the minimum in $d(t_0, \delta, r) = \min\{d_1(t_0, r)$, $d_2(t_0, \delta, r)$,$d_3(t_0, \delta, r)$, $d_4(t_0, \delta, r)\}$. As a result, the argument of the infimum $\inf_{(a_{01}, a_{02}, a_{12}) \in \mathcal{O}_+}(a_{01} + a_{02} + a_{12})$ coincides necessarily with a triple (a_{01}, a_{02}, a_{12}) from the following subset.

- $\boldsymbol{a_{01} \leq 1, a_{02} \leq 1}$. Two categories of triples (a_{01}, a_{02}, a_{12}) should be considered.

  - $1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1 - a_{02})^+ < 0$.

    As done before, it is straightforward to verify that there is no triples $(a_{01}, a_{02}, a_{12}) \in \mathcal{O}^+$ that fall under this category.

- $\circ\ 1 + \left(1 - \frac{r}{t_0}\right)^+ - a_{12} - (1 - a_{02})^+ \geq 0.$

The third inequality in (81) leads in this case to

$$a_{02} > \frac{r}{t_1} - \left(1 - \frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta\,. \tag{85}$$

In order to evaluate the first inequality in (81), two subcategories of triples (a_{01}, a_{02}, a_{12}) should be further examined.

1) $a_{02} < a_{01} + \delta$. For triples $(a_{01}, a_{02}, a_{12}) \in \mathcal{O}^+$ under this category, the first inequality in (81) leads to $a_{02} > (1 - r)^+$.
2) $a_{02} \geq a_{01} + \delta$. The first inequality results in this case in $a_{02} + \frac{t_0}{t_1} a_{01} > \frac{1-r}{t_1} - \frac{t_0}{t_1}\delta$.

Referring to Figures 8 and 9 reveals that $\inf_{a_{01} \leq 1, a_{02} \leq 1}(a_{01} + a_{02} + a_{12})$ coincides with the rhs of (53). We have thus proved that $d_2(t_0, \delta, r)$ is indeed given by (53).

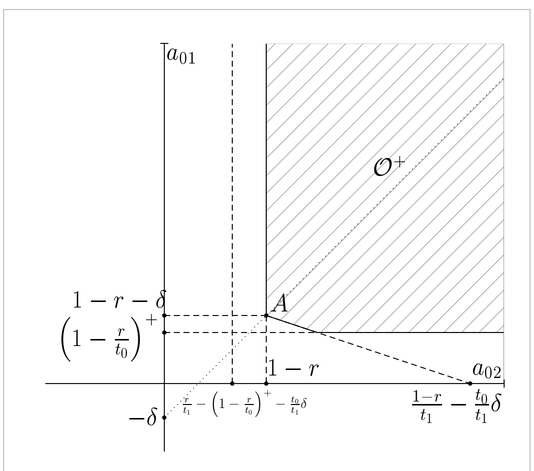

Figure 8. Outage region for the DoQF protocol in the case $\frac{r}{t_1} - \left(1 - \frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta \leq 1 - r$.

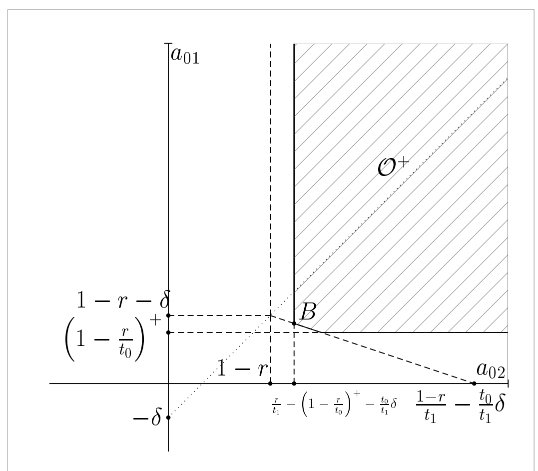

Figure 9. Outage region for the DoQF protocol in the case $1 - r < \frac{r}{t_1} - \left(1 - \frac{r}{t_0}\right)^+ - \frac{t_0}{t_1}\delta$.

Now consider the case $\delta > 1 - \left(1 - \frac{r}{t_0}\right)^+$ in order to prove that (54) holds. To that end, refer to the second and the fourth inequalities in the definition of $\mathcal{O}$ given by (81), that is $(1 - a_{01})^+ < \frac{r}{t_0}$ and $\delta \leq (1 - a_{01})^+$. Note that $(1 - a_{01})^+ \leq 1$ since $a_{01} > 0$. A necessary condition for a_{01} to satisfy the second and the fourth inequalities in (81), and consequently to belong to $\mathcal{O}_+$ is thus

$\delta \leq \min\left\{1, \frac{r}{t_0}\right\} = 1 - \left(1 - \frac{r}{t_0}\right)^+$. This means that if we choose δ such that $\delta > 1 - \left(1 - \frac{r}{t_0}\right)^+$, the set $\mathcal{O}_+$ will be empty. In this case, $P_{o,2}(\rho) = 0$ for sufficiently large ρ. In other words, there exists $\rho_0 > 0$ such that $\forall \rho \geq \rho_0$, the event $\overline{\mathcal{E}}\&\mathcal{S}$ cannot be realized and the relay will not be able to quantize, reducing the DoQF to a classical DF scheme. The corresponding DMT $d_2(t_0, \delta, r)$ will have no effect in this case on the final DMT of the protocol. We can give it for convenience the value $d_2(t_0, \delta, r) = 2(1-r)^+$, which is the upper-bound on the DMT of any single-relay protocol.

B. Proof of Lemma 1

Assume that parameters t_0 and δ of the DoQF protocol are fixed such that $t_0 = t^*_{0,\mathrm{DF}}(r)$ and $\delta = 1 - \left(1 - \frac{r}{t^*_{\mathrm{DF}}(r)}\right)^+ = \frac{r}{t^*_{\mathrm{DF}}(r)}$, where $t^*_{0,\mathrm{DF}}(r)$ is defined by (63). In this case, equations (52), (53), (57) and (58) lead to $d_1(t_0, r) = d_4(t_0, \delta, r) = d^*_{\mathrm{DF}}(r)$ and $d_2(t_0, \delta, r) = d_3(t_0, \delta, r) = 2(1-r)^+$, meaning that $d(t_0, \delta, r) = d^*_{\mathrm{DF}}(r)$.

We conclude that the DoQF can be reduced to have the performance of DF by choosing $t_0 = t^*_{0,\mathrm{DF}}(r)$ and $\delta = \frac{r}{t^*_{0,\mathrm{DF}}(r)}$. The final DMT $d^*_{\mathrm{DoQF}}(r)$ of the DoQF is therefore necessarily greater or equal to $d^*_{\mathrm{DF}}(r)$. The proof of Lemma 1 is thus completed.

C. Proof of Lemma 2

Proving Lemma 2 requires proving that the following three inequalities hold: $r \leq t^*_{0,\mathrm{DoQF}}(r)$, $t^*_{0,\mathrm{DoQF}}(r) \leq t^*_{0,\mathrm{DF}}(r)$ and $0.5 \leq t^*_{0,\mathrm{DoQF}}(r)$. Let us begin with the proof of the inequality $r \leq t^*_{0,\mathrm{DoQF}}(r)$. Assume to the contrary that $r > t^*_{0,\mathrm{DoQF}}(r)$. In this case, $d_3(t^*_{0,\mathrm{DoQF}}(r), \delta^*_{\mathrm{DoQF}}(r), r) = 0$ due to (57). This implies that the DMT of the DoQF satisfies $d(t^*_{0,\mathrm{DoQF}}(r), \delta^*_{\mathrm{DoQF}}(r), r) = d_3(t^*_{0,\mathrm{DoQF}}(r), \delta^*_{\mathrm{DoQF}}(r), r) = 0$, which is in contradiction with Lemma 1. We conclude that $r \leq t^*_{0,\mathrm{DoQF}}(r)$ holds true.

We now show that the inequality $t^*_{0,\text{DoQF}}(r) \leq t^*_{0,\text{DF}}(r)$ also holds true. For that sake, note that the DMT $d^*_{\text{DF}}(r)$ of DF given by (62) can be written as a function of $t^*_{0,\text{DF}}(r)$ defined by (63):

$$d^*_{\text{DF}}(r) = 2 - \frac{r}{1 - t^*_{0,\text{DF}}(r)} = d_1\left(t^*_{0,\text{DF}}(r), r\right) \ , \tag{86}$$

where the second equality in (86) can be easily checked by referring to (52). On the other hand,

$$d_1\left(t^*_{0,\text{DoQF}}(r), r\right) \geq d^*_{\text{DoQF}}(r) \tag{87}$$

due to (50). Furthermore, Lemma 1 states that

$$d^*_{\text{DoQF}}(r) \geq d^*_{\text{DF}}(r) \ . \tag{88}$$

Combining (86), (87) and (88) leads to $d_1\left(t^*_{0,\text{DoQF}}(r), r\right) \geq d_1\left(t^*_{0,\text{DF}}(r), r\right)$. Since $d_1(t_0, r) = 2 - \frac{r}{1-t_0}$, we conclude that $t^*_{0,\text{DoQF}}(r) \leq t^*_{0,\text{DF}}(r)$ holds.

In order to prove that inequality $t^*_{0,\text{DoQF}}(r) \geq 0.5$ holds, we will show that the best DMT that can be achieved with $t_0 < 0.5$ ***i.e.***, $\max_{t_0<0.5} d(t_0, \delta, r)$, is less or equal to the DMT that can be achieved by choosing $t_0 \geq 0.5$. It can be shown after some algebra that

$$\forall u \geq 0.5, \forall v < 0.5, \quad d_2(v, \delta, r) \leq d_2(u, \delta, r) \ ,$$

where $d_2(u, \delta, r)$ is given by (53) and $d_2(v, \delta, r)$ is given by (56). Furthermore, it is straightforward to show that functions $t \mapsto d_3(t, \delta, r)$ and $t \mapsto d_4(t, \delta, r)$ defined respectively by (57) and (58) are increasing w.r.t t. Finally, since $d_1(v, r) = 2(1-r)^+$ for any $v < 0.5$ due to (52), then $d(v, \delta, r) = \min\{d_2(v, \delta, r), d_3(v, \delta, r), d_4(v, \delta, r)\}$. Putting all pieces together, we conclude that

$$\forall u \geq 0.5, \forall v < 0.5, \quad d(v, \delta, r) \leq d(u, \delta, r) \,,$$

which in turn means that $t^*_{0,\text{DoQF}} \geq 0.5$.

D. Proof of Lemma 3

Lemma 3 states that the following two inequalities hold true for $r > 0.25$:

$$\delta^*_{\text{DoQF}}(r) < 1 - \left(1 - \frac{r}{t^*_{0,\text{DoQF}}(r)}\right)^+ \text{ and } 0 < \delta^*_{\text{DoQF}}(r).$$

Recall from our discussion in Appendix D-A that the first inequality is a necessary condition for the DMT of the DoQF protocol to be greater or equal to the DMT of DF. We thus only need to prove the second inequality. To that end, we will resort to Lemma 1 which implies that

$$d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) \geq d^*_{\text{DF}}(r) \,, \tag{89}$$

where $d_3\left(t^*_{0,\text{DoQF}}(r), \delta^*_{\text{DoQF}}(r), r\right) = 4 + \frac{t^*_{0,\text{DoQF}}(r)}{1-t^*_{0,\text{DoQF}}(r)}\delta^*_{\text{DoQF}}(r) - \left(4 + \frac{t^*_{0,\text{DoQF}}(r)}{1-t^*_{0,\text{DoQF}}(r)}\right)\frac{r}{t^*_{0,\text{ DoQF}}(r)}$ due to (70). Consider first the case $\frac{\sqrt{5}-1}{\sqrt{5}+1} < r \leq 1$. In this case, $d^*_{\text{DF}}(r) = (1-r)(2-r)$ due to [21]. Inequality (89) is therefore equivalent to

$$4 + \frac{t^*_{0,\text{DoQF}}(r)}{1 - t^*_{0,\text{DoQF}}(r)}\delta^*_{\text{DoQF}}(r) - \left(4 + \frac{t^*_{0,\text{DoQF}}(r)}{1 - t^*_{0,\text{DoQF}}(r)}\right)\frac{r}{t^*_{0,\text{ DoQF}}(r)} \geq (1-r)(2-r) \,.$$

It is straightforward to show that the above inequality is equivalent to

$$\frac{t^*_{0,\text{DoQF}}(r)}{1 - t^*_{0,\text{DoQF}}(r)}\delta^*_{\text{DoQF}}(r) \geq r^2 + \left(\frac{4}{t^*_{0,\text{DoQF}}(r)} + \frac{1}{1 - t^*_{0,\text{DoQF}}(r)} - 3\right) r - 2 \,. \tag{90}$$

One can check after some algebra that the rhs of (90) is strictly positive for $\frac{\sqrt{5}-1}{\sqrt{5}+1} < r \leq 1$. We conclude that $\delta^*_{\text{DoQF}}(r) > 0$ on this interval. The proof of the strict positivity of $\delta^*_{\text{DoQF}}(r)$

for $0.25 < r \leq \frac{\sqrt{5}-1}{\sqrt{5}+1}$ can be done without difficulty in the same way, completing the proof of Lemma 3.