# DF-based Sum-rate Optimization for Multicarrier Multiple Access Relay Channel



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# DF-based Sum-rate Optimization for Multicarrier Multiple Access Relay Channel

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**Abstract**

We consider a system that consists of two sources, a half-duplex relay, and a destination. The sources want to transmit their messages reliably to the destination with the help of the relay. We study and analyze the performance of two transmission schemes in which the relay implements decode-and-forward strategy. In the first scheme, we incorporate Orthogonal Frequency Division Multiple Access (OFDMA) transmission into the system. In this scheme, there is only one source node transmitting on each subcarrier. The transmission can be either with or without the help of the relay. In the second scheme, we implement Orthogonal Frequency Division Multiplexing (OFDM) transmission. In this scheme, both sources can transmit their messages using all subcarriers. The relay can help none, only one or both sources. For both schemes, we discuss the design criteria and evaluate the achievable sum-rate. Next, for each scheme, we study and solve the problem of resources (powers and subcarriers) allocation aiming at maximizing the allowed sum-rate. For the first scheme, we develop a duality-based algorithm that finds a globally optimum solution. For the second scheme, we propose an iterative coordinate-descent algorithm that finds a suboptimum solution. We show through numerical examples the effectiveness of these algorithms and illustrate the benefits of OFDM transmission over OFDMA for the model that we study.



**Index Terms**

OFDMA, OFDM, Decode-and-forward, relay channel, decoding order, optimization.

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# I. Introduction

Relaying has been introduced to extend system coverage, enhance spectrum efficiency and improve the performance of wireless systems. Cooperative relay networks have been studied extensively for many wireless systems [1], [2], [3]. In a typical relay system, the relay helps the transmitters by forwarding the transmitted messages to the destination. Different efficient relaying protocols have been proposed in the literature, including amplifying-and-forwarding (AF), decoding-and-forwarding (DF), and compressing-and-forwarding (CF) [2], [4]. Each protocol has its advantages and its disadvantages; and which scheme outperforms the others depends on the network topology and channel conditions. Capacity bounds and rate regions have been established in [5] for the standard three-terminal gaussian relay channel and in [4], [6] for the gaussian multiple access relay channel (MARC). The reader may also refer to [7], [8], [9], [10] for some related works.

In the context of cooperative communication, multicarrier transmission techniques, such as the popular Orthogonal Frequency Division Multiplexing (OFDM) and its multi-user version Orthogonal Frequency-Division Multiple Access (OFDMA), constitute promising tools that can offer high data rate. In particular, this is due to the fact that these techniques permit to handle frequency selectivity and harness multi-user diversity. Essentially for these reasons, these techniques have been adopted in most next-generation wireless standards, and are generally considered in the context of relay-aided communications in frequency selective channels.

In this paper, we consider communication over a multicarrier two-source multiaccess channel in which the transmission is aided by a relay node, i.e., a multicarrier two-source multiaccess relay channel (MARC). The communication takes place in two transmission periods or time slots. The sources transmit only during the first transmission period. The relay is half-duplex, implements decode-and-forward protocol and transmits only during the second transmission period. We propose two multicarrier transmission schemes, based respectively on OFDMA and OFDM. In the first scheme, each subcarrier can only be used by at most one source at a time. In the second scheme, each subcarrier can be used by both sources simultaneously. For both schemes, we derive the allowed sum-rate. Also, we study the problem of allocating the resources (i.e., powers and subcarriers) and selecting the relay operation mode (i.e., active or idle) optimally in a way to maximize the obtained sum-rate. Some of the key issues that we consider are related to the way the subcarriers are assigned among the two sources, the selection of appropriate relay operation mode for every subcarrier, and the allocation of power among the two sources and the relay. Because of the presence of the relay node, such a resources allocation problem is more involved comparatively than those for conventional OFDM systems that do not involve relays.

## A. Connection with Related Works

For a point-to-point OFDM transmission aided by a DF relay node, some resource allocation algorithms have been proposed and studied in the literature. For example, in [11] the authors investigate the problem of maximizing the sum-rate for an OFDM transmission protocol that uses a half-duplex DF relay node. Depending on the fading coefficients, on each subcarrier the relay node can be either idle or active. If the relay is idle, the source transmits a new independent symbol in the second time slot. This transmission protocol is extended for the scenarios in which the transmission involve multiple relays, and the related resource allocation problems are solved in [12], [13], [14]. The problem of resources allocation for OFDM transmission over a two-way channel that is aided by a DF relay has been investigated as well, and addressed in [15], [16].

For OFDMA systems without relaying, some resources allocation problems have been studied in [17], [18], [19]. For OFDMA systems that involve relays, some related contributions have been proposed in the literature. These include [20] and

[21], in which the authors consider respectively the maximization of the allowed sum-rate and the maximization of a weighted sum goodput. Also, in [22] the authors maximize a metric depending on the rates and queue lengths of the source and relays. In [23], the authors jointly optimize the relay strategies and physical-layer resources in a multiple users network, where each user can act as a relay. In [24], the authors consider an optimal resources allocation strategy for cooperative relaying-enabled OFDMA multi-hop wireless systems. In [25] and [26], the authors study capacity regions of OFDMA multiple access networks that comprise AF and DF relays. They also investigate a problem of subcarriers assignment for given powers at the sources and the relay. In [27], a throughput maximization problem with fairness constraint is solved for a cooperative OFDMA network. The authors propose an efficient algorithm with low computational complexity that assigns appropriately subcarriers and powers. The reader may also refer to [28], [29], [30] for some related works.

For multiaccess relay networks, in [31] the authors investigate a problem of power allocation among two sources and a relay. Also, in [32] the authors study the problem of resources allocation for a multi-user DF-based relay network with orthogonal channel access that uses OFDMA. In this work, the setting that we consider is somehow connected to [31] and [32], with the following differences. First, in comparison to [31], in our case we consider frequency selective channels by means of multi-carrier; and we address the problem of maximizing the offered sum-rate under a total sum power constraint. Also, in our setting the relay uses the same codebook as that used by the sources and thus it transmits the same codewords that are sent by the sources. This explains the use of maximum ratio combining (MRC) at the destination in our work. Furthermore, we study the optimization problem by considering different optimization parameters. Second, in comparison with [32], we mention that the setup in [32] does not consider the case in which the sources are allowed to transmit their messages using the same subcarrier. Moreover, comparing the transmission scheme of [32] with the OFDMA transmission scheme that we consider in this paper, we note that in [32] the case in which the destination gets information from *only* the direct links (i.e, the relay is idle) is not considered explicitly therein, and the problem of allocating the powers in a way to maximize the obtained sum-rate is considered under individual power constraint.

### B. Contributions

The main contributions of this paper can be summarized as follows. For the multicarrier multiaccess relay network that we consider, we propose two transmission schemes that use respectively OFDMA and OFDM. For each of these transmission schemes, we first derive the allowed sum-rate; and then we study and solve the problem of maximizing the offered sum-rate under a total sum power constraint. The optimization problems involve subcarriers assignment as well as power allocation among the sources and the relay.

In the OFDMA-based scheme, each subcarrier is used by only one source at a time; and so each source transmits its codeword or symbol free of interference, to the relay and destination. The relay can be either idle or active; and the selection of the appropriate operation mode depends on the channel coefficients. In the case in which the relay remains idle, the destination recovers the transmitted codeword using the signal from the source. In the case in which the relay is active, it uses the *same* subcarrier employed by the source to forward the decoded codeword to the destination. The destination then performs maximum-ratio combining of the outputs from the source and relay to recover the transmitted codeword.

In the OFDM-based scheme, both sources utilize all subcarriers to transmit their codewords to the relay and destination. That is, each subcarrier can be shared by both sources simultaneously. The relay can help none, only one, or both sources. In all cases, whenever it is active, the relay transmits on the *same* subcarrier as that utilized by the source or sources. Also, if, for a given subcarrier, the relay helps both sources simultaneously, it re-encodes the decoded sources' codewords via superposition coding. The decoding procedures at the relay and destination are based on successive decoding and maximum-ratio combining. At this level, we should mention that, by opposition to a standard multiple access channel in which the allowed sum-rate does not depend on which decoding order is considered, in presence of relay nodes, i.e., for multiple access relay networks, different decoding orders at the relay and destination generally yield different allowed sum-rates. Taking this aspect into consideration, we consider all possible decoding orders combinations, and select the appropriate combination that offers the largest sum-rate. In addition to the decoding orders, the relay operation modes (i.e, helping none, only one, or both sources simultaneously) obviously also influences the sum-rate that is allowed per subcarrier, and, so, thereby the total offered sum-rate.

For each of the multicarrier transmission schemes that we consider, we study and solve the problem of maximizing the offered sum-rate under a total sum power constraint. The total sum power constraint comprises the powers used by all transmitting terminals, on all subcarriers. For the OFDMA-based transmission scheme, the optimization problem consists of i) partitioning the available subcarriers among the two sources, ii) selecting the appropriate relay operation mode (i.e., transmitting or not-transmitting) for every subcarrier, and iii) allocating the powers on each subcarrier and among the transmitting terminals. We show that the resulting optimization problem is convex, and we provide an efficient algorithm that finds a global solution optimally. For the OFDM-based transmission scheme, the optimization problem comprises i) selecting the appropriate relay operation mode (i.e., helping none, only one, or both sources simultaneously) for every subcarrier, ii) choosing the best decoding orders at the relay (if active) and destination for every subcarrier, and iii) allocating the powers on each subcarrier and among the transmitting terminals. We show that the resulting optimization problem can be seen as of mixed-integer linear programming type. Also, we propose an iterative algorithm that is based on a coordinate descent approach and that, for every subcarrier, finds the best relay operation mode and decoding orders at the relay (if active) and destination, and appropriate powers for the terminals transmitting on that subcarrier, alternately. The iterations stop when convergence to a stationary point is obtained. For given relay operation mode and decoding orders combination, the problem of allocating the powers appropriately is non-convex. In order to solve this problem, we propose a geometric programming approach. Also, we utilize a successive convex approximation method that is similar to in [33].

For both schemes, we illustrate our results through some numerical examples. In particular, our analysis shows that by allowing the sources to possibly transmit on the same subcarrier simultaneously, one can afford a larger sum-rate, i.e., the OFDM-based transmission scheme offers a substantial sum-rate gain over the one that is based on OFDMA.

### C. *Outline and Notation*

An outline of the remainder of this paper is as follows. Section II describes in more details the system model that we consider in this work. In Section III, we analyze the sum-rates that are achievable using these schemes. Section IV contains the optimization problems formulations for both schemes as well as the algorithms that we propose to solve these problems. Section V contains some numerical examples, and Section VI concludes the paper.

The following notations are used throughout the paper. Lowercase boldface letters are used to denote vectors, e.g., $\mathbf{x}$. Calligraphic letters designate alphabets, i.e., $\mathcal{X}$. The cardinality of a set $\mathcal{X}$ is denoted by $|\mathcal{X}|$. For vectors, we write $\mathbf{x} \in \mathbb{A}^n$, e.g., $\mathbb{A} = \mathbb{R}$ or $\mathbb{A} = \mathbb{C}$, to mean that $\mathbf{x}$ is a column vector of size $n$, with its elements taken from the set $\mathbb{A}$. For a vector $\mathbf{x} \in \mathbb{R}^n$, $\|\mathbf{x}\|$ designates the norm of $\mathbf{x}$ in terms of Euclidean distance. The Gaussian distribution with mean $\mu$ and variance

$\sigma^2$ is denoted by $\mathcal{N}(\mu, \sigma^2)$. We use $[x]^+$ to denote $\max\{0, x\}$. For a given $a \in \mathbb{R}$ and $b \in \mathbb{R}$, $\mathbf{1}_{a>b} = 1$ if $a > b$ and $\mathbf{1}_{a>b} = 0$ if $a \leq b$. Finally, for a complex-valued number $z = x + jy \in \mathbb{C}$, the notations $\text{Re}\{z\}$ and $\text{Im}\{z\}$ refer repectively to the real part and imaginary part of $z \in \mathbb{C}$, i.e., $\text{Re}\{z\} = x$ and $\text{Im}\{z\} = y$ and the notation $z^*$ refer to the complex conjugate of $z$, i.e., $z^* = x - jy$.

## II. System Model and Multicarrier Transmission Schemes

### A. System Model

We consider a multiaccess relay network that comprises two sources (**A** and **B**), a relay node (**R**) and a destination (**D**), as shown in Figure 1. The sources **A** and **B** want to transmit two messages, $W_a \in \mathcal{W}_a$ and $W_b \in \mathcal{W}_b$, to the destination with the help of the relay. The relay is half-duplex and implements DF strategy. The communication takes place in $n$ channel uses, and is divided into two periods or time slots with equal durations. Furthermore, the transmission is performed using multiple carriers. In what follows, we will consider both OFDMA and OFDM multicarrier transmissions. As usually assumed in similar settings, we assume that appropriate cycle prefixing is employed, turning the channel into a number of, say $K$, parallel subchannels.

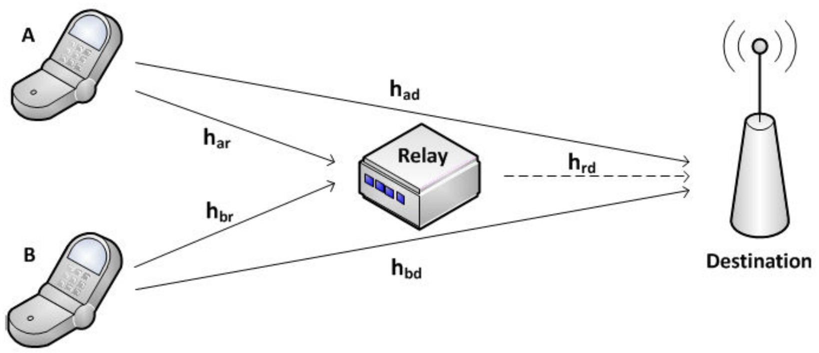


Fig. 1. Multiple-access relay channel with a half-duplex relay

Also, we assume that the states of the channel are known perfectly to all terminals, i.e., perfect channel state information at receivers (CSIR) and perfect channel state information at the transmitters (CSIT); and that they remain constant over a transmission period. That is, during each transmission period, each receiver has perfect knowledge of all channel coefficients on all subcarriers on which it receives, and each transmitter has perfect knowledge of all channel coefficients on all subcarriers on which it transmits. Furthermore, the noise signals at the relay and destination are independent from each others, and independently and identically distributed (i.i.d) circular complex Gaussian, with zero mean and variance $N$. Also, we consider the following constraint on the transmitted power,

$$\Big(\sum_{k=1}^{K}\mathbb{E}[\|\mathbf{x}_a[k]\|^2]\Big) + \Big(\sum_{k=1}^{K}\mathbb{E}[\|\mathbf{x}_b[k]\|^2]\Big) + \Big(\sum_{k=1}^{K}\mathbb{E}[\|\tilde{\mathbf{x}}_r[k]\|^2]\Big) \leq nP_t, \tag{1}$$

where $P_t \geq 0$ is the total per-channel use power imposed on the system, the first sum is the total power used by Source **A** during the whole transmission, the second sum is the total power used by Source **B** during the whole transmission, and the third sum is the total power used by Relay **R** during the whole transmission. Also, the inputs $\mathbf{x}_a[k]$, $\mathbf{x}_b[k]$ and $\tilde{\mathbf{x}}_r[k]$ denote respectively the codeword or symbol sent by Source **A** on subcarrier $k$ during the first transmission period, the codeword sent Source **B** on subcarrier $k$ during the first transmission period, and the codeword sent by Relay **R** on subcarrier $k$ during the second transmission period.

For convenience, let $\beta_a[k] \geq 0$ and $\beta_b[k] \geq 0$ be nonnegative scalars such that $\beta_a^2[k]P_t$ and $\beta_b^2[k]P_t$ be the per-channel use powers used at Source **A** and Source **B** on subcarrier $k$, respectively. Similarly, let $\beta_r[k] \geq 0$ be a nonnegative scalar such that $\beta_r^2[k]P_t$ be the per-channel use power used by Relay **R** on subcarrier $k$. Also, let $\beta_{ar}^2[k]P_t$ be the fraction of the power that the relay uses to help Source **A**, and $\beta_{br}^2[k]P_t$ be the fraction of the power that the relay uses to help Source **B**, with $\beta_{ar}^2[k] + \beta_{br}^2[k] = \beta_r^2[k]$. The aforementioned constraint on the available sum power can be rewritten equivalently as

$$\sum_{k=1}^{K} \left( \beta_a^2[k] + \beta_b^2[k] + \beta_{ar}^2[k] + \beta_{br}^2[k] \right) \leq 1. \tag{2}$$

Moreover, for convenience we will sometimes use the shorthand vector notation $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T \in \mathbb{R}^4$. Finally, the signal-to-noise ratio will be denoted as $\mathrm{snr} = P_t/N$ in the linear scale, and by $\mathrm{SNR} = 10\log_{10}(\mathrm{snr})$ in decibels.

## B. Multicarrier Transmission Schemes

There are in total $K$ subcarriers that can be used by the sources for the transmission. In what follows, we describe the input-output relations obtained using an OFDMA-based transmission and an OFDM-based transmission. In the OFDMA-based transmission, a subcarrier can be used by only one source at a time; and in the OFDM-based transmission, both sources can transmit simultaneously on every subcarrier.

*1) OFDMA Transmission:* The encoding and transmission scheme on subcarrier $k$, $1 \leq k \leq K$, is as follows. As we mentioned previously, only one of the two sources sends on this subcarrier. Let $\mathbf{x}_i[k]$, $i = a$ or $i = b$, be the input of the source transmitting on this subcarrier, during the first transmission period. During this period, the outputs at the relay and destination on subcarrier $k$ are given by

$$\begin{aligned} \mathbf{y}_r[k] &= h_{\mathrm{ir}}[k]\mathbf{x}_i[k] + \mathbf{z}_r[k] \\ \mathbf{y}_d[k] &= h_{\mathrm{id}}[k]\mathbf{x}_i[k] + \mathbf{z}_d[k] \end{aligned} \tag{3}$$

where $h_{\mathrm{ar}}[k]$ and $h_{\mathrm{br}}[k]$ are the channel gains on the links to the relay; $h_{\mathrm{ad}}[k]$ and $h_{\mathrm{bd}}[k]$ are the channel gains on the links to the destination; the vector $\mathbf{z}_r[k]$ is the additive noise at the relay, and the vector $\mathbf{z}_d[k]$ is the additive noise at the destination. These noise vectors are mutually independent, and are with components drawn i.i.d. according to the circular complex Gaussian distribution with zero mean and variance $N$.

Assuming that it decodes correctly the transmitted codeword, during the second transmission period the relay re-encodes this codeword using the same codebook as that used by the source. Thus, the destination receives

$$\tilde{\mathbf{y}}_d[k] = h_{\mathrm{rd}}[k]\tilde{\mathbf{x}}_r[k] + \tilde{\mathbf{z}}_d[k] \tag{4}$$

during the second transmission period, where $h_{\text{rd}}[k]$ is the channel gain on the link to the destination; and the vector $\tilde{\mathbf{z}}_d[k]$ is the additive noise at the destination during this period, assumed to be independent from all other noise vectors, and with components drawn i.i.d. according to a circular complex Gaussian distribution with zero mean and variance $N$.

*2) OFDM Transmission:* The encoding and transmission scheme on subcarrier $k$, $1 \leq k \leq K$, is as follows. As we mentioned previously, both sources transmit simultaneously on the same subcarrier $k$ in this case. During the first transmission period, Source $\mathbf{A}$ transmits the codeword $\mathbf{x}_a[k]$ over the channel. Similarly, Source $\mathbf{B}$ transmits the codeword $\mathbf{x}_b[k]$ over the channel. During this period, the outputs at the relay and destination on subcarrier $k$ are given by

$$\begin{aligned} \mathbf{y}_r[k] &= h_{\text{ar}}[k]\mathbf{x}_a[k] + h_{\text{br}}[k]\mathbf{x}_b[k] + \mathbf{z}_r[k] \\ \mathbf{y}_d[k] &= h_{\text{ad}}[k]\mathbf{x}_a[k] + h_{\text{bd}}[k]\mathbf{x}_b[k] + \mathbf{z}_d[k], \end{aligned} \tag{5}$$

where $h_{\text{ar}}[k]$ and $h_{\text{br}}[k]$ are the channel gains on the links to the relay; $h_{\text{ad}}[k]$ and $h_{\text{bd}}[k]$ are the channel gains on the links to the destination; the vector $\mathbf{z}_r[k]$ is the additive noise at the relay, and the vector $\mathbf{z}_d[k]$ is the additive noise at the destination. These noise vectors are mutually independent, and are with components drawn i.i.d. according to the circular complex Gaussian distribution with zero mean and variance $N$.

Assuming that it decodes correctly the codewords transmitted by the sources, during the second transmission period the relay re-encodes the codewords using the same codebook employed by the sources. Thus, during this period, the output at the destination on subcarrier $k$ is given by

$$\tilde{\mathbf{y}}_d[k] = h_{\text{rd}}[k]\tilde{\mathbf{x}}_r[k] + \tilde{\mathbf{z}}_d[k], \tag{6}$$

where $h_{\text{rd}}[k]$ is the channel gain on the link to the destination; and the vector $\tilde{\mathbf{z}}_d[k]$ is the additive noise at the destination during this period, assumed to be independent from all other noise vectors, and with components drawn i.i.d. according to a circular complex Gaussian distribution with zero mean and variance $N$.

## III. Sum-Rate Analysis

In this section, we analyze the OFDMA and OFDM multicarrier transmission schemes that we described in the previous section, from the allowed sum-rate viewpoint.

### A. Sum-Rate Analysis for the OFDMA-based Transmission

The following proposition provides an achievable sum-rate for the multiaccess relay model of Figure 1, using OFDMA multicarrier transmission.

***Proposition** 1:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$, the following sum-rate is achievable for the multiaccess relay channel of Figure 1

$$R_{\text{sum}}^{\text{OFDMA}} = \max \sum_{k=1}^{K} \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right) \tag{7}$$

where $h_{\text{eq}}[k]$ is such that

$$\begin{aligned} |h_{\text{eq}}[k]|^2 = \max\Bigg\{ &\frac{|h_{\text{ar}}[k]|^2|h_{\text{rd}}[k]|^2}{|h_{\text{ar}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{ad}}[k]|^2}\mathbf{1}_{|h_{\text{ar}}[k]|>|h_{\text{ad}}[k]|}, |h_{\text{ad}}[k]|^2, |h_{\text{bd}}[k]|^2, \\ &\frac{|h_{\text{br}}[k]|^2|h_{\text{rd}}[k]|^2}{|h_{\text{br}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{bd}}[k]|^2}\mathbf{1}_{|h_{\text{br}}[k]|>|h_{\text{bd}}[k]|}\Bigg\} \end{aligned} \tag{8}$$

and the maximization is over $\{\beta_s[k]\}_{k=1}^{K}$, satisfying

$$\sum_{k=1}^{K} \beta_s^2[k] \leq 1. \tag{9}$$

**Proof:** Recall the OFDMA-based transmission scheme of Section II-B. In what follows, we describe the decoding procedures at the relay and destination; and we analyze the allowed sum-rate.

Fix a power policy $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (3). The relay utilizes joint typicality decoding to decode the transmitted codeword. The rate (per channel use) at which the relay can perform this reliably on subcarrier $k$ can be shown easily to be

$$R_{\mathrm{ir}}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\mathrm{ir}}[k]|^2 P_t}{N}\right), \tag{10}$$

where $i = a$ if subcarrier $k$ is used for transmission by Source $\mathbf{A}$, and $i = b$ if subcarrier $k$ is used for transmission by Source $\mathbf{B}$. (See below a procedure that selects optimally the source that should transmit on subcarrier $k$).

At the end of the transmission, the destination utilizes the output vector $\mathbf{y}_d[k]$ from the direct transmission by the source given by (3) and the output vector $\tilde{\mathbf{y}}_d[k]$ from the transmission by the relay given by (4) to get an estimate of the transmitted codeword. In doing this, the destination performs a maximum-ratio combining of the two output components. The rate (per channel use) at which the destination can perform this reliably on subcarrier $k$ can be shown easily to be

$$R_{\mathrm{id}}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\mathrm{id}}[k]|^2 P_t}{N} + \frac{\beta_{ir}^2[k]|h_{\mathrm{rd}}[k]|^2 P_t}{N}\right). \tag{11}$$

With the help of the relay node, the destination gets the information transmitted on subcarrier $k$ correctly as long as this information is sent at a rate that is no larger than the minimum among $R_{\mathrm{ir}}[k]$ as given by (10) and $R_{\mathrm{id}}[k]$ as given by (11), i.e.,

$$R[k] = \min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\mathrm{ir}}[k]|^2 P_t}{N}\right), \frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\mathrm{id}}[k]|^2 P_t}{N} + \frac{\beta_{ir}^2[k]|h_{\mathrm{rd}}[k]|^2 P_t}{N}\right)\right\}. \tag{12}$$

As shown in [11], at the optimum, the constraint associated with the minimization in (12) should be saturated, i.e.,

$$\beta_i^2[k] = \frac{\beta_{ir}^2[k]|h_{\mathrm{rd}}[k]|^2}{|h_{\mathrm{ir}}[k]|^2 - |h_{\mathrm{id}}[k]|^2}. \tag{13}$$

Let $\beta_s^2[k] = \beta_i^2[k] + \beta_{ir}^2[k]$, the rate $R[k]$ on subcarrier $k$ can be rewritten equivalently as,

$$R[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\mathrm{ir}}[k]|^2|h_{\mathrm{rd}}[k]|^2 P_t}{N(|h_{\mathrm{ir}}[k]|^2 + |h_{\mathrm{rd}}[k]|^2 - |h_{\mathrm{id}}[k]|^2)}\right). \tag{14}$$

From the above, it follows that it is beneficial that the relay be active on subcarrier $k$, i.e., the relay decodes and forwards the source's codeword, if and only if (iff) the following two conditions hold

$$\begin{cases} |h_{\mathrm{id}}[k]|^2 & < |h_{\mathrm{ir}}[k]|^2 \\ |h_{\mathrm{id}}[k]|^2 & < \frac{|h_{\mathrm{ir}}[k]|^2|h_{\mathrm{rd}}[k]|^2}{|h_{\mathrm{ir}}[k]|^2 + |h_{\mathrm{rd}}[k]|^2 - |h_{\mathrm{id}}[k]|^2}. \end{cases} \tag{15}$$

If it is more advantageous that the relay remains idle on subcarrier $k$, the destination decodes the transmitted codeword using only its output component from the direct transmission, i.e., from the source. In this case, one can get a larger rate by allocating all the available power $\beta_s^2[k]P_t$ for transmission on subcarrier $k$ to the transmitting source. The destination decodes the transmitted codeword at rate

$$R[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\mathrm{id}}[k]|^2 P_t}{N}\right). \tag{16}$$

*Summarizing:* for every subcarrier $k$, $1 \leq k \leq K$, the appropriate relay operation mode (i.e., transmitting or not-transmitting on that subcarrier) can be selected optimally based on the actual channel states. More precisely, the relay helps the source that transmits on subcarrier $k$ iff the two conditions in (15) hold simultaneously; otherwise it remains idle. Investigating (14) and (16), we introduce the following *equivalent* channel gains for the transmission on subcarrier $k$,

$$|h_i[k]|^2 = \max\left\{|h_{\text{id}}[k]|^2, \frac{|h_{\text{ir}}[k]|^2|h_{\text{rd}}[k]|^2}{|h_{\text{ir}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{id}}[k]|^2}\mathbf{1}_{|h_{\text{ir}}[k]|>|h_{\text{id}}[k]|}\right\}, \qquad i = a, b. \tag{17}$$

In order to maximize the allowed sum-rate, subcarrier $k$ should be assigned to the source that has the largest equivalent channel gain among $|h_a[k]|^2$ and $|h_b[k]|^2$. Then, defining the equivalent channel coefficient $h_{\text{eq}}[k]$ for subcarrier $k$ to be

$$|h_{\text{eq}}[k]|^2 = \max\left\{|h_a[k]|^2, |h_b[k]|^2\right\}, \tag{18}$$

the rate that is allowed on subcarrier $k$, $1 \leq k \leq K$, can be put into the compact form

$$R[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right), \tag{19}$$

where $h_{\text{eq}}[k]$ is given by (18).

For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$ and power policy $\{\beta_s[k]\}_{k=1}^{K}$, since OFDMA transforms the channel into a set of $K$ *parallel* subchannels, the sum-rate that is offered through the transmission is obtained by simply summing over all subchannels the individual rate $R[k]$, $k = 1, \cdots, K$ [34]. Finally, the following larger sum-rate can be obtained by maximizing over all allowable power policies, i.e.,

$$R_{\text{sum}}^{\text{OFDMA}} = \max \sum_{k=1}^{K} \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right) \tag{20}$$

where the maximization is over $\{\beta_s[k]\}_{k=1}^{K}$ such that $\sum_{k=1}^{K}\beta_s^2[k] \leq 1$.

This completes the proof of Proposition 1. □

## *B. Sum-rate Analysis for the OFDM-based Transmission*

In this section we describe the OFDM transmission scheme where sources **A** and **B** transmit their codewords simultaneously using all subcarriers.

For convenience, we define the quantities given in Definition 1 and Definition 2, which we will use extensively throughout this section.

***Definition** 1:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$, and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, with for $1 \leq k \leq K$, $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, let

$$\Theta_{\text{b}}^{(1)}[k] = \frac{N(\beta_b^2[k]|h_{\text{bd}}[k]|^2 + \beta_{br}^2[k]|h_{\text{rd}}[k]|^2)P_t + \beta_b^2[k]|h_{\text{bd}}[k]|^2\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_t^2}{N^2 + \beta_a^2[k]|h_{\text{ad}}[k]|^2P_tN + \beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_tN} \tag{21}$$

$$\Theta_{\text{b}}^{(2)}[k] = \frac{\beta_{br}^2[k]|h_{\text{rd}}[k]|^2\beta_a^2[k]|h_{\text{ad}}[k]|^2P_t^2 - 2\beta_a[k]\beta_b[k]\beta_{ar}[k]\beta_{br}[k]\text{Re}\{h_{\text{bd}}^*[k]h_{\text{ad}}[k]\}|h_{\text{rd}}[k]|^2P_t^2}{N^2 + \beta_a^2[k]|h_{\text{ad}}[k]|^2P_tN + \beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_tN} \tag{22}$$

$$\text{snr}_{\text{b}}[k] = \Theta_{\text{b}}^{(1)}[k] + \Theta_{\text{b}}^{(2)}[k]. \tag{23}$$

Also, let $\Theta_{\text{a}}^{(1)}[k]$, $\Theta_{\text{a}}^{(2)}[k]$, and $\text{snr}_{\text{a}}[k]$ be obtained by swapping the indices $a$ and $b$ in $\Theta_{\text{b}}^{(1)}[k]$, $\Theta_{\text{b}}^{(2)}[k]$, and $\text{snr}_{\text{b}}[k]$, respectively.

***Definition** 2:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$, and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, with for $1 \le k \le K$, $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, let

$$R_1[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t}{N} + \frac{\beta_b^2[k]|h_{\text{bd}}[k]|^2 P_t}{N}\right) \tag{24}$$

$$\begin{aligned} R_2[k] = \min&\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}\right), \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{bd}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t} + \frac{\beta_{br}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right)\right\} \\ &+ \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t}{N}\right) \end{aligned} \tag{25}$$

$$\begin{aligned} R_3[k] = \min&\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}{N}\right), \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t}{N} + \frac{\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right)\right\} \\ &+ \min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}\right), \frac{1}{2}\log_2\left(1 + \text{snr}_{\text{b}}[k]\right)\right\} \end{aligned} \tag{26}$$

$$\begin{aligned} R_4[k] = \min&\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}{N + \beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}\right), \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t}{N} + \frac{\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right)\right\} \\ &+ \min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N}\right), \frac{1}{2}\log_2\left(1 + \text{snr}_{\text{b}}[k]\right)\right\}. \end{aligned} \tag{27}$$

Also, let $R_5[k]$, $R_6[k]$, and $R_7[k]$ be obtained by swapping the indices $a$ and $b$ in $R_2[k]$, $R_3[k]$, and $R_4[k]$, respectively.

The following proposition provides an achievable sum-rate for the multiaccess relay model of Figure 1, using OFDM multicarrier transmission.

***Proposition** 2:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$, the following sum-rate is achievable for the multiaccess relay channel of Figure 1,

$$R_{\text{sum}}^{\text{OFDM}} = \max \sum_{k=1}^{K} \max_{1 \le l \le 7} R_l[k], \tag{28}$$

where, for $1 \le k \le K$, and $1 \le l \le 7$, $R_l[k]$ is defined as in Definition 2; and the outer maximization is over $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, with $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, such that

$$\sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \le 1. \tag{29}$$

The proof of Proposition 2 will follow. The following remark reveals certain aspects related to the coding scheme, and is useful for a better understanding of the proof and its structure.

*Remark 1:* The proof is based on the OFDM multicarrier transmission scheme of Section II-B. In this scheme, by opposition to that of Proposition 1, both sources are allowed to transmit simultaneously on every subcarrier. The relay is half-duplex, and implements decode-and-forward strategy on the symbols transmitted on each subcarrier. It helps none, only one, or both sources simultaneously. In the case in which the relay helps both sources simultaneously, on the same subcarrier, it shares its power among the two and superimposes the information that is intended to help Source **A** and the one that is intended to help Source **B**. The destination decodes the sources's codewords successively, and the decoding operations are based on maximum-ratio combining. The relay decodes both codewords only if it helps both sources to transmit their codewords; and, if so, it also decodes the sources's codewords successively. As we mentioned previously, different decoding orders combinations (at the relay, if applicable, and at the destination) generally result in different achievable sum-rates. That is, in general no decoding order outperforms the others; and the selection of the appropriate decoding order depends on the fading coefficients.

| | Decoding order at the relay | Decoding order at the destination | Case |
|---|---|---|---|
| Direct Transmission | N.A. | No Decoding order | 1 |
| The relay forwards $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | 2 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | 3 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | 4 |
| The relay forwards $\mathbf{x}_a[k]$ | $\mathbf{x}_a[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | 5 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | 6 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | 7 |

TABLE I

DIFFERENT USEFUL CASES FOR THE OFDM MULTICARRIER TRANSMISSION



In addition to the decoding orders at the relay and destination, the relay operation mode (i.e, helping none, only one or both sources) influences the allowed sum-rate. This leads to thirteen different cases if all possible combinations are considered using the decoding orders and the relay operation modes. However, it can be easily seen that whenever the relay helps only one of the sources (by decoding and forwarding the codeword transmitted by that source), this codeword should be decoded first at the destination. When the relay helps the two sources simultaneously, a total of four possible decoding orders need to be investigated and compared (two possible decoding orders at the destination for each possible decoding order at the relay). Hence, out of the thirteen apriori possible cases only seven actually attribute to be of interest. These cases are summarized in Table I.

**Proof:** Recall the OFDM-based transmission scheme of Section II-B. Also, recall the seven possible cases that we mentioned in Remark 1, summarized in Table I. In what follows, because of symmetry, we only analyze the following four cases for the transmission on subcarrier $k$, $1 \le k \le K$: **Case** 1) transmission to the destination on subcarrier $k$ utilizes only the direct links, i.e., the relay remains idle on subcarrier $k$, **Case** 2) the relay helps only one source on subcarrier $k$, e.g., Source **B** by decoding and forwarding the transmitted symbol $\mathbf{x}_b[k]$, **Case** 3) the relay helps both sources simultaneously on subcarrier $k$, and the codeword $\mathbf{x}_b[k]$ of Source **B** is decoded first at both relay and destination, and **Case** 4) the relay helps both sources simultaneously on subcarrier $k$, with the codeword $\mathbf{x}_a[k]$ of Source **A** decoded first at the relay and the codeword $\mathbf{x}_b[k]$ of Source **B** decoded first at the destination. The analysis of the remaining three cases (obtained respectively from Case 2, Case 3 and Case 4 by swapping the roles of the sources) can be obtained straightforwardly by symmmetry. For each of the four cases that will be analyzed, we first describe the decoding procedures at the relay and destination and then analyze the allowed sum-rate.

Case 1 *Transmission using only direct links:* This scenario corresponds to a regular MAC, and the sum-rate that is achievable on subcarrier $k$, $1 \le k \le K$, can be shown easily [34] to be $R_1[k]$ as given by (24) in Definition 2.

Case 2 *The relay helps only Source **B**:* At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (5). The relay utilizes joint typicality decoding to decode the codeword $\mathbf{x}_b[k]$ transmitted by Source **B** on subcarrier $k$, $1 \le k \le K$. In doing so, the relay treats the codeword $\mathbf{x}_a[k]$ transmitted by Source **A** as unknown noise. For large $n$, the decoding can be done reliably at rate

$$R_{\text{br}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}\right), \tag{30}$$

where the upper script refers to the case in hand. The relay then forwards the decoded codeword on the same subcarrier $k$ to the destination, during the second transmission period. To this end, the relay sends

$$\tilde{\mathbf{x}}_r[k] = \sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k]. \tag{31}$$

Using (31), the destination's output components $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$ from the two transmission periods, given by (5) and (6), can be rewritten equivalently as

$$\mathbf{y}_d[k] = h_{\text{ad}}[k]\mathbf{x}_a[k] + h_{\text{bd}}\mathbf{x}_b[k] + \mathbf{z}_d[k]$$
$$\tilde{\mathbf{y}}_d[k] = h_{\text{rd}}[k]\sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k] + \tilde{\mathbf{z}}_d[k]. \tag{32}$$

The destination decodes the codewords transmitted by both sources successively. Given that the relay helps only Source **B**, it can be shown relatively straightforwardly that, in this case, decoding the relayed codeword $\mathbf{x}_b[k]$ first, i.e., before canceling out its contribution and decoding the non-relayed codeword $\mathbf{x}_a[k]$, results in a sum-rate that is larger than the one that would be allowed if the decoding of the codewords at the destination is performed in the reverse order. Thus, the destination first decodes codeword $\mathbf{x}_b[k]$, cancels its contribution out and then decodes codeword $\mathbf{x}_a[k]$. In order to decode codeword $\mathbf{x}_b[k]$, the destination combines the output components $\mathbf{y}_d[k]$ and $\tilde{\mathbf{y}}_d[k]$ to their maximum ratio, i.e., using standard maximum ratio combining (MRC). It can be shown easily that, for large $n$, the decoding of codeword $\mathbf{x}_b[k]$ can be decoded reliably at rate

$$R_{\text{bd}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{bd}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t} + \frac{\beta_{br}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right). \tag{33}$$

Next, the destination subtracts out the contribution of $\mathbf{x}_b[k]$ from $\mathbf{y}_d[k]$ and, so, decodes the codeword $\mathbf{x}_a[k]$ free of interference. It can be shown easily that, for large $n$, this can be done reliably at rate

$$R_{\text{ad}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2}{N}\right). \tag{34}$$

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier $k$, $1 \le k \le K$, as long as $n$ is large and these codewords are sent at a sum-rate that is no larger than the sum of $R_{\text{ad}}^{(2)}[k]$ and the minimum among $R_{\text{br}}^{(2)}[k]$ and $R_{\text{bd}}^{(2)}[k]$, i.e., $R_2[k]$ as given by (25) in Definition 2.

<u>Case</u> 3 *The relay helps both sources, and the decoding orders at the relay and destination are identical:* In this case we assume that the relay helps both sources, and that both the relay and destination first decode codeword $\mathbf{x}_b[k]$, cancel out its contribution and then decode codeword $\mathbf{x}_a[k]$.

Consider first the decoding operations at the relay. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (5). The relay utilizes joint typicality decoding to decode the transmitted codeword $\mathbf{x}_b[k]$ from the output vector $\mathbf{y}_r[k]$. In doing so, the relay treats the codeword $\mathbf{x}_a[k]$ transmitted by Source **A** as unknown noise. It can be shown easily that, for large $n$, the decoding can be done reliably at rate

$$R_{\text{br}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}\right), \tag{35}$$

where the upper script refers to the case in hand. The relay then subtracts out the contribution of $\mathbf{x}_b[k]$ from $\mathbf{y}_r[k]$ and then decodes codeword $\mathbf{x}_a[k]$, again using a joint typicality decoding. Similarly, for large $n$, this can be done reliably at rate

$$R_{\text{ar}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}{N}\right). \tag{36}$$

During the second transmission period, the relay helps both sources and transmits their codewords simultaneously on subcarrier $k$. To this end, the relay shares its power among re-transmitting codeword $\mathbf{x}_a[k]$ and re-transmitting codeword $\mathbf{x}_b[k]$, on the same subcarrier $k$, using superposition coding. That is, the relay sends

$$\tilde{\mathbf{x}}_r[k] = \sqrt{\frac{\beta_{ar}^2[k]}{\beta_a^2[k]}}\mathbf{x}_a[k] + \sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k] \tag{37}$$

on subcarrier $k$, where $\beta_{ar}[k]$ and $\beta_{br}[k]$ are nonnegative scalars chosen to adjust power and are such that $\beta_{ar}^2[k] + \beta_{br}^2[k] = \beta_r^2[k]$. (The way this power sharing needs to be performed appropriately will be addressed in Section IV-B).
Using (37), the destination's output components $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$ from the two transmission periods, given by (5) and (6), can be rewritten equivalently as

$$\begin{aligned}\mathbf{y}_d[k] &= h_{\text{ad}}[k]\mathbf{x}_a[k] + h_{\text{bd}}[k]\mathbf{x}_b[k] + \mathbf{z}_d[k],\\ \tilde{\mathbf{y}}_d[k] &= h_{\text{rd}}[k]\sqrt{\frac{\beta_{ar}^2[k]}{\beta_a^2[k]}}\mathbf{x}_a[k] + h_{\text{rd}}[k]\sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k] + \tilde{\mathbf{z}}_d[k].\end{aligned} \tag{38}$$

The destination decodes the codewords transmitted by both sources successively, in the same order this is performed at the relay. More precisely, the destination first decodes codeword $\mathbf{x}_b[k]$, cancels its contribution out and then decodes codeword $\mathbf{x}_a[k]$. In order to decode codeword $\mathbf{x}_b[k]$, the destination combines the output components $\mathbf{y}_d[k]$ and $\tilde{\mathbf{y}}_d[k]$ to their maximum ratio. Through straightforward algebra, which we omit for brevity, it can be shown that, for large $n$, the destination can get the correct $\mathbf{x}_b[k]$ at rate

$$R_{\text{bd}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \text{snr}_\text{b}[k]\right), \tag{39}$$

where $\text{snr}_\text{b}[k]$ is given in Definition 1.

Next, the destination subtracts out the contribution of codeword $\mathbf{x}_b[k]$ from $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$, and combines the resulting equivalent output components using MRC to decode codeword $\mathbf{x}_a[k]$. Again, through straightforward algebra, which we omit for brevity, it can be shown that, for large $n$, the destination can get the correct $\mathbf{x}_a[k]$ at rate

$$R_{\text{ad}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t}{N} + \frac{\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right). \tag{40}$$

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier $k$, $1 \le k \le K$, as long as $n$ is large and these codewords are sent at a sum-rate that is no larger than the sum of the minimum among $R_{\text{ar}}^{(3)}[k]$ and $R_{\text{ad}}^{(3)}[k]$ and the minimum among $R_{\text{br}}^{(3)}[k]$ and $R_{\text{bd}}^{(3)}[k]$, i.e., $R_3[k]$ as given by (26) in Definition 2.

Case 4 *The relay helps both sources, and the decoding orders at the relay and destination are different:* In this case we assume that the relay helps both sources, and that the relay and destination decode the sources' codewords in different orders. In particular, in what follows we analyze the case in which the decoding order at the relay is such that codeword $\mathbf{x}_a[k]$ is decoded first, and the decoding at the destination is maintained as in Case 3 above.

Consider first the decoding operations at the relay. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (5). Proceeding along the lines in the analysis of Case 3 above, but the roles of codewords $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ swapped, it can be shown easily that, for large $n$, the relay can get the correct $\mathbf{x}_a[k]$ at rate

$$R_{\text{ar}}^{(4)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}{N + \beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}\right) \tag{41}$$

and the correct $\mathbf{x}_b[k]$ at rate

$$R_{\text{br}}^{(4)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N}\right), \tag{42}$$

where the upper scripts refer to the case in hand.

The decoding at the destination is exactly as in Case 3. Thus, for large $n$, the destination can first get the correct $\mathbf{x}_b[k]$ at rate $R_{\text{bd}}^{(4)}[k] = R_{\text{bd}}^{(3)}[k]$ as given by (39); and then subtract its contribution out and get the correct codeword $\mathbf{x}_a[k]$ at rate $R_{\text{ad}}^{(4)}[k] = R_{\text{ad}}^{(3)}[k]$ as given by (40).

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier $k$, $1 \le k \le K$, as long as $n$ is large and these codewords are sent at a sum-rate that is no larger than the sum of the minimum among $R_{\text{ar}}^{(4)}[k]$ and $R_{\text{ad}}^{(4)}[k]$ and the minimum among $R_{\text{br}}^{(4)}[k]$ and $R_{\text{bd}}^{(4)}[k]$, i.e., $R_4[k]$ as given by (27) in Definition 2.

This completes the analysis of Cases 1-4. The analysis of Case 5, Case 6 and Case 7 in Table I can be obtained straightforwardly respectively from the analysis of Case 2, Case 3 and Case 4, by swapping the roles of Source **A** and Source **B**. This leads to the associated sum-rates $R_5[k]$, $R_6[k]$ and $R_7[k]$ as given in Definition 2.

*Summarizing:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$ and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, sum-rates of $R_l[k]$ bits per second, $1 \le l \le 7$, are achievable on subcarrier $k$, $1 \le k \le K$, using the OFDM-based transmission that we described. Thus, the sum-rate $R[k] = \max_{1\le l\le 7} R_l[k]$ on subcarrier $k$, i.e., the maximum among the seven sum-rates $\{R_l[k]\}_{l=1}^{7}$, is obtained by selecting for subcarrier $k$ the coding scheme that offers the larger per-subcarrier sum-rate among those of the aforementioned seven cases. Next, since OFDM transforms the channel into a set of $K$ *parallel* subchannels, the total sum-rate that is offered through the transmission, over all subchannels, is obtained by simply summing over all subchannels the individual allowed per-subcarrier sum-rates [34]. Finally, the larger sum-rate $R_{\text{sum}}^{\text{OFDM}}$ as given in the statement of Proposition 2 can be obtained by maximizing the obtained total sum-rate over all allowable power policies.

This completes the proof of Proposition 2. □

## IV. Sum Rate Optimization

In this section, for each of the multicarrier transmission schemes that we consider, we study and solve the problem of maximizing the offered sum-rate under a total sum power constraint.

### A. OFDMA Sum-rate Optimization

*1) Problem Formulation:* Consider the sum-rate $R_{\text{sum}}^{\text{OFDMA}}$ as given by (7) in Proposition 1. The optimization problem is stated as:

$$\text{(A)} \; : \; \max \quad \sum_{k=1}^{K} \frac{1}{2} \log_2 \left( 1 + \frac{\beta_s^2[k] |h_{\text{eq}}[k]|^2 P_t}{N} \right) \tag{43a}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} \beta_s^2[k] \leq 1, \tag{43b}$$

$$\beta_s^2[k] \geq 0, \tag{43c}$$

$$|h_{\text{eq}}[k]|^2 = \max \left\{ \frac{|h_{\text{ar}}[k]|^2 |h_{\text{rd}}[k]|^2}{|h_{\text{ar}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{ad}}[k]|^2} \mathbf{1}_{|h_{\text{ar}}[k]| > |h_{\text{ad}}[k]|}, |h_{\text{ad}}[k]|^2, |h_{\text{bd}}[k]|^2, \frac{|h_{\text{br}}[k]|^2 |h_{\text{rd}}[k]|^2}{|h_{\text{br}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{bd}}[k]|^2} \mathbf{1}_{|h_{\text{br}}[k]| > |h_{\text{bd}}[k]|} \right\}. \tag{43d}$$

*Remark 2:* we note that, as described in Section III-A, in order to maximize the allowed sum-rate $R_{\text{sum}}^{\text{OFDMA}}$, subcarrier $k$ should be assigned to the source that has the largest equivalent channel gain among $|h_{\text{a}}[k]|^2$ and $|h_{\text{b}}[k]|^2$ given by (17). This is a subcarrier allocation based on a greedy algorithm in which a subcarrier $k$ is assigned to the source that has the largest equivalent channel gain. Thus, the maximization of problem (A) is only over $\{\beta_s[k]\}_{k=1}^{K}$.

The optimization problem (A) is concave. In what follows, we provide an efficient algorithm that finds a global solution optimally.

*2) Power Allocation:* In this section, we focus on the problem of finding appropriate power values $\{\beta_s[k]\}_{k=1}^{K}$. We solve this problem using dual decomposition approach. The Lagrangian function can be defined as:

$$L(\mu, \beta_s[k]) = \sum_{k=1}^{K} \frac{1}{2} \log_2 \left( 1 + \frac{\beta_s^2[k] |h_{\text{eq}}[k]|^2 P_t}{N} \right) - \mu \left[ \sum_{k=1}^{K} \beta_s^2[k] - 1 \right], \tag{44}$$

where $\mu$ is the Lagrange multiplier associated with the global power constraint. The solution can be found by applying and solving the KKT conditions [35]. This leads to a waterfilling solution adjusted to the equivalent channel $|h_{\text{eq}}[k]|^2$:

$$\beta_s^2[k] = \left[ \frac{1}{\mu} - \frac{N}{P_t |h_{\text{eq}}[k]|^2} \right]^+. \tag{45}$$

We should note that any subcarrier can be excluded from transmission if its allocated power is zero. The water-level, i.e. $\mu$, has to be chosen such that the power constraint (43b) is fulfilled, and is given by

$$\frac{1}{\mu} = \frac{1}{K'} + \frac{1}{K'} \sum_{k=1}^{K'} \frac{N}{|h_{\text{eq}}[k]|^2 P_t}, \tag{46}$$

where $K'$ is the number of subcarriers with a non zero positive power value. To compute $R_{\text{sum}}^{\text{OFDMA}}$ as given by (7), we develop the following algorithm, to which we refer to as "Algorithm A" in reference to the optimization problem (A). The iterative algorithm (Algorithm A) terminates if $|\sum_{k=1}^{K} \beta_s^2[k] - 1|$ is smaller than a prescribed small strictly positive constant $\epsilon$.

**Algorithm A** Power Allocation for $R_{\text{sum}}^{\text{OFDMA}}$ as given by (7)

1: Calculate $\mu$ using (46) for $K' = K$
2: Solve the power allocation problem using (45), and calculate the power values $\{\beta_s[k]\}_{k=1}^{K}$
3: Decrease the number of subcarriers $K = K - 1$ by removing the subcarrier that has the smallest equivalent channel $|h_{\text{eq}}[k]|^2$ and then go to step 1
4: Terminate if $|\sum_{k=1}^{K} \beta_s^2[k] - 1| \leq \epsilon$

### B. OFDM Sum-rate Optimization

This section is devoted to maximize the sum-rate of the objective function given in (28). the optimization problem comprises i) selecting the appropriate relay operation mode (i.e., helping none, only one, or both sources simultaneously) for every subcarrier, ii) choosing the best decoding orders at the relay (if active) and destination for every subcarrier, and iii) allocating the powers on each subcarrier and among the transmitting terminals.

*1) Problem Formulation:* Consider the sum-rate $R_{\text{sum}}^{\text{OFDM}}$ as given by (28) in Proposition 2. The optimization problem can be equivalently stated as

$$\text{(B)} \ : \ \max \qquad \sum_{k=1}^{K} \sum_{l=1}^{7} a_l[k] R_l[k], \tag{47}$$

where, for $1 \le k \le K$, and $1 \le l \le 7$, $a_l[k]$ is an indicator whose value should be 0 or 1, and $R_l[k]$ is defined as in Definition 2; and the maximization is over $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, with $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, satisfying

$$\sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \le 1, \tag{48}$$

and over $\{\mathbf{a}[k]\}_{k=1}^{K}$, with $\mathbf{a}[k] = [a_1[k], a_2[k], .., a_7[k]]^T$, such that

$$\|\mathbf{a}[k]\|^2 \le 1, \ \text{ for } 1 \le k \le K. \tag{49}$$

The optimization problem (B) is a mixed integer linear programming problem; and, so, it is not easy to solve it optimally. In what follows, we solve this optimization problem iteratively, by finding appropriate powers $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ and indicators $\{\mathbf{a}[k]\}_{k=1}^{K}$ alternately. We note that the selection of $\{\mathbf{a}[k]\}_{k=1}^{K}$ determine the decoding orders at the relay and destination, and the relay operation mode (i.e., helping none, only one, or both sources simultaneously).

Let us, with a slight abuse of notation, denote by $R_{\text{sum}}^{\text{OFDM}}[\iota]$ the value of the sum-rate at some iteration $\iota \ge 0$. To compute $R_{\text{sum}}^{\text{OFDM}}$ as given by (28) iteratively, we develop the following algorithm, to which we refer to as "Algorithm B" in reference to the optimization problem (B).

**Algorithm B** Iterative algorithm for computing $R_{\text{sum}}^{\text{OFDM}}$ as given by (28)

1: Initialization: set $\iota = 1$
2: Set $\{\boldsymbol{\beta}[k] = \boldsymbol{\beta}^{(\iota-1)}[k]\}_{k=1}^{K}$ in (47), and solve the obtained problem as we will describe in Section IV-B2 given below. Denote by $\{\mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ the found $\{\mathbf{a}[k]\}_{k=1}^{K}$
3: Set $\{\mathbf{a}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ in (47), and solve the obtained problem using "Algorithm B-1" given below. Denote by $\{\boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ the found $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$
4: Increment the iteration index as $\iota = \iota + 1$, and go back to Step 2
5: Terminate if $|R_{\text{sum}}^{\text{OFDM}}[\iota] - R_{\text{sum}}^{\text{OFDM}}[\iota - 1]| \le \epsilon$

As described in "Algorithm B", we compute the power values given by $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ and the indicator values given by $\{\mathbf{a}[k]\}_{k=1}^{K}$, alternately. More specifically, at iteration $\iota \ge 1$, the algorithm computes appropriate indicator values $\{\mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ that correspond to a maximum of (47) with the choice of the power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ set to their values obtained from the previous iteration, i.e., $\{\boldsymbol{\beta}[k] = \boldsymbol{\beta}^{(\iota-1)}[k]\}_{k=1}^{K}$ (for the initialization, set $\{\boldsymbol{\beta}^{(0)}[k]\}_{k=1}^{K}$ to an appropriate value). This sub-problem is an integer linear programming (ILP) problem [36] and we solve it by selecting the largest sum-rate ($R_l[k]$)

on each subcarrier $k$. Next, the power values $\{\boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ can be computed in order to maximize (47) with the choice of $\{\mathbf{a}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$. This sub-problem can be formulated as a Complementary geometric programming problem. We solve it through a geometric programming and successive convex optimization approach (see "Algorithm B-1" below). The iterative algorithm ("Algorithm B") terminates if the following condition holds: $|R_{\text{sum}}^{\text{OFDM}}[\iota] - R_{\text{sum}}^{\text{OFDM}}[\iota-1]|$ is smaller than a prescribed small strictly positive constant $\epsilon$ — in this case, the optimized value of the sum-rate is $R_{\text{sum}}^{\text{OFDM}}[\iota]$, and is attained using the power values $\{\tilde{\boldsymbol{\beta}}[k] = \boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ and indicator values $\{\tilde{\mathbf{a}}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$.

In the following two sections, we study the aforementioned two sub-problems of problem (B), and describe the algorithms that we propose to solve them.

*2) Indicator Allocation:* In this section, we focus on the problem of finding the indicator values $\{\mathbf{a}[k]\}_{k=1}^{K}$ for a given choice of power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$. Investigating the objective function in (47), it can be stated as

$$\max \quad \sum_{k=1}^{K}\sum_{l=1}^{7} a_l[k] R_l[k], \tag{50a}$$

$$\text{s. t.} \quad \|\mathbf{a}[k]\|^2 \le 1, \ \text{for } 1 \le k \le K \tag{50b}$$

$$a_l[k] \in \{0,1\}, \ l \in \{1,2,..,7\}, \ \text{for } 1 \le k \le K. \tag{50c}$$

It can be easily seen from (50a), that the optimum value of $\mathbf{a}[k]$, at a subcarrier $k$, can be obtained by investigating the sum-rate $R_l[k]$. The indicator $\mathbf{a}[k]$ is calculated in such a way that the largest sum-rate $R_l[k]$ is selected, and it is given by

$$a_l[k] = \begin{cases} 1, & l = \arg\max_{1\le l\le 7} \ R_l[k] \\ 0, & \text{otherwise.} \end{cases}$$

Hence, the largest sum-rate at subcarrrier $k$ is

$$\tilde{R}[k] = \max_{1\le l\le 7} \ R_l[k]. \tag{51}$$

*3) Power Allocation:* In this section, we focus on the problem of calculating $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ for a given choice of $\{\mathbf{a}[k]\}_{k=1}^{K}$. Investigating the objective function in (47), it can be stated as

$$\max \quad \sum_{k=1}^{K} \tilde{R}[k], \tag{52a}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \le 1, \tag{52b}$$

$$\beta_i^2[k] \ge 0, \ i \in \{a, b, ar, br\}, \ \text{for } 1 \le k \le K \tag{52c}$$

| | $(f_1(\boldsymbol{\beta}[k]))^{-1}$ | $(f_2(\boldsymbol{\beta}[k]))^{-1}$ | $(f_3(\boldsymbol{\beta}[k]))^{-1}$ | $(f_4(\boldsymbol{\beta}[k]))^{-1}$ |
|---|---|---|---|---|
| Case 1 | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}{N}$ | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\vert h_{\mathrm{bd}}[k]\vert^2 P_t}{N+\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}$ | $1+\frac{\beta_b^2[k]\vert h_{\mathrm{bd}}[k]\vert^2 P_t}{N+\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}$ |
| Case 2 | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}{N}$ | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\vert h_{\mathrm{br}}[k]\vert^2 P_t}{N+\beta_a^2[k]\vert h_{\mathrm{ar}}[k]\vert^2 P_t}$ | $1+\frac{\beta_b^2[k]\vert h_{\mathrm{bd}}[k]\vert^2 P_t}{N+\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}+\frac{\beta_{br}^2[k]\vert h_{\mathrm{rd}}[k]\vert^2 P_t}{N}$ |
| Case 3 | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ar}}[k]\vert^2 P_t}{N}$ | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}{N}+\frac{\beta_{ar}^2[k]\vert h_{\mathrm{rd}}[k]\vert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\vert h_{\mathrm{br}}[k]\vert^2 P_t}{N+\beta_a^2[k]\vert h_{\mathrm{ar}}[k]\vert^2 P_t}$ | $1+\mathrm{snr_b}[k]$ |
| Case 4 | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ar}}[k]\vert^2 P_t}{N+\beta_b^2[k]\vert h_{\mathrm{br}}[k]\vert^2 P_t}$ | $1+\frac{\beta_a^2[k]\vert h_{\mathrm{ad}}[k]\vert^2 P_t}{N}+\frac{\beta_{ar}^2[k]\vert h_{\mathrm{rd}}[k]\vert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\vert h_{\mathrm{br}}[k]\vert^2 P_t}{N}$ | $1+\mathrm{snr_b}[k]$ |

TABLE II

USEFUL FUNCTIONS FOR THE ANALYSIS OF THE CASES DESCRIBED IN SECTION III-B.



where $\tilde{R}[k]$ is given in (51). It can be easily seen that this problem can be equivalently stated as

$$\min \quad \prod_{k=1}^{K} \Delta_a[k]\Delta_b[k], \tag{53a}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \leq 1, \tag{53b}$$

$$\Delta_a[k] \geq f_1(\boldsymbol{\beta}[k]), \quad \Delta_a[k] \geq f_2(\boldsymbol{\beta}[k]) \qquad \text{if } f_1(\boldsymbol{\beta}[k]) \neq f_2(\boldsymbol{\beta}[k]) \tag{53c}$$

$$\Delta_a[k] = f_1(\boldsymbol{\beta}[k]) \qquad \text{if } f_1(\boldsymbol{\beta}[k]) = f_2(\boldsymbol{\beta}[k]) \tag{53d}$$

$$\Delta_b[k] \geq f_3(\boldsymbol{\beta}[k]), \quad \Delta_b[k] \geq f_4(\boldsymbol{\beta}[k]) \qquad \text{if } f_3(\boldsymbol{\beta}[k]) \neq f_4(\boldsymbol{\beta}[k]) \tag{53e}$$

$$\Delta_b[k] = f_3(\boldsymbol{\beta}[k]) \qquad \text{if } f_3(\boldsymbol{\beta}[k]) = f_4(\boldsymbol{\beta}[k]) \tag{53f}$$

$$\beta_i^2[k] \geq 0, \; i \in \{a, b, ar, br\}, \tag{53g}$$

$$\Delta_a[k] \geq 0, \; \Delta_b[k] \geq 0 \in \mathbb{R}, \text{ for } 1 \leq k \leq K \tag{53h}$$

where $\Delta_a[k]$ and $\Delta_b[k]$ are simultaneously extra optimization variables and the objective function in (53a). Also, it is easy to see that the power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ that achieve the minimum value of $\prod_{k=1}^{K} \Delta_a[k]\Delta_b[k]$ also achieve the maximum value of the objective function in (47). The functions $f_l(\boldsymbol{\beta}[k])$ are given in Table II for the cases described in section III-B. For brevity, we did not include the functions of the remaining three cases (case 5, case 6 and case 7) in Table II, as these can be easily obtained by swapping the indices a and b in the functions of case 2, case 3 and case 4, respectively.

*Remark 3:* we note that $\Delta_a[k]$ acts as an upper bound for the two functions $f_1(\boldsymbol{\beta}[k])$ and $f_2(\boldsymbol{\beta}[k])$, and whenever the two functions $f_1(\boldsymbol{\beta}[k])$ and $f_2(\boldsymbol{\beta}[k])$ are identical, for example Case 1 and 2 given in Table II, $\Delta_a[k]$ is not anymore an upper bound. Thus, the inequalities in (53c) should be considered as equality as given in (53d). Similarly, $\Delta_b[k]$ acts as an upper bound for the two functions $f_3(\boldsymbol{\beta}[k])$ and $f_4(\boldsymbol{\beta}[k])$, and whenever the two functions $f_3(\boldsymbol{\beta}[k])$ and $f_4(\boldsymbol{\beta}[k])$ are identical, for example Case 1 given in Table II, $\Delta_b[k]$ is not anymore an upper bound. Thus, the inequalities in (53e) should be considered as equality as given in (53f).

The optimization problem (53a) is non-linear and non-convex. Therefore, we consider geometric programming (GP) which is a special form of convex optimization for which efficient algorithms have been developed [33], [37]. There are two forms of GP: the standard form and the convex form. In its standard form, a GP optimization problem is generally written as [33]

$$\text{minimize} \quad f_0(\boldsymbol{\beta}[k], \Delta_a[k], \Delta_b[k]) \tag{54a}$$

$$\text{subject to} \quad f_j(\boldsymbol{\beta}[k], \Delta_a[k], \Delta_b[k]) \leq 1, \quad j = 1, \cdots, J, \tag{54b}$$

$$g_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_a[k], \Delta_b[k]) = 1, \quad l = 1, \cdots, L, \tag{54c}$$

where the functions $f_0$ and $f_j$, $j = 1, \cdots, J$, are posynomials and the functions $g_l$, $l = 1, \cdots, L$, are monomials in $\boldsymbol{\beta}[k]$, $\Delta_a[k]$ and $\Delta_b[k]$. In its standard form, (54) is not a convex optimization problem. However, a careful application of an appropriate logarithmic transformation of the involved variables and constants generally turns the problem (54) into one that is equivalent and convex. That is, (54) is a GP nonlinear, nonconvex optimization problem that can be transformed into a nonlinear, convex optimization problem.

We can rewrite the optimization problem (53a) in a way such that we have ratios of posynomial functions, given by

$$\min \quad \prod_{k=1}^{K} \Delta_a[k]\Delta_b[k], \tag{55a}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \leq 1, \tag{55b}$$

$$\frac{p_1(\boldsymbol{\beta}[k], \Delta_a[k])}{g_1(\boldsymbol{\beta}[k], \Delta_a[k])} \leq 1, \ \frac{p_2(\boldsymbol{\beta}[k], \Delta_a[k])}{g_2(\boldsymbol{\beta}[k], \Delta_a[k])} \leq 1 \qquad \text{if } f_1(\boldsymbol{\beta}[k]) \neq f_2(\boldsymbol{\beta}[k]) \tag{55c}$$

$$\frac{p_1(\boldsymbol{\beta}[k], \Delta_a[k])}{g_1(\boldsymbol{\beta}[k], \Delta_a[k])} = 1, \qquad \text{if } f_1(\boldsymbol{\beta}[k]) = f_2(\boldsymbol{\beta}[k]) \tag{55d}$$

$$\frac{p_3(\boldsymbol{\beta}[k], \Delta_b[k])}{g_3(\boldsymbol{\beta}[k], \Delta_b[k])} \leq 1, \ \frac{p_4(\boldsymbol{\beta}[k], \Delta_b[k])}{g_4(\boldsymbol{\beta}[k], \Delta_b[k])} \leq 1 \qquad \text{if } f_3(\boldsymbol{\beta}[k]) \neq f_4(\boldsymbol{\beta}[k]) \tag{55e}$$

$$\frac{p_3(\boldsymbol{\beta}[k], \Delta_b[k])}{g_3(\boldsymbol{\beta}[k], \Delta_b[k])} = 1, \qquad \text{if } f_3(\boldsymbol{\beta}[k]) = f_4(\boldsymbol{\beta}[k]) \tag{55f}$$

$$\beta_i^2[k] \geq 0, \ \ i \in \{a, b, ar, br\}, \tag{55g}$$

$$\Delta_a[k] \geq 0, \ \Delta_b[k] \geq 0 \in \mathbb{R}, \text{ for } 1 \leq k \leq K \tag{55h}$$

In our case, the constraints in (55a) contain functions that are non posynomial as a ratio of two posynomials is not a posynomial. Minimizing or upper bounding a ratio between two posynomials belongs to a non-convex class of problems known as Complementary GP [33]. One can transform a Complementary GP problem into a GP problem using series of approximations. In order to get posynomial functions, we approximate the denominator of the functions $g_l(\boldsymbol{\beta}[k], \Delta_i[k])$, $l \in \{1, 2, 3, 4\}$, $i \in \{a, b\}$, with monomials, by using *lemma 1* [37].

*Remark 4:* we should note that whenever the two functions $f_1(\boldsymbol{\beta}[k])$ and $f_2(\boldsymbol{\beta}[k])$ are identical, the constraints (55d) should contain functions that are monomial— recall that a ratio between posynomial and monomial is in general non monomial. In order to get monomial functions, we approximate both the numerator $p_1(\boldsymbol{\beta}[k], \Delta_a[k])$ and denominator $g_1(\boldsymbol{\beta}[k], \Delta_a[k])$ with monomial functions, by using *lemma 1*. Similarly, whenever the two functions $f_3(\boldsymbol{\beta}[k])$ and $f_4(\boldsymbol{\beta}[k])$ are identical, the constraints (55f) should contain functions that are monomial. In order to get monomial functions, we approximate both the numerator $p_3(\boldsymbol{\beta}[k], \Delta_b[k])$ and denominator $g_3(\boldsymbol{\beta}[k], \Delta_b[k])$ with monomial functions, by using *lemma 1*.

*Lemma 1:* Let $g_l(\boldsymbol{\beta}[k], \Delta_i[k]) = \sum_j u_j(\boldsymbol{\beta}[k], \Delta_i[k])$ be a posynomial. Then

$$g_l(\boldsymbol{\beta}[k], \Delta_i[k]) \geq \tilde{g}_l(\boldsymbol{\beta}[k], \Delta_i[k]) = \prod_j \left( \frac{u_j(\boldsymbol{\beta}[k], \Delta_i[k])}{\gamma_j} \right)^{\gamma_j}. \tag{56}$$

Here, $\gamma_j = u_j(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) / g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$, $\forall j$, for any fixed positive $\boldsymbol{\beta}^{(0)}[k]$ and $\Delta_i^{(0)}[k]$ then $\tilde{g}_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) = g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$, and $\tilde{g}(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$ is the best local monomial approximation to $g(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$ near $\boldsymbol{\beta}^{(0)}[k]$ and $\Delta_i^{(0)}[k]$. □

Let $\tilde{g}_l(\boldsymbol{\beta}[k], \Delta_i[k])$ be the monomial approximation of the function $g_l(\boldsymbol{\beta}[k], \Delta_i[k])$ obtained using Lemma 1. Using this monomial approximation, the ratios of posynomials involved in the constraint (55a) can be upper bounded by posynomials. The optimal solution of the problem obtained using the convex approximations is also optimal for the original problem, i.e., satisfies the KKT conditions of the original problem (55a), if the applied approximations satisfy the following three properties [38], [37]:

1) $g_l(\boldsymbol{\beta}[k], \Delta_i[k]) \leq \tilde{g}_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_i[k])$ for all $\boldsymbol{\beta}[\mathrm{k}]$ and $\Delta_i[k]$ where $\tilde{g}_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_i[k])$ is the approximation of $g_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_i[k])$.
2) $g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) = \tilde{g}_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$ where $\boldsymbol{\beta}^{(0)}[k]$ and $\Delta_i^{(0)}[k]$ are the optimal solution of the approximated problem in the previous iteration.
3) $\triangledown g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) = \triangledown \tilde{g}_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$, where $\triangledown g_l(\cdot)$ stands for the gradient of function $g_l(\cdot)$.

In summary, applying the aforementioned transformations, we transformed the original optimization problem (53a) first into a Complementary GP problem and then into a GP problem by applying the convex approximations (56). Finally, the obtained GP problem can be solved easily for instance using an interior point approach. More specifically, the problem of finding the appropriate $\{\boldsymbol{\beta}[k]\}_{k=1}^K$ can be solved using "Algorithm B-1" hereinafter.

## V. Numerical Examples

Throughout this section, we set the number of subcarriers to $K = 128$ and we model the channel coefficients as independent and randomly generated variables. The channel impulse response (CIR) is modeled as a delay line with length $L = 32$ taps. The taps are generated from i.i.d circular complex Gaussian distributions with zero mean and the variance is chosen according to the strength of the corresponding link. More specifically, the link from source **A** to the relay has a variance $\sigma_{\text{ar}}^2$; that from source **B** to the relay has a variance $\sigma_{\text{br}}^2$; and that from the relay to the destination has a variance $\sigma_{\text{rd}}^2$. Similar assumptions and notations are used for the direct links from the sources to the destination. The channel state information (CSI) $\{h_{\text{ar}}, h_{\text{br}}\}$, $\{h_{\text{ad}}, h_{\text{bd}}\}$, and $\{h_{\text{rd}}\}$ are computed by taking $K$-points Fast Fourier Transform of the CIR. Furthermore, we assume that, at every time instant, all the nodes know, or can estimate with high accuracy, the values taken by the channel coefficients at that time, i.e., full CSI. Also, we set $P_t = 30$ dBW.

In order to illustrate the theoretical analysis and the effectiveness of the OFDM transmission scheme of Proposition 2 given in (28), we compare it with the OFDMA transmission scheme of Proposition 1 given in (7).

It can be easily seen, from the theoretical analysis of OFDM and OFDMA, that the complexity of finding the optimum power values for the OFDM is larger than that for the OFDMA. In Figure 2 we observe that the maximum sum-rate for the OFDM transmission scheme can be obtained with only 180 iterations.

**Algorithm B-1** Power allocation policy for $R_{\text{sum}}^{\text{OFDM}}$ as given by (28)

1: Set $\{\boldsymbol{\beta}^{(0)}[k]\}_{k=1}^{K}$ to an initial value. Compute $\{\Delta_a^{(0)}[k]\}_{k=1}^{K}$ and $\{\Delta_b^{(0)}[k]\}_{k=1}^{K}$ for the value of $\{\boldsymbol{\beta}^{(0)}[k]\}_{k=1}^{K}$ and set $\iota_1 = 1$ and $k = 1$
2: **While** $k \leq K$ **do**
3: **If** $f_1(\boldsymbol{\beta}[k]) \neq f_2(\boldsymbol{\beta}[k])$ **then**
4: Approximate $g_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{g}_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ and $g_2(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{g}_2(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_a^{(\iota_1-1)}[k]$ using (56)
5: **else**
6: Approximate $p_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{p}_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ and $g_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{g}_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_a^{(\iota_1-1)}[k]$ using (56)
7: **end if**
8: **If** $f_3(\boldsymbol{\beta}[k]) \neq f_4(\boldsymbol{\beta}[k])$ **then**
9: Approximate $g_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{g}_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ and $g_4(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{g}_4(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_b^{(\iota_1-1)}[k]$ using (56)
10: **else**
11: Approximate $p_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{p}_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ and $g_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{g}_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_b^{(\iota_1-1)}[k]$ using (56)
12: **end if**
13: Increment the subcarrier $k$ as $k = k + 1$
14: **end while**
15: Solve the resulting approximated GP problem using an interior point approach. Denote the found solutions as $\{\boldsymbol{\beta}^{(\iota_1)}[k]\}_{k=1}^{K}$, $\{\Delta_a^{(\iota_1)}[k]\}_{k=1}^{K}$, and $\{\Delta_b^{(\iota_1)}[k]\}_{k=1}^{K}$.
16: Increment the iteration index as $\iota_1 = \iota_1 + 1$ and go back to Step 2 using $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, $\{\Delta_a[k]\}_{k=1}^{K}$ and $\{\Delta_b[k]\}_{k=1}^{K}$ of step 15
17: Terminate if $\|\boldsymbol{\beta}^{(\iota_1)}[k] - \boldsymbol{\beta}^{(\iota_1-1)}[k]\| \leq \epsilon$, for $1 \leq k \leq K$

As discussed before, OFDMA-based scheme allows only one source to transmit on each subcarrier. On the contrary, OFDM-based scheme does not have such a restriction and thus it has higher spectral efficiency than OFDMA.

Figure 3 depicts the evolution of the sum-rate obtained using the OFDMA transmission scheme, i.e., the sum-rate $R_{\text{sum}}^{\text{OFDMA}}$ of Proposition 1; and the sum-rate obtained using the OFDM transmission scheme, i.e., the sum-rate $R_{\text{sum}}^{\text{OFDM}}$ of Proposition 2, as functions of the signal-to-noise ratio SNR= $10\log(P_t/N)$ (in decibels). Note that the curves correspond to numerical values of channel coefficients chosen such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 26$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 0$ dBW. The figure also shows the rate obtained using the OFDMA transmission scheme of source **A**, i.e., $R_{\text{A}}^{\text{OFDMA}}$, and source **B**, i.e., $R_{\text{B}}^{\text{OFDMA}}$ and the rate obtained using the OFDM transmission scheme of source **A**, i.e., $R_{\text{A}}^{\text{OFDM}}$, and source **B**, i.e., $R_{\text{B}}^{\text{OFDM}}$.

For the example shown in Figure 3, we observe that the transmission scheme of Proposition 2 gives larger sum-rate than the transmission scheme of Proposition 1 at SNR values higher than 5 dB, and gives almost the same sum-rate at low SNR, between 0 and 5 dB. Also, we observe that the rate of source **A** and source **B** using the OFDM transmission scheme give larger value than the rate of source **A** and source **B** using the OFDMA transmission scheme respectively.

Figure 4 depicts the same curves for other combinations of channel coefficients, chosen such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 20$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 0$ dBW. In this case, we observe that the curves show behaviors that are generally similar to those of Figure 3. Also, note that the gap between the sum-rate of Proposition 1 and the sum-rate of Proposition 2 decreases comparing with the results of Figure 3. This is precisely due to that the sources-to-relay links chosen for the example shown in Figure 4 are of weaker quality than the sources-to-relay links chosen for the example shown in Figure 3.

Figure 5 shows similar curves for other combinations of channel coefficients, chosen that such that $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = 20$ dBW, $\sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 26$ dBW. Note that the gap between the sum-rate of Proposition 1 and the sum-rate of Proposition 2 increases at low SNR, between 0 and 10 dB. This is precisely due to that the sources-to-destination links chosen for the example shown in Figure 5 are of better quality than the sources-to-destination links chosen for the example shown in Figure 4.

As discussed in Section IV-B, for OFDM transmission scheme it is not possible to a priori select on each subcarrier the case which outperforms the others. In order to observe which of the cases are being considered in the optimization for the OFDM transmission scheme, Figure 6 shows the probability of occurrence for the different cases given in Table I. Note that the histogram corresponds to 8000 carriers, SNR = 15 dB and numerical values of channel coefficients chosen such that $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = 26$ dBW, $\sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 0$ dBW.

For the example shown in Figure 6, we observe that whenever the sources-to-relay links and relay-destination links are of better quality than the sources-to-destination links, it is more probable that the relay helps both sources, i.e., cases 3, 4, 6, and 7.

Figure 7 shows the same histogram for other combinations of channel coefficients, chosen such that $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = 20$ dBW, $\sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 20$ dBW. Comparing with Figure 6, we observe that when the sources-to-relay, sources-to-destination, and relay-to-destination links have the same strength, the probability that the relay helps both sources (cases 3, 4, 6, and 7) decreases, the probability that the relay helps one source (cases 2 and 5) increases, and the probability that the relay is idle (case 1) increases.

Figure 8 shows the same histogram for other combinations of channel coefficients, chosen such that $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = 20$ dBW, $\sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 26$ dBW. Comparing with Figure 7, we observe that when the sources-to-destination links have better quality than the other links, the probability that the relay helps both sources (cases 3, 4, 6, and 7) decreases, the probability that the relay helps one source (cases 2 and 5) increases, and the probability that the relay is idle (case 1) increases.

Figure 9 shows the rate obtained using the OFDMA transmission scheme of source **A** and source **B** on each subcarrier $k$, and the sum-rate on each subcarrier $k$. Figure 10 shows the rate obtained using the OFDM transmission scheme of source **A** and source **B** on each subcarrier $k$, and the sum-rate on each subcarrier $k$. Note that, for Figures 9 and 10, the curves corresponds to SNR = 20 dB and numerical values of channel coefficients chosen such that $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = 20$ dBW, $\sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 7$ dBW. We observe that on average the OFDM scheme gives larger sum-rate on each subcarrier than the OFDMA scheme, and on some subcarriers, for example $k = 20$, both schemes give the same sum-rate.

## VI. Conclusion

We consider communication over a multicarrier two-source multiaccess channel in which the transmission is aided by a half-duplex relay node. We study and analyze the performance of two transmission schemes in which the relay implements decode-and-forward strategy. We propose two multicarrier transmission schemes, based respectively on OFDMA and OFDM. In the first scheme, each subcarrier can only be used by at most one source at a time. In the second scheme, each subcarrier can be used by both sources simultaneously. For both schemes, we derive the allowed sum-rate. Also, we study the problem of allocating the resources (i.e., powers and subcarriers), selecting the relay operation modes and decoding orders at the relay and destination (for OFDM transmission) optimally in a way to maximize the obtained sum-rate. For the OFDMA-based transmission, we develop a duality-based algorithm that finds a globally optimum solution. For the OFDM-based transmission, we propose an iterative coordinate-descent algorithm that finds a suboptimum solution. For both schemes, we illustrate our results through some numerical examples. In particular, our analysis shows that by allowing the sources to possibly transmit on the same subcarrier simultaneously, one can afford a larger sum-rate, i.e., the OFDM-based transmission scheme offers a substantial sum-rate gain over the one that is based on OFDMA.

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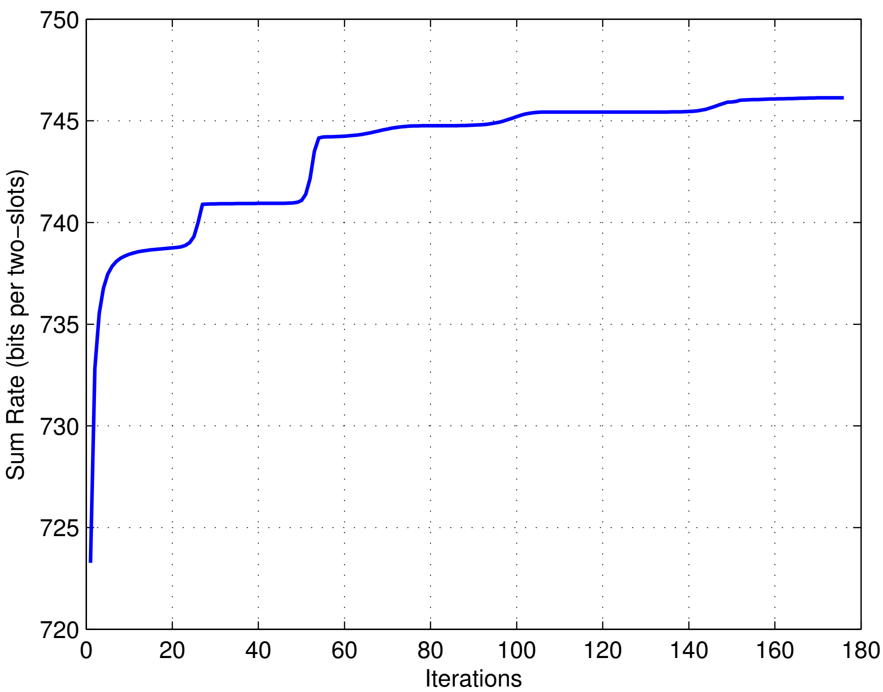


Fig. 2. Convergence of "Algorithm B". Numerical values are $K = 128$, SNR = 20 dB, $\sigma^2_{\rm ar} = \sigma^2_{\rm br} = \sigma^2_{\rm rd} = 20$ dBW, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd} = 14$ dBW.

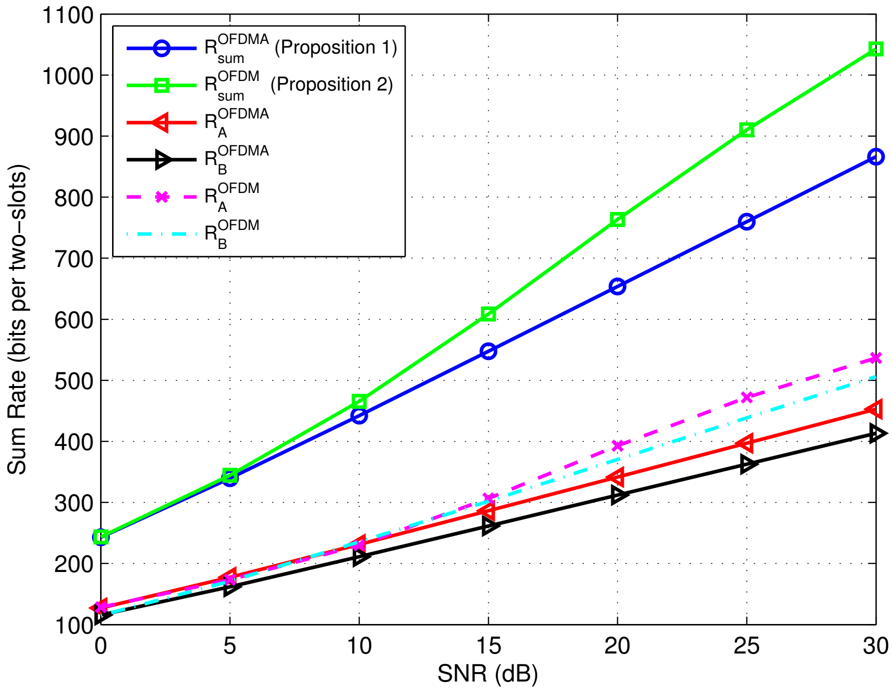


Fig. 3. Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 26$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 0$ dBW.

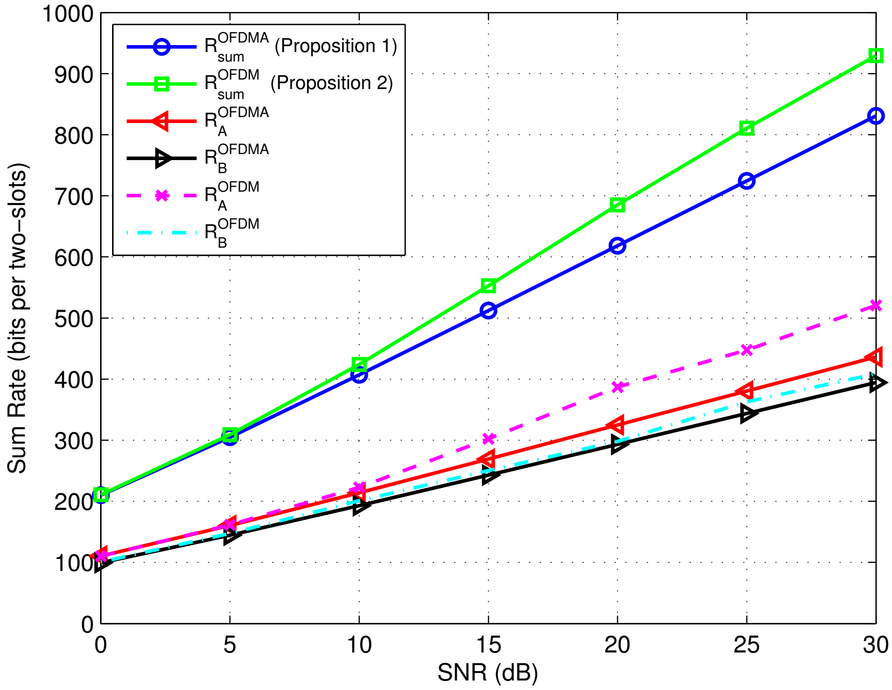


Fig. 4. Sum-rate comparison. Numerical values are $K = 128$, $\sigma_{\rm ar}^2 = \sigma_{\rm br}^2 = 20$ dBW, $\sigma_{\rm rd}^2 = 20$ dBW, and $\sigma_{\rm ad}^2 = \sigma_{\rm bd}^2 = 0$ dBW.

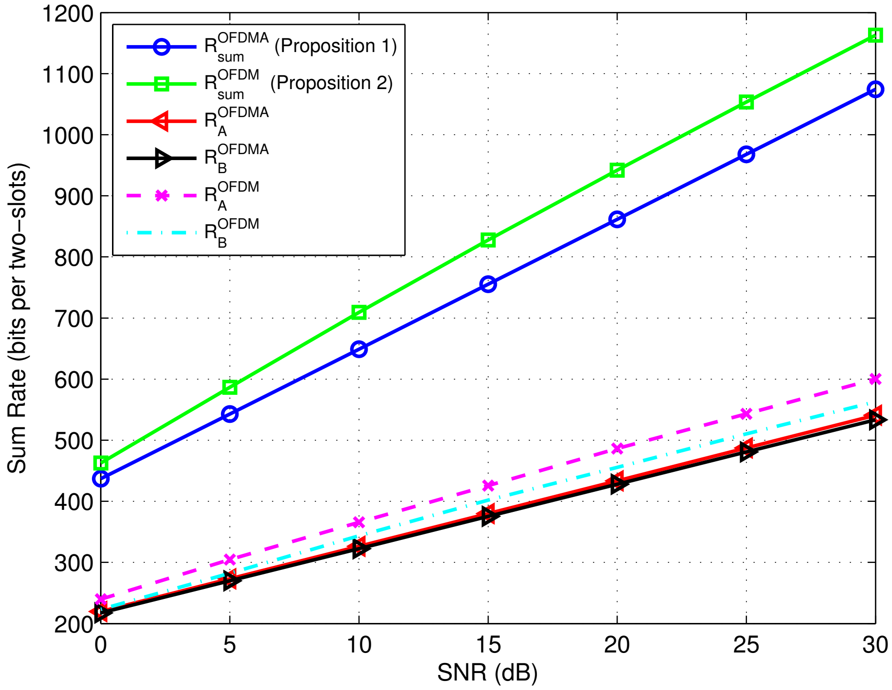


Fig. 5. Sum-rate comparison. Numerical values are $K = 128$, $\sigma^2_{\mathrm{ar}} = \sigma^2_{\mathrm{br}} = 20$ dBW, $\sigma^2_{\mathrm{rd}} = 20$ dBW, and $\sigma^2_{\mathrm{ad}} = \sigma^2_{\mathrm{bd}} = 26$ dBW.

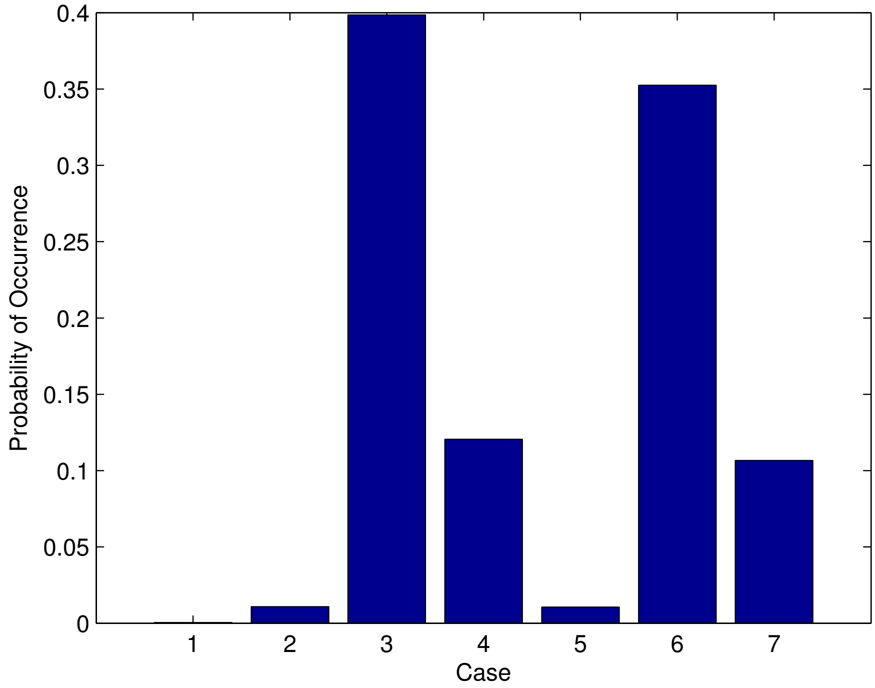


Fig. 6. Selection of relay operation modes and decoding orders at the relay and destination for the cases given in Table I. Numerical values are $8000$ carriers, $SNR$ = 15 dB, $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}}$ = 26 dBW, $\sigma^2_{\text{rd}}$ = 20 dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}}$ = 0 dBW.

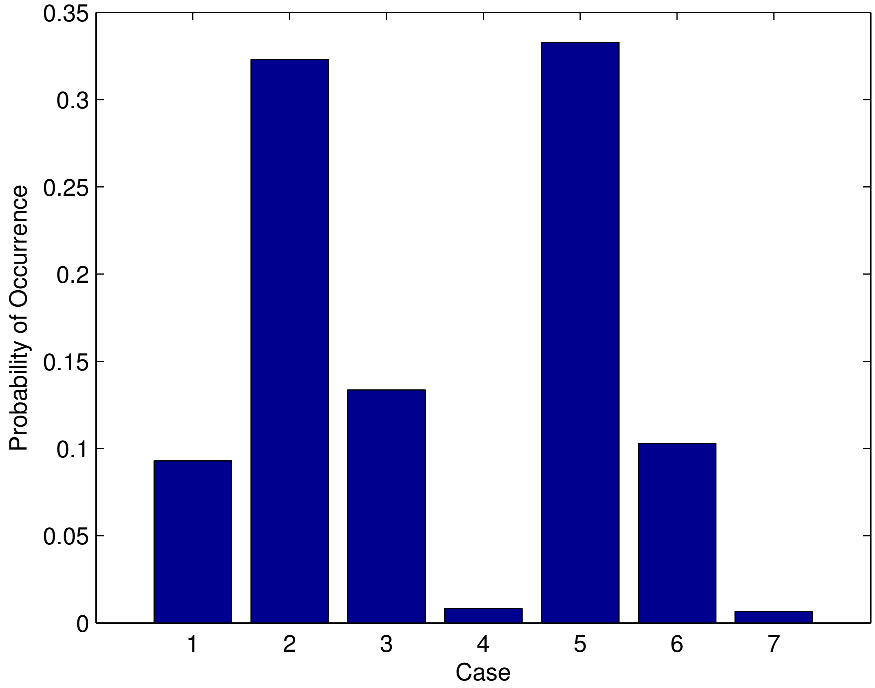


Fig. 7. Selection of relay operation modes and decoding orders at the relay and destination for the cases given in Table I. Numerical values are 8000 carriers, $SNR$ = 15 dB, $\sigma^2_{\mathrm{ar}} = \sigma^2_{\mathrm{br}}$ = 20 dBW, $\sigma^2_{\mathrm{rd}}$ = 20 dBW, and $\sigma^2_{\mathrm{ad}} = \sigma^2_{\mathrm{bd}}$ = 20 dBW.

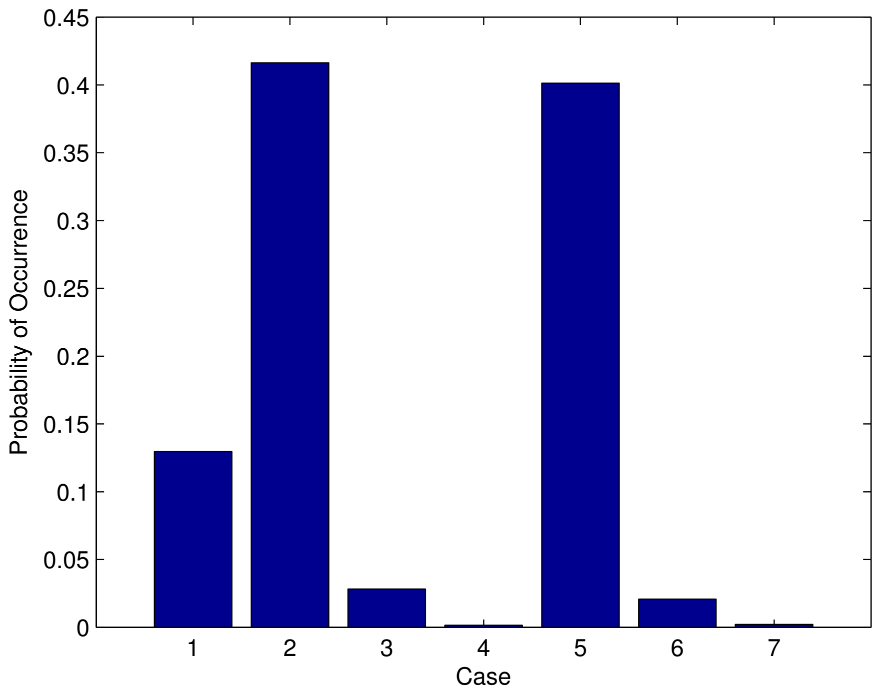


Fig. 8. Selection of relay operation modes and decoding orders at the relay and destination for the cases given in Table I. Numerical values are 8000 carriers, $SNR$ = 15 dB, $\sigma^2_{\mathrm{ar}} = \sigma^2_{\mathrm{br}}$ = 20 dBW, $\sigma^2_{\mathrm{rd}}$ = 20 dBW, and $\sigma^2_{\mathrm{ad}} = \sigma^2_{\mathrm{bd}}$ = 26 dBW.

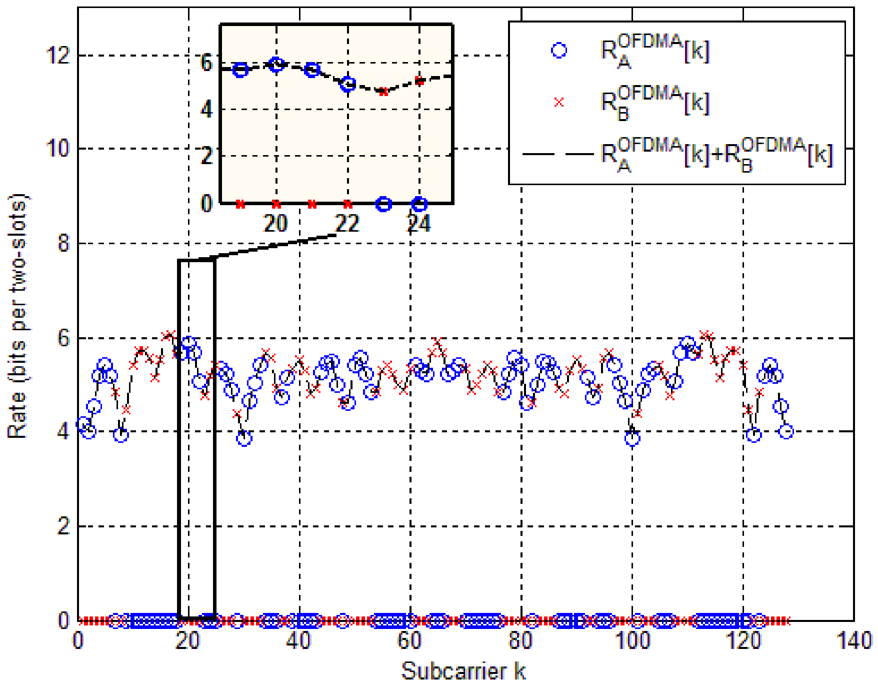


Fig. 9. Rate values per subcarrier for the OFDMA multicarrier transmission. Numerical values are $K = 128$, $SNR = 20$ dB, $\sigma_{\rm ar}^2 = \sigma_{\rm br}^2 = 20$ dBW, $\sigma_{\rm rd}^2 = 20$ dBW, and $\sigma_{\rm ad}^2 = \sigma_{\rm bd}^2 = 7$ dBW.

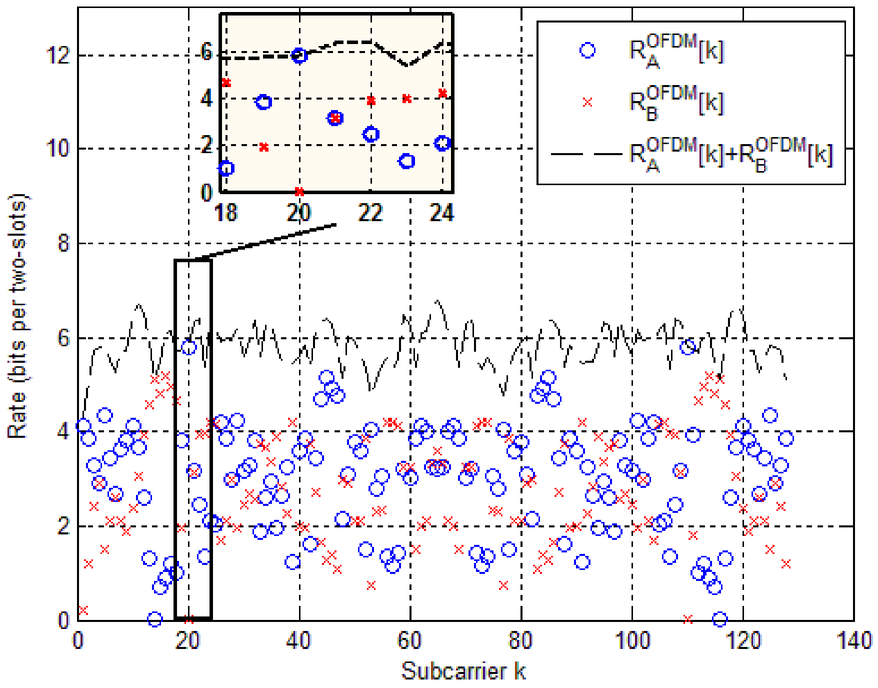


Fig. 10. Rate values per subcarrier for the OFDM multicarrier transmission. Numerical values are $K = 128$, $SNR = 20$ dB, $\sigma_{\mathrm{ar}}^2 = \sigma_{\mathrm{br}}^2 = 20$ dBW, $\sigma_{\mathrm{rd}}^2 = 20$ dBW, and $\sigma_{\mathrm{ad}}^2 = \sigma_{\mathrm{bd}}^2 = 7$ dBW.

# DF-based Sum-rate Optimization for Multicarrier Multiple Access Relay Channel

Mohieddine El Soussi Abdellatif Zaidi Luc Vandendorpe

***Abstract*—We consider a system that consists of two sources, a half-duplex relay, and a destination. The sources want to transmit their messages reliably to the destination with the help of the relay. We study and analyze the performance of two transmission schemes in which the relay implements decode-and-forward strategy. In the first scheme, we incorporate Orthogonal Frequency Division Multiple Access (OFDMA) transmission into the system. In this scheme, there is only one source node transmitting on each subcarrier. The transmission can be either with or without the help of the relay. In the second scheme, we implement Orthogonal Frequency Division Multiplexing (OFDM) transmission. In this scheme, both sources can transmit their messages using all subcarriers. The relay can help none, only one or both sources. For both schemes, we discuss the design criteria and evaluate the achievable sum-rate. Next, for each scheme, we study and solve the problem of resources (powers and subcarriers) allocation aiming at maximizing the allowed sum-rate. For the first scheme, we develop a duality-based algorithm that finds a globally optimum solution. For the second scheme, we propose an iterative coordinate-descent algorithm that finds a suboptimum solution. We show through numerical examples the effectiveness of these algorithms and illustrate the benefits of OFDM transmission over OFDMA for the model that we study.**

***Index Terms*—OFDMA, OFDM, Decode-and-forward, relay channel, decoding order, optimization.**

## I. Introduction

RELAYING has been introduced to extend system coverage, enhance spectrum efficiency and improve the performance of wireless systems. Cooperative relay networks have been studied extensively for many wireless systems [1], [2], [3]. In a typical relay system, the relay helps the transmitters by forwarding the transmitted messages to the destination. Different efficient relaying protocols have been proposed in the literature, including amplifying-and-forwarding (AF), decoding-and-forwarding (DF), and compressing-and-forwarding (CF) [2], [4]. Each protocol has its advantages and its disadvantages; and which scheme outperforms the others depends on the network topology and channel conditions. Capacity bounds and rate regions have been established in [5] for the standard three-terminal gaussian relay channel and in [4], [6] for the gaussian multiple access relay channel (MARC). The reader may also refer to [7], [8], [9], [10] for some related works.

In the context of cooperative communication, multicarrier transmission techniques, such as the popular Orthogonal Frequency Division Multiplexing (OFDM) and its multi-user version Orthogonal Frequency-Division Multiple Access (OFDMA), constitute promising tools that can offer high data rate. In particular, this is due to the fact that these techniques permit to handle frequency selectivity and harness multi-user diversity. Essentially for these reasons, these techniques have been adopted in most next-generation wireless standards, and are generally considered in the context of relay-aided communications in frequency selective channels.

In this paper, we consider communication over a multicarrier two-source multiaccess channel in which the transmission is aided by a relay node, i.e., a multicarrier two-source multiaccess relay channel (MARC). The communication takes place in two transmission periods or time slots. The sources transmit only during the first transmission period. The relay is half-duplex, implements decode-and-forward protocol and transmits only during the second transmission period. We propose two multicarrier transmission schemes, based respectively on OFDMA and OFDM. In the first scheme, each subcarrier can only be used by at most one source at a time. In the second scheme, each subcarrier can be used by both sources simultaneously. For both schemes, we derive the allowed sum-rate. Also, we study the problem of allocating the resources (i.e., powers and subcarriers) and selecting the relay operation mode (i.e., active or idle) optimally in a way to maximize the obtained sum-rate. Some of the key issues that we consider are related to the way the subcarriers are assigned among the two sources, the selection of appropriate relay operation mode for every subcarrier, and the allocation of power among the two sources and the relay. Because of the presence of the relay node, such a resources allocation problem is more involved comparatively than those for conventional OFDM systems that do not involve relays.

### A. Connection with Related Works

For a point-to-point OFDM transmission aided by a DF relay node, some resource allocation algorithms have been proposed and studied in the literature. For example, in [11] the authors investigate the problem of maximizing the sum-rate for an OFDM transmission protocol that uses a half-duplex DF relay node. Depending on the fading coefficients, on each subcarrier the relay node can be either idle or active. If the relay is idle, the source transmits a new independent symbol in the second time slot. This transmission protocol is extended for the scenarios in which the transmission involve multiple relays, and the related resource allocation problems are solved in [12], [13], [14]. The problem of resources allocation for OFDM transmission over a two-way channel that is aided by a DF relay has been investigated as well, and addressed in [15], [16].

For OFDMA systems without relaying, some resources allocation problems have been studied in [17], [18], [19]. For OFDMA systems that involve relays, some related contributions have been proposed in the literature. These include [20] and [21], in which the authors consider respectively the maximization of the allowed sum-rate and the maximization of a weighted sum goodput. Also, in [22]

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the authors maximize a metric depending on the rates and queue lengths of the source and relays. In [23], the authors jointly optimize the relay strategies and physical-layer resources in a multiple users network, where each user can act as a relay. In [24], the authors consider an optimal resources allocation strategy for cooperative relaying-enabled OFDMA multi-hop wireless systems. In [25] and [26], the authors study capacity regions of OFDMA multiple access networks that comprise AF and DF relays. They also investigate a problem of subcarriers assignment for given powers at the sources and the relay. In [27], a throughput maximization problem with fairness constraint is solved for a cooperative OFDMA network. The authors propose an efficient algorithm with low computational complexity that assigns appropriately subcarriers and powers. The reader may also refer to [28], [29], [30] for some related works.

For multiaccess relay networks, in [31] the authors investigate a problem of power allocation among two sources and a relay. Also, in [32] the authors study the problem of resources allocation for a multi-user DF-based relay network with orthogonal channel access that uses OFDMA. In this work, the setting that we consider is somehow connected to [31] and [32], with the following differences. First, in comparison to [31], in our case we consider frequency selective channels by means of multi-carrier; and we address the problem of maximizing the offered sum-rate under a total sum power constraint. Also, in our setting the relay uses the same codebook as that used by the sources and thus it transmits the same codewords that are sent by the sources. This explains the use of maximum ratio combining (MRC) at the destination in our work. Furthermore, we study the optimization problem by considering different optimization parameters. Second, in comparison with [32], we mention that the setup in [32] does not consider the case in which the sources are allowed to transmit their messages using the same subcarrier. Moreover, comparing the transmission scheme of [32] with the OFDMA transmission scheme that we consider in this paper, we note that in [32] the case in which the destination gets information from *only* the direct links (i.e, the relay is idle) is not considered explicitly therein, and the problem of allocating the powers in a way to maximize the obtained sum-rate is considered under individual power constraint.

### B. Contributions

The main contributions of this paper can be summarized as follows. For the multicarrier multiaccess relay network that we consider, we propose two transmission schemes that use respectively OFDMA and OFDM. For each of these transmission schemes, we first derive the allowed sum-rate; and then we study and solve the problem of maximizing the offered sum-rate under a total sum power constraint. The optimization problems involve subcarriers assignment as well as power allocation among the sources and the relay.

In the OFDMA-based scheme, each subcarrier is used by only one source at a time; and so each source transmits its codeword or symbol free of interference, to the relay and destination. The relay can be either idle or active; and the selection of the appropriate operation mode depends on the channel coefficients. In the case in which the relay remains idle, the destination recovers the transmitted codeword using the signal from the source. In the case in which the relay is active, it uses the *same* subcarrier employed by the source to forward the decoded codeword to the destination. The destination then performs maximum-ratio combining of the outputs from the source and relay to recover the transmitted codeword.

In the OFDM-based scheme, both sources utilize all subcarriers to transmit their codewords to the relay and destination. That is, each subcarrier can be shared by both sources simultaneously. The relay can help none, only one, or both sources. In all cases, whenever it is active, the relay transmits on the *same* subcarrier as that utilized by the source or sources. Also, if, for a given subcarrier, the relay helps both sources simultaneously, it re-encodes the decoded sources' codewords via superposition coding. The decoding procedures at the relay and destination are based on successive decoding and maximum-ratio combining. At this level, we should mention that, by opposition to a standard multiple access channel in which the allowed sum-rate does not depend on which decoding order is considered, in presence of relay nodes, i.e., for multiple access relay networks, different decoding orders at the relay and destination generally yield different allowed sum-rates. Taking this aspect into consideration, we consider all possible decoding orders combinations, and select the appropriate combination that offers the largest sum-rate. In addition to the decoding orders, the relay operation modes (i.e, helping none, only one, or both sources simultaneously) obviously also influences the sum-rate that is allowed per subcarrier, and, so, thereby the total offered sum-rate.

For each of the multicarrier transmission schemes that we consider, we study and solve the problem of maximizing the offered sum-rate under a total sum power constraint. The total sum power constraint comprises the powers used by all transmitting terminals, on all subcarriers. For the OFDMA-based transmission scheme, the optimization problem consists of i) partitioning the available subcarriers among the two sources, ii) selecting the appropriate relay operation mode (i.e., transmitting or not-transmitting) for every subcarrier, and iii) allocating the powers on each subcarrier and among the transmitting terminals. We show that the resulting optimization problem is convex, and we provide an efficient algorithm that finds a global solution optimally. For the OFDM-based transmission scheme, the optimization problem comprises i) selecting the appropriate relay operation mode (i.e., helping none, only one, or both sources simultaneously) for every subcarrier, ii) choosing the best decoding orders at the relay (if active) and destination for every subcarrier, and iii) allocating the powers on each subcarrier and among the transmitting terminals. We show that the resulting optimization problem can be seen as of mixed-integer linear programming type. Also, we propose an iterative algorithm that is based on a coordinate descent approach and that, for every subcarrier, finds the best relay operation mode and decoding orders at the relay (if active) and destination, and appropriate powers for the terminals transmitting on that subcarrier, alternately. The iterations stop when convergence to a stationary point is obtained. For given relay operation mode and decoding orders combination, the problem of allocating the powers appropriately is non-convex. In order to solve this problem, we propose a geometric programming approach. Also, we utilize a successive convex approximation method that is similar to in [33].

For both schemes, we illustrate our results through some numerical examples. In particular, our analysis shows that by allowing the sources to possibly transmit on the same subcarrier simultaneously, one can afford a larger sum-rate, i.e., the OFDM-based transmission scheme offers a substantial sum-rate gain over the one that is based on OFDMA.

### C. Outline and Notation

An outline of the remainder of this paper is as follows. Section II describes in more details the system model that we consider in this work. In Section III, we analyze the sum-rates that are achievable using these schemes. Section IV contains the optimization problems formulations for both schemes as well as the algorithms that we propose to solve these problems. Section V contains some numerical examples, and Section VI concludes the paper.

The following notations are used throughout the paper. Lowercase boldface letters are used to denote vectors, e.g., $\mathbf{x}$. Calligraphic letters designate alphabets, i.e., $\mathcal{X}$. The cardinality of a set $\mathcal{X}$ is denoted by $|\mathcal{X}|$. For vectors, we write $\mathbf{x} \in \mathbb{A}^n$, e.g., $\mathbb{A} = \mathbb{R}$ or $\mathbb{A} = \mathbb{C}$, to mean that $\mathbf{x}$ is a column vector of size $n$, with its elements taken from the set $\mathbb{A}$. For a vector $\mathbf{x} \in \mathbb{R}^n$, $\|\mathbf{x}\|$ designates the norm of $\mathbf{x}$ in terms of Euclidean distance. The Gaussian distribution with mean $\mu$ and variance $\sigma^2$ is denoted by $\mathcal{N}(\mu, \sigma^2)$. We use $[x]^+$ to denote $\max\{0, x\}$. For a given $a \in \mathbb{R}$ and $b \in \mathbb{R}$, $\mathbf{1}_{a>b} = 1$ if $a > b$ and $\mathbf{1}_{a>b} = 0$ if $a \leq b$. Finally, for a complex-valued number $z = x + jy \in \mathbb{C}$, the notations $\text{Re}\{z\}$ and $\text{Im}\{z\}$ refer repectively to the real part and imaginary part of $z \in \mathbb{C}$, i.e., $\text{Re}\{z\} = x$ and $\text{Im}\{z\} = y$ and the notation $z^*$ refer to the complex conjugate of $z$, i.e., $z^* = x - jy$.

## II. System Model and Multicarrier Transmission Schemes

### A. System Model

We consider a multiaccess relay network that comprises two sources ($\mathbf{A}$ and $\mathbf{B}$), a relay node ($\mathbf{R}$) and a destination ($\mathbf{D}$), as shown in Figure 1. The sources $\mathbf{A}$ and $\mathbf{B}$ want to transmit two messages, $W_a \in \mathcal{W}_a$ and $W_b \in \mathcal{W}_b$, to the destination with the help of the relay. The relay is half-duplex and implements DF strategy. The communication takes place in $n$ channel uses, and is divided into two periods or time slots with equal durations. Furthermore, the transmission is performed using multiple carriers. In what follows, we will consider both OFDMA and OFDM multicarrier transmissions. As usually assumed in similar settings, we assume that appropriate cycle prefixing is employed, turning the channel into a number of, say $K$, parallel subchannels.

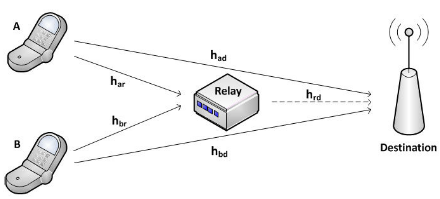


Fig. 1. Multiple-access relay channel with a half-duplex relay

Also, we assume that the states of the channel are known perfectly to all terminals, i.e., perfect channel state information at receivers (CSIR) and perfect channel state information at the transmitters (CSIT); and that they remain constant over a transmission period. That is, during each transmission period, each receiver has perfect knowledge of all channel coefficients on all subcarriers on which it receives, and each transmitter has perfect knowledge of all channel coefficients on all subcarriers on which it transmits. Furthermore, the noise signals at the relay and destination are independent from each others, and independently and identically distributed (i.i.d) circular complex Gaussian, with zero mean and variance $N$. Also, we consider the following constraint on the transmitted power,

$$\Big(\sum_{k=1}^{K} \mathbb{E}[\|\mathbf{x}_a[k]\|^2]\Big) + \Big(\sum_{k=1}^{K} \mathbb{E}[\|\mathbf{x}_b[k]\|^2]\Big) + \Big(\sum_{k=1}^{K} \mathbb{E}[\|\tilde{\mathbf{x}}_r[k]\|^2]\Big) \leq nP_t, \tag{1}$$

where $P_t \geq 0$ is the total per-channel use power imposed on the system, the first sum is the total power used by Source $\mathbf{A}$ during the whole transmission, the second sum is the total power used by Source $\mathbf{B}$ during the whole transmission, and the third sum is the total power used by Relay $\mathbf{R}$ during the whole transmission. Also, the inputs $\mathbf{x}_a[k]$, $\mathbf{x}_b[k]$ and $\tilde{\mathbf{x}}_r[k]$ denote respectively the codeword or symbol sent by Source $\mathbf{A}$ on subcarrier $k$ during the first transmission period, the codeword sent Source $\mathbf{B}$ on subcarrier $k$ during the first transmission period, and the codeword sent by Relay $\mathbf{R}$ on subcarrier $k$ during the second transmission period.
For convenience, let $\beta_a[k] \geq 0$ and $\beta_b[k] \geq 0$ be nonnegative scalars such that $\beta_a^2[k]P_t$ and $\beta_b^2[k]P_t$ be the per-channel use powers used at Source $\mathbf{A}$ and Source $\mathbf{B}$ on subcarrier $k$, respectively. Similarly, let $\beta_r[k] \geq 0$ be a nonnegative scalar such that $\beta_r^2[k]P_t$ be the per-channel use power used by Relay $\mathbf{R}$ on subcarrier $k$. Also, let $\beta_{ar}^2[k]P_t$ be the fraction of the power that the relay uses to help Source $\mathbf{A}$, and $\beta_{br}^2[k]P_t$ be the fraction of the power that the relay uses to help Source $\mathbf{B}$, with $\beta_{ar}^2[k] + \beta_{br}^2[k] = \beta_r^2[k]$. The aforementioned constraint on the available sum power can be rewritten equivalently as

$$\sum_{k=1}^{K} \Big(\beta_a^2[k] + \beta_b^2[k] + \beta_{ar}^2[k] + \beta_{br}^2[k]\Big) \leq 1. \tag{2}$$

Moreover, for convenience we will sometimes use the shorthand vector notation $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T \in \mathbb{R}^4$. Finally, the signal-to-noise ratio will be denoted as $\text{snr} = P_t/N$ in the linear scale, and by $\text{SNR} = 10\log_{10}(\text{snr})$ in decibels.

### B. Multicarrier Transmission Schemes

There are in total $K$ subcarriers that can be used by the sources for the transmission. In what follows, we describe the input-output relations obtained using an OFDMA-based transmission and an OFDM-based transmission. In the OFDMA-based transmission, a subcarrier can be used by only one source at a time; and in the OFDM-based transmission, both sources can transmit simultaneously on every subcarrier.

*1) OFDMA Transmission:* The encoding and transmission scheme on subcarrier $k$, $1 \leq k \leq K$, is as follows. As we mentioned previously, only one of the two sources sends on this subcarrier. Let $\mathbf{x}_i[k]$, $i = a$ or $i = b$, be the input of the source transmitting on this subcarrier, during the first transmission period. During this period, the outputs at the relay and destination on subcarrier $k$ are given by

$$\begin{aligned} \mathbf{y}_r[k] &= h_{\text{ir}}[k]\mathbf{x}_i[k] + \mathbf{z}_r[k] \\ \mathbf{y}_d[k] &= h_{\text{id}}[k]\mathbf{x}_i[k] + \mathbf{z}_d[k] \end{aligned} \tag{3}$$

where $h_{\text{ar}}[k]$ and $h_{\text{br}}[k]$ are the channel gains on the links to the relay; $h_{\text{ad}}[k]$ and $h_{\text{bd}}[k]$ are the channel gains on the links to the destination; the vector $\mathbf{z}_r[k]$ is the additive noise at the relay, and the vector $\mathbf{z}_d[k]$ is the additive noise at the destination. These noise vectors are mutually independent, and are with components drawn i.i.d. according to the circular complex Gaussian distribution with zero mean and variance $N$.
Assuming that it decodes correctly the transmitted codeword, during the second transmission period the relay re-encodes this codeword using the same codebook as that used by the source. Thus, the

destination receives

$$\tilde{\mathbf{y}}_d[k] = h_{\text{rd}}[k]\tilde{\mathbf{x}}_r[k] + \tilde{\mathbf{z}}_d[k] \tag{4}$$

during the second transmission period, where $h_{\text{rd}}[k]$ is the channel gain on the link to the destination; and the vector $\tilde{\mathbf{z}}_d[k]$ is the additive noise at the destination during this period, assumed to be independent from all other noise vectors, and with components drawn i.i.d. according to a circular complex Gaussian distribution with zero mean and variance $N$.

*2) OFDM Transmission:* The encoding and transmission scheme on subcarrier $k$, $1 \leq k \leq K$, is as follows. As we mentioned previously, both sources transmit simultaneously on the same subcarrier $k$ in this case. During the first transmission period, Source **A** transmits the codeword $\mathbf{x}_a[k]$ over the channel. Similarly, Source **B** transmits the codeword $\mathbf{x}_b[k]$ over the channel. During this period, the outputs at the relay and destination on subcarrier $k$ are given by

$$\begin{aligned} \mathbf{y}_r[k] &= h_{\text{ar}}[k]\mathbf{x}_a[k] + h_{\text{br}}[k]\mathbf{x}_b[k] + \mathbf{z}_r[k] \\ \mathbf{y}_d[k] &= h_{\text{ad}}[k]\mathbf{x}_a[k] + h_{\text{bd}}[k]\mathbf{x}_b[k] + \mathbf{z}_d[k], \end{aligned} \tag{5}$$

where $h_{\text{ar}}[k]$ and $h_{\text{br}}[k]$ are the channel gains on the links to the relay; $h_{\text{ad}}[k]$ and $h_{\text{bd}}[k]$ are the channel gains on the links to the destination; the vector $\mathbf{z}_r[k]$ is the additive noise at the relay, and the vector $\mathbf{z}_d[k]$ is the additive noise at the destination. These noise vectors are mutually independent, and are with components drawn i.i.d. according to the circular complex Gaussian distribution with zero mean and variance $N$.

Assuming that it decodes correctly the codewords transmitted by the sources, during the second transmission period the relay re-encodes the codewords using the same codebook employed by the sources. Thus, during this period, the output at the destination on subcarrier $k$ is given by

$$\tilde{\mathbf{y}}_d[k] = h_{\text{rd}}[k]\tilde{\mathbf{x}}_r[k] + \tilde{\mathbf{z}}_d[k], \tag{6}$$

where $h_{\text{rd}}[k]$ is the channel gain on the link to the destination; and the vector $\tilde{\mathbf{z}}_d[k]$ is the additive noise at the destination during this period, assumed to be independent from all other noise vectors, and with components drawn i.i.d. according to a circular complex Gaussian distribution with zero mean and variance $N$.

## III. Sum-Rate Analysis

In this section, we analyze the OFDMA and OFDM multicarrier transmission schemes that we described in the previous section, from the allowed sum-rate viewpoint.

### *A. Sum-Rate Analysis for the OFDMA-based Transmission*

The following proposition provides an achievable sum-rate for the multiaccess relay model of Figure 1, using OFDMA multicarrier transmission.

***Proposition 1:*** For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$, the following sum-rate is achievable for the multiaccess relay channel of Figure 1

$$R_{\text{sum}}^{\text{OFDMA}} = \max \sum_{k=1}^{K} \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right) \tag{7}$$

where $h_{\text{eq}}[k]$ is such that

$$\begin{aligned} |h_{\text{eq}}[k]|^2 = \max\Bigg\{ & \frac{|h_{\text{ar}}[k]|^2|h_{\text{rd}}[k]|^2}{|h_{\text{ar}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{ad}}[k]|^2}\mathbf{1}_{|h_{\text{ar}}[k]|>|h_{\text{ad}}[k]|}, \\ & |h_{\text{ad}}[k]|^2, |h_{\text{bd}}[k]|^2, \\ & \frac{|h_{\text{br}}[k]|^2|h_{\text{rd}}[k]|^2}{|h_{\text{br}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{bd}}[k]|^2}\mathbf{1}_{|h_{\text{br}}[k]|>|h_{\text{bd}}[k]|}\Bigg\} \end{aligned} \tag{8}$$

and the maximization is over $\{\beta_s[k]\}_{k=1}^{K}$, satisfying

$$\sum_{k=1}^{K} \beta_s^2[k] \leq 1. \tag{9}$$

**Proof:** Recall the OFDMA-based transmission scheme of Section II-B. In what follows, we describe the decoding procedures at the relay and destination; and we analyze the allowed sum-rate.

Fix a power policy $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (3). The relay utilizes joint typicality decoding to decode the transmitted codeword. The rate (per channel use) at which the relay can perform this reliably on subcarrier $k$ can be shown easily to be

$$R_{\text{ir}}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\text{ir}}[k]|^2 P_t}{N}\right), \tag{10}$$

where $i = a$ if subcarrier $k$ is used for transmission by Source **A**, and $i = b$ if subcarrier $k$ is used for transmission by Source **B**. (See below a procedure that selects optimally the source that should transmit on subcarrier $k$).

At the end of the transmission, the destination utilizes the output vector $\mathbf{y}_d[k]$ from the direct transmission by the source given by (3) and the output vector $\tilde{\mathbf{y}}_d[k]$ from the transmission by the relay given by (4) to get an estimate of the transmitted codeword. In doing this, the destination performs a maximum-ratio combining of the two output components. The rate (per channel use) at which the destination can perform this reliably on subcarrier $k$ can be shown easily to be

$$R_{\text{id}}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\text{id}}[k]|^2 P_t}{N} + \frac{\beta_{ir}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right). \tag{11}$$

With the help of the relay node, the destination gets the information transmitted on subcarrier $k$ correctly as long as this information is sent at a rate that is no larger than the minimum among $R_{\text{ir}}[k]$ as given by (10) and $R_{\text{id}}[k]$ as given by (11), i.e.,

$$\begin{aligned} R[k] = \min\Bigg\{ & \frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\text{ir}}[k]|^2 P_t}{N}\right), \\ & \frac{1}{2}\log_2\left(1 + \frac{\beta_i^2[k]|h_{\text{id}}[k]|^2 P_t}{N} + \frac{\beta_{ir}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right)\Bigg\}. \end{aligned} \tag{12}$$

As shown in [11], at the optimum, the constraint associated with the minimization in (12) should be saturated, i.e.,

$$\beta_i^2[k] = \frac{\beta_{ir}^2[k]|h_{\text{rd}}[k]|^2}{|h_{\text{ir}}[k]|^2 - |h_{\text{id}}[k]|^2}. \tag{13}$$

Let $\beta_s^2[k] = \beta_i^2[k] + \beta_{ir}^2[k]$, the rate $R[k]$ on subcarrier $k$ can be rewritten equivalently as,

$$R[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{ir}}[k]|^2|h_{\text{rd}}[k]|^2 P_t}{N(|h_{\text{ir}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{id}}[k]|^2)}\right). \tag{14}$$

From the above, it follows that it is beneficial that the relay be active on subcarrier $k$, i.e., the relay decodes and forwards the source's codeword, if and only if (iff) the following two conditions hold

$$\begin{cases} |h_{\text{id}}[k]|^2 & < |h_{\text{ir}}[k]|^2 \\ |h_{\text{id}}[k]|^2 & < \frac{|h_{\text{ir}}[k]|^2 |h_{\text{rd}}[k]|^2}{|h_{\text{ir}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{id}}[k]|^2}. \end{cases} \tag{15}$$

If it is more advantageous that the relay remains idle on subcarrier $k$, the destination decodes the transmitted codeword using only its output component from the direct transmission, i.e., from the source. In this case, one can get a larger rate by allocating all the available power $\beta_s^2[k]P_t$ for transmission on subcarrier $k$ to the transmitting source. The destination decodes the transmitted codeword at rate

$$R[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{id}}[k]|^2 P_t}{N}\right). \tag{16}$$

*Summarizing:* for every subcarrier $k$, $1 \le k \le K$, the appropriate relay operation mode (i.e., transmitting or not-transmitting on that subcarrier) can be selected optimally based on the actual channel states. More precisely, the relay helps the source that transmits on subcarrier $k$ iff the two conditions in (15) hold simultaneously; otherwise it remains idle. Investigating (14) and (16), we introduce the following *equivalent* channel gains for the transmission on subcarrier $k$,

$$|h_i[k]|^2 = \max\left\{\frac{|h_{\text{ir}}[k]|^2 |h_{\text{rd}}[k]|^2}{|h_{\text{ir}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{id}}[k]|^2}\mathbf{1}_{|h_{\text{ir}}[k]| > |h_{\text{id}}[k]|},\right.$$
$$\left. |h_{\text{id}}[k]|^2\right\}, \qquad i = a, b. \tag{17}$$

In order to maximize the allowed sum-rate, subcarrier $k$ should be assigned to the source that has the largest equivalent channel gain among $|h_a[k]|^2$ and $|h_b[k]|^2$. Then, defining the equivalent channel coefficient $h_{\text{eq}}[k]$ for subcarrier $k$ to be

$$|h_{\text{eq}}[k]|^2 = \max\left\{|h_a[k]|^2, |h_b[k]|^2\right\}, \tag{18}$$

the rate that is allowed on subcarrier $k$, $1 \le k \le K$, can be put into the compact form

$$R[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right), \tag{19}$$

where $h_{\text{eq}}[k]$ is given by (18).

For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^K$ and power policy $\{\beta_s[k]\}_{k=1}^K$, since OFDMA transforms the channel into a set of $K$ *parallel* subchannels, the sum-rate that is offered through the transmission is obtained by simply summing over all subchannels the individual rate $R[k]$, $k = 1, \cdots, K$ [34]. Finally, the following larger sum-rate can be obtained by maximizing over all allowable power policies, i.e.,

$$R_{\text{sum}}^{\text{OFDMA}} = \max \sum_{k=1}^{K} \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right) \tag{20}$$

where the maximization is over $\{\beta_s[k]\}_{k=1}^K$ such that $\sum_{k=1}^K \beta_s^2[k] \le 1$.

This completes the proof of Proposition 1. □

## B. Sum-rate Analysis for the OFDM-based Transmission

In this section we describe the OFDM transmission scheme where sources **A** and **B** transmit their codewords simultaneously using all subcarriers.

For convenience, we define the quantities given in Definition 1 and Definition 2, which we will use extensively throughout this section.

The following proposition provides an achievable sum-rate for the multiaccess relay model of Figure 1, using OFDM multicarrier transmission.

***Proposition 2:*** For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^K$, the following sum-rate is achievable for the multiaccess relay channel of Figure 1,

$$R_{\text{sum}}^{\text{OFDM}} = \max \sum_{k=1}^{K} \max_{1 \le l \le 7} R_l[k], \tag{28}$$

where, for $1 \le k \le K$, and $1 \le l \le 7$, $R_l[k]$ is defined as in Definition 2; and the outer maximization is over $\{\boldsymbol{\beta}[k]\}_{k=1}^K$, with $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, such that

$$\sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \le 1. \tag{29}$$

The proof of Proposition 2 will follow. The following remark reveals certain aspects related to the coding scheme, and is useful for a better understanding of the proof and its structure.

*Remark 1:* The proof is based on the OFDM multicarrier transmission scheme of Section II-B. In this scheme, by opposition to that of Proposition 1, both sources are allowed to transmit simultaneously on every subcarrier. The relay is half-duplex, and implements decode-and-forward strategy on the symbols transmitted on each subcarrier. It helps none, only one, or both sources simultaneously. In the case in which the relay helps both sources simultaneously, on the same subcarrier, it shares its power among the two and superimposes the information that is intended to help Source **A** and the one that is intended to help Source **B**. The destination decodes the sources's codewords successively, and the decoding operations are based on maximum-ratio combining. The relay decodes both codewords only if it helps both sources to transmit their codewords; and, if so, it also decodes the sources's codewords successively. As we mentioned previously, different decoding orders combinations (at the relay, if applicable, and at the destination) generally result in different achievable sum-rates. That is, in general no decoding order outperforms the others; and the selection of the appropriate decoding order depends on the fading coefficients. In addition to the decoding orders at the relay and destination, the relay operation mode (i.e, helping none, only one or both sources) influences the allowed sum-rate. This leads to thirteen different cases if all possible combinations are considered using the decoding orders and the relay operation modes. However, it can be easily seen that whenever the relay helps only one of the sources (by decoding and forwarding the codeword transmitted by that source), this codeword should be decoded first at the destination. When the relay helps the two sources simultaneously, a total of four possible decoding orders need to be investigated and compared (two possible decoding orders at the destination for each possible decoding order at the relay). Hence, out of the thirteen apriori possible cases only seven actually attribute to be of interest. These cases are summarized in Table I.

**Proof:** Recall the OFDM-based transmission scheme of Section II-B. Also, recall the seven possible cases that we mentioned in Remark 1, summarized in Table I. In what follows, because

***Definition** 1:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$, and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, with for $1 \leq k \leq K$, $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, let

$$\Theta_{\text{b}}^{(1)}[k] = \frac{N(\beta_b^2[k]|h_{\text{bd}}[k]|^2 + \beta_{br}^2[k]|h_{\text{rd}}[k]|^2)P_t + \beta_b^2[k]|h_{\text{bd}}[k]|^2\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_t^2}{N^2 + \beta_a^2[k]|h_{\text{ad}}[k]|^2P_tN + \beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_tN} \tag{21}$$

$$\Theta_{\text{b}}^{(2)}[k] = \frac{\beta_{br}^2[k]|h_{\text{rd}}[k]|^2\beta_a^2[k]|h_{\text{ad}}[k]|^2P_t^2 - 2\beta_a[k]\beta_b[k]\beta_{ar}[k]\beta_{br}[k]\text{Re}\{h_{\text{bd}}^*[k]h_{\text{ad}}[k]\}|h_{\text{rd}}[k]|^2P_t^2}{N^2 + \beta_a^2[k]|h_{\text{ad}}[k]|^2P_tN + \beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_tN} \tag{22}$$

$$\text{snr}_{\text{b}}[k] = \Theta_{\text{b}}^{(1)}[k] + \Theta_{\text{b}}^{(2)}[k]. \tag{23}$$

Also, let $\Theta_{\text{a}}^{(1)}[k]$, $\Theta_{\text{a}}^{(2)}[k]$, and $\text{snr}_{\text{a}}[k]$ be obtained by swapping the indices $a$ and $b$ in $\Theta_{\text{b}}^{(1)}[k]$, $\Theta_{\text{b}}^{(2)}[k]$, and $\text{snr}_{\text{b}}[k]$, respectively.

***Definition** 2:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^{K}$, and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, with for $1 \leq k \leq K$, $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, let

$$R_1[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2P_t}{N} + \frac{\beta_b^2[k]|h_{\text{bd}}[k]|^2P_t}{N}\right) \tag{24}$$

$$\begin{aligned} R_2[k] = &\min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2P_t}\right), \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{bd}}[k]|^2P_t}{N + \beta_a^2[k]|h_{\text{ad}}[k]|^2P_t} + \frac{\beta_{br}^2[k]|h_{\text{rd}}[k]|^2P_t}{N}\right)\right\} \\ &+ \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2P_t}{N}\right) \end{aligned} \tag{25}$$

$$\begin{aligned} R_3[k] = &\min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2P_t}{N}\right), \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2P_t}{N} + \frac{\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_t}{N}\right)\right\} \\ &+ \min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2P_t}\right), \frac{1}{2}\log_2\left(1 + \text{snr}_{\text{b}}[k]\right)\right\} \end{aligned} \tag{26}$$

$$\begin{aligned} R_4[k] = &\min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2P_t}{N + \beta_b^2[k]|h_{\text{br}}[k]|^2P_t}\right), \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2P_t}{N} + \frac{\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2P_t}{N}\right)\right\} \\ &+ \min\left\{\frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2P_t}{N}\right), \frac{1}{2}\log_2\left(1 + \text{snr}_{\text{b}}[k]\right)\right\}. \end{aligned} \tag{27}$$

Also, let $R_5[k]$, $R_6[k]$, and $R_7[k]$ be obtained by swapping the indices $a$ and $b$ in $R_2[k]$, $R_3[k]$, and $R_4[k]$, respectively.

| | Decoding order at the relay | Decoding order at the destination | Case |
|---|---|---|---|
| Direct Transmission | N.A. | No Decoding order | 1 |
| The relay forwards $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | 2 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | 3 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | 4 |
| The relay forwards $\mathbf{x}_a[k]$ | $\mathbf{x}_a[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | 5 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | 6 |
| The relay forwards $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ | $\mathbf{x}_b[k] \to \mathbf{x}_a[k]$ | $\mathbf{x}_a[k] \to \mathbf{x}_b[k]$ | 7 |

TABLE I
DIFFERENT USEFUL CASES FOR THE OFDM MULTICARRIER TRANSMISSION

of symmetry, we only analyze the following four cases for the transmission on subcarrier $k$, $1 \leq k \leq K$: **Case** 1) transmission to the destination on subcarrier $k$ utilizes only the direct links, i.e., the relay remains idle on subcarrier $k$, **Case** 2) the relay helps only one source on subcarrier $k$, e.g., Source **B** by decoding and forwarding the transmitted symbol $\mathbf{x}_b[k]$, **Case** 3) the relay helps both sources simultaneously on subcarrier $k$, and the codeword $\mathbf{x}_b[k]$ of Source **B** is decoded first at both relay and destination, and **Case** 4) the relay helps both sources simultaneously on subcarrier $k$, with the codeword $\mathbf{x}_a[k]$ of Source **A** decoded first at the relay and the codeword $\mathbf{x}_b[k]$ of Source **B** decoded first at the destination. The analysis of the remaining three cases (obtained respectively from Case 2, Case 3 and Case 4 by swapping the roles of the sources) can be obtained straightforwardly by symmmetry. For each of the four cases that will be analyzed, we first describe the decoding procedures at the relay and destination and then analyze the allowed sum-rate.

Case 1 *Transmission using only direct links:* This scenario corresponds to a regular MAC, and the sum-rate that is achievable on subcarrier $k$, $1 \leq k \leq K$, can be shown easily [34] to be $R_1[k]$ as given by (24) in Definition 2.

Case 2 *The relay helps only Source **B**:* At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (5). The relay utilizes joint typicality decoding to decode the codeword $\mathbf{x}_b[k]$ transmitted by Source **B** on subcarrier $k$, $1 \leq k \leq K$. In doing so, the relay treats the codeword $\mathbf{x}_a[k]$ transmitted by Source **A** as unknown noise. For large $n$, the decoding can be done reliably at rate

$$R_{\text{br}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}\right), \tag{30}$$

where the upper script refers to the case in hand. The relay then forwards the decoded codeword on the same subcarrier $k$ to the destination, during the second transmission period. To this end, the relay sends

$$\tilde{\mathbf{x}}_r[k] = \sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k]. \tag{31}$$

Using (31), the destination's output components $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$ from the two transmission periods, given by (5) and (6), can be rewritten equivalently as

$$\begin{aligned}\mathbf{y}_d[k] &= h_{\text{ad}}[k]\mathbf{x}_a[k] + h_{\text{bd}}\mathbf{x}_b[k] + \mathbf{z}_d[k]\\ \tilde{\mathbf{y}}_d[k] &= h_{\text{rd}}[k]\sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k] + \tilde{\mathbf{z}}_d[k].\end{aligned} \tag{32}$$

The destination decodes the codewords transmitted by both sources successively. Given that the relay helps only Source **B**, it can be shown relatively straightforwardly that, in this case, decoding the relayed codeword $\mathbf{x}_b[k]$ first, i.e., before canceling out its contribution and decoding the non-relayed codeword $\mathbf{x}_a[k]$, results in a sum-rate that is larger than the one that would be allowed if the decoding of the codewords at the destination is performed in the reverse order. Thus, the destination first decodes codeword $\mathbf{x}_b[k]$, cancels its contribution out and then decodes codeword $\mathbf{x}_a[k]$. In order to decode codeword $\mathbf{x}_b[k]$, the destination combines the output components $\mathbf{y}_d[k]$ and $\tilde{\mathbf{y}}_d[k]$ to their maximum ratio, i.e., using standard maximum ratio combining (MRC). It can be shown easily that, for large $n$, the decoding of codeword $\mathbf{x}_b[k]$ can be decoded reliably at rate

$$R_{\text{bd}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{bd}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t} + \frac{\beta_{br}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right). \tag{33}$$

Next, the destination subtracts out the contribution of $\mathbf{x}_b[k]$ from $\mathbf{y}_d[k]$ and, so, decodes the codeword $\mathbf{x}_a[k]$ free of interference. It can be shown easily that, for large $n$, this can be done reliably at rate

$$R_{\text{ad}}^{(2)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2}{N}\right). \tag{34}$$

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier $k$, $1 \leq k \leq K$, as long as $n$ is large and these codewords are sent at a sum-rate that is no larger than the sum of $R_{\text{ad}}^{(2)}[k]$ and the minimum among $R_{\text{br}}^{(2)}[k]$ and $R_{\text{bd}}^{(2)}[k]$, i.e., $R_2[k]$ as given by (25) in Definition 2.

Case 3 *The relay helps both sources, and the decoding orders at the relay and destination are identical:* In this case we assume that the relay helps both sources, and that both the relay and destination first decode codeword $\mathbf{x}_b[k]$, cancel out its contribution and then decode codeword $\mathbf{x}_a[k]$.

Consider first the decoding operations at the relay. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (5). The relay utilizes joint typicality decoding to decode the transmitted codeword $\mathbf{x}_b[k]$ from the output vector $\mathbf{y}_r[k]$. In doing so, the relay treats the codeword $\mathbf{x}_a[k]$ transmitted by Source **A** as unknown noise. It can be shown easily that, for large $n$, the decoding can be done reliably at rate

$$R_{\text{br}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N + \beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}\right), \tag{35}$$

where the upper script refers to the case in hand. The relay then subtracts out the contribution of $\mathbf{x}_b[k]$ from $\mathbf{y}_r[k]$ and then decodes codeword $\mathbf{x}_a[k]$, again using a joint typicality decoding. Similarly, for large $n$, this can be done reliably at rate

$$R_{\text{ar}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}{N}\right). \tag{36}$$

During the second transmission period, the relay helps both sources and transmits their codewords simultaneously on subcarrier $k$. To this end, the relay shares its power among re-transmitting codeword $\mathbf{x}_a[k]$ and re-transmitting codeword $\mathbf{x}_b[k]$, on the same subcarrier $k$, using superposition coding. That is, the relay sends

$$\tilde{\mathbf{x}}_r[k] = \sqrt{\frac{\beta_{ar}^2[k]}{\beta_a^2[k]}}\mathbf{x}_a[k] + \sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k] \tag{37}$$

on subcarrier $k$, where $\beta_{ar}[k]$ and $\beta_{br}[k]$ are nonnegative scalars chosen to adjust power and are such that $\beta_{ar}^2[k] + \beta_{br}^2[k] = \beta_r^2[k]$. (The way this power sharing needs to be performed appropriately will be addressed in Section IV-B).

Using (37), the destination's output components $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$ from the two transmission periods, given by (5) and (6), can be rewritten equivalently as

$$\begin{aligned}\mathbf{y}_d[k] &= h_{\text{ad}}[k]\mathbf{x}_a[k] + h_{\text{bd}}[k]\mathbf{x}_b[k] + \mathbf{z}_d[k],\\ \tilde{\mathbf{y}}_d[k] &= h_{\text{rd}}[k]\sqrt{\frac{\beta_{ar}^2[k]}{\beta_a^2[k]}}\mathbf{x}_a[k] + h_{\text{rd}}[k]\sqrt{\frac{\beta_{br}^2[k]}{\beta_b^2[k]}}\mathbf{x}_b[k] + \tilde{\mathbf{z}}_d[k].\end{aligned} \tag{38}$$

The destination decodes the codewords transmitted by both sources successively, in the same order this is performed at the relay. More precisely, the destination first decodes codeword $\mathbf{x}_b[k]$, cancels its contribution out and then decodes codeword $\mathbf{x}_a[k]$. In order to decode codeword $\mathbf{x}_b[k]$, the destination combines the output components $\mathbf{y}_d[k]$ and $\tilde{\mathbf{y}}_d[k]$ to their maximum ratio. Through straightforward algebra, which we omit for brevity, it can be shown that, for large $n$, the destination can get the correct $\mathbf{x}_b[k]$ at rate

$$R_{\text{bd}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \text{snr}_{\text{b}}[k]\right), \tag{39}$$

where $\text{snr}_{\text{b}}[k]$ is given in Definition 1.

Next, the destination subtracts out the contribution of codeword $\mathbf{x}_b[k]$ from $(\mathbf{y}_d[k], \tilde{\mathbf{y}}_d[k])$, and combines the resulting equivalent output components using MRC to decode codeword $\mathbf{x}_a[k]$. Again, through straightforward algebra, which we omit for brevity, it can be shown that, for large $n$, the destination can get the correct $\mathbf{x}_a[k]$ at rate

$$R_{\text{ad}}^{(3)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ad}}[k]|^2 P_t}{N} + \frac{\beta_{ar}^2[k]|h_{\text{rd}}[k]|^2 P_t}{N}\right). \tag{40}$$

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier $k$, $1 \leq k \leq K$, as long as $n$ is large and these codewords are sent at a sum-rate that is no larger than the sum of the minimum among $R_{\text{ar}}^{(3)}[k]$ and $R_{\text{ad}}^{(3)}[k]$ and the minimum among $R_{\text{br}}^{(3)}[k]$ and $R_{\text{bd}}^{(3)}[k]$, i.e., $R_3[k]$ as given by (26) in Definition 2.

Case 4 *The relay helps both sources, and the decoding orders at the relay and destination are different:* In this case we assume that the relay helps both sources, and that the relay and destination decode the sources' codewords in different orders. In particular, in what follows we analyze the case in which the decoding order at the relay is such that codeword $\mathbf{x}_a[k]$ is decoded first, and the decoding at the destination is maintained as in Case 3 above.

Consider first the decoding operations at the relay. At the end of the first transmission period, the relay gets the output vector $\mathbf{y}_r[k]$ given by (5). Proceeding along the lines in the analysis of Case 3 above, but the roles of codewords $\mathbf{x}_a[k]$ and $\mathbf{x}_b[k]$ swapped, it can be shown easily that, for large $n$, the relay can get the correct $\mathbf{x}_a[k]$ at rate

$$R_{\text{ar}}^{(4)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_a^2[k]|h_{\text{ar}}[k]|^2 P_t}{N + \beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}\right) \quad (41)$$

and the correct $\mathbf{x}_b[k]$ at rate

$$R_{\text{br}}^{(4)}[k] = \frac{1}{2}\log_2\left(1 + \frac{\beta_b^2[k]|h_{\text{br}}[k]|^2 P_t}{N}\right), \quad (42)$$

where the upper scripts refer to the case in hand.

The decoding at the destination is exactly as in Case 3. Thus, for large $n$, the destination can first get the correct $\mathbf{x}_b[k]$ at rate $R_{\text{bd}}^{(4)}[k] = R_{\text{bd}}^{(3)}[k]$ as given by (39); and then subtract its contribution out and get the correct codeword $\mathbf{x}_a[k]$ at rate $R_{\text{ad}}^{(4)}[k] = R_{\text{ad}}^{(3)}[k]$ as given by (40).

From the above, it follows that, in this case, the destination can decode reliably the sources' codewords that are transmitted on subcarrier $k$, $1 \le k \le K$, as long as $n$ is large and these codewords are sent at a sum-rate that is no larger than the sum of the minimum among $R_{\text{ar}}^{(4)}[k]$ and $R_{\text{ad}}^{(4)}[k]$ and the minimum among $R_{\text{br}}^{(4)}[k]$ and $R_{\text{bd}}^{(4)}[k]$, i.e., $R_4[k]$ as given by (27) in Definition 2.

This completes the analysis of Cases 1-4. The analysis of Case 5, Case 6 and Case 7 in Table I can be obtained straightforwardly respectively from the analysis of Case 2, Case 3 and Case 4, by swapping the roles of Source **A** and Source **B**. This leads to the associated sum-rates $R_5[k]$, $R_6[k]$ and $R_7[k]$ as given in Definition 2.

*Summarizing:* For given channel states $\{h_{\text{ar}}[k], h_{\text{br}}[k], h_{\text{ad}}[k], h_{\text{bd}}[k], h_{\text{rd}}[k]\}_{k=1}^K$ and power policy $\{\boldsymbol{\beta}[k]\}_{k=1}^K$, sum-rates of $R_l[k]$ bits per second, $1 \le l \le 7$, are achievable on subcarrier $k$, $1 \le k \le K$, using the OFDM-based transmission that we described. Thus, the sum-rate $R[k] = \max_{1\le l\le 7} R_l[k]$ on subcarrier $k$, i.e., the maximum among the seven sum-rates $\{R_l[k]\}_{l=1}^7$, is obtained by selecting for subcarrier $k$ the coding scheme that offers the larger per-subcarrier sum-rate among those of the aforementioned seven cases. Next, since OFDM transforms the channel into a set of $K$ *parallel* subchannels, the total sum-rate that is offered through the transmission, over all subchannels, is obtained by simply summing over all subchannels the individual allowed per-subcarrier sum-rates [34]. Finally, the larger sum-rate $R_{\text{sum}}^{\text{OFDM}}$ as given in the statement of Proposition 2 can be obtained by maximizing the obtained total sum-rate over all allowable power policies.

This completes the proof of Proposition 2. □

## IV. Sum Rate Optimization

In this section, for each of the multicarrier transmission schemes that we consider, we study and solve the problem of maximizing the offered sum-rate under a total sum power constraint.

### A. OFDMA Sum-rate Optimization

*1) Problem Formulation:* Consider the sum-rate $R_{\text{sum}}^{\text{OFDMA}}$ as given by (7) in Proposition 1. The optimization problem is stated as:

$$\text{(B)} \; : \; \max \quad \sum_{k=1}^K \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right) \quad (43a)$$

$$\text{s. t.} \quad \sum_{k=1}^K \beta_s^2[k] \le 1, \quad (43b)$$

$$\beta_s^2[k] \ge 0, \quad (43c)$$

$$|h_{\text{eq}}[k]|^2 = \max\Big\{\frac{|h_{\text{ar}}[k]|^2|h_{\text{rd}}[k]|^2}{|h_{\text{ar}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{ad}}[k]|^2}\mathbf{1}_{|h_{\text{ar}}[k]|>|h_{\text{ad}}[k]|}, |h_{\text{ad}}[k]|^2, |h_{\text{bd}}[k]|^2, \frac{|h_{\text{br}}[k]|^2|h_{\text{rd}}[k]|^2}{|h_{\text{br}}[k]|^2 + |h_{\text{rd}}[k]|^2 - |h_{\text{bd}}[k]|^2}\mathbf{1}_{|h_{\text{br}}[k]|>|h_{\text{bd}}[k]|}\Big\}. \quad (43d)$$

*Remark 2:* we note that, as described in Section III-A, in order to maximize the allowed sum-rate $R_{\text{sum}}^{\text{OFDMA}}$, subcarrier $k$ should be assigned to the source that has the largest equivalent channel gain among $|h_a[k]|^2$ and $|h_b[k]|^2$ given by (17). This is a subcarrier allocation based on a greedy algorithm in which a subcarrier $k$ is assigned to the source that has the largest equivalent channel gain. Thus, the maximization of problem (A) is only over $\{\beta_s[k]\}_{k=1}^K$.

The optimization problem (A) is concave. In what follows, we provide an efficient algorithm that finds a global solution optimally.

*2) Power Allocation:* In this section, we focus on the problem of finding appropriate power values $\{\beta_s[k]\}_{k=1}^K$. We solve this problem using dual decomposition approach. The Lagrangian function can be defined as:

$$L(\mu, \beta_s[k]) = \sum_{k=1}^K \frac{1}{2}\log_2\left(1 + \frac{\beta_s^2[k]|h_{\text{eq}}[k]|^2 P_t}{N}\right) - \mu\left[\sum_{k=1}^K \beta_s^2[k] - 1\right], \quad (44)$$

where $\mu$ is the Lagrange multiplier associated with the global power constraint. The solution can be found by applying and solving the KKT conditions [35]. This leads to a waterfilling solution adjusted to the equivalent channel $|h_{\text{eq}}[k]|^2$:

$$\beta_s^2[k] = \left[\frac{1}{\mu} - \frac{N}{P_t|h_{\text{eq}}[k]|^2}\right]^+. \quad (45)$$

We should note that any subcarrier can be excluded from transmission if its allocated power is zero. The water-level, i.e. $\mu$, has to be chosen such that the power constraint (43b) is fulfilled, and is given by

$$\frac{1}{\mu} = \frac{1}{K'} + \frac{1}{K'}\sum_{k=1}^{K'} \frac{N}{|h_{\text{eq}}[k]|^2 P_t}, \quad (46)$$

where $K'$ is the number of subcarriers with a non zero positive power value. To compute $R_{\text{sum}}^{\text{OFDMA}}$ as given by (7), we develop the following algorithm, to which we refer to as "Algorithm A" in reference to the optimization problem (A). The iterative algorithm (Algorithm A) terminates if $|\sum_{k=1}^K \beta_s^2[k] - 1|$ is smaller than a prescribed small strictly positive constant $\epsilon$.

**Algorithm A** Power Allocation for $R_{\text{sum}}^{\text{OFDMA}}$ as given by (7)

1: Calculate $\mu$ using (46) for $K' = K$
2: Solve the power allocation problem using (45), and calculate the power values $\{\beta_s[k]\}_{k=1}^{K}$
3: Decrease the number of subcarriers $K = K - 1$ by removing the subcarrier that has the smallest equivalent channel $|h_{\text{eq}}[k]|^2$ and then go to step 1
4: Terminate if $|\sum_{k=1}^{K} \beta_s^2[k] - 1| \leq \epsilon$

### *B. OFDM Sum-rate Optimization*

This section is devoted to maximize the sum-rate of the objective function given in (28). the optimization problem comprises i) selecting the appropriate relay operation mode (i.e., helping none, only one, or both sources simultaneously) for every subcarrier, ii) choosing the best decoding orders at the relay (if active) and destination for every subcarrier, and iii) allocating the powers on each subcarrier and among the transmitting terminals.

*1) Problem Formulation:* Consider the sum-rate $R_{\text{sum}}^{\text{OFDM}}$ as given by (28) in Proposition 2. The optimization problem can be equivalently stated as

$$\text{(B)} \;:\; \max \quad \sum_{k=1}^{K} \sum_{l=1}^{7} a_l[k] R_l[k], \tag{47}$$

where, for $1 \leq k \leq K$, and $1 \leq l \leq 7$, $a_l[k]$ is an indicator whose value should be 0 or 1, and $R_l[k]$ is defined as in Definition 2; and the maximization is over $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$, with $\boldsymbol{\beta}[k] = [\beta_a[k], \beta_b[k], \beta_{ar}[k], \beta_{br}[k]]^T$, satisfying

$$\sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \leq 1, \tag{48}$$

and over $\{\mathbf{a}[k]\}_{k=1}^{K}$, with $\mathbf{a}[k] = [a_1[k], a_2[k], .., a_7[k]]^T$, such that

$$\|\mathbf{a}[k]\|^2 \leq 1, \quad \text{for } 1 \leq k \leq K. \tag{49}$$

The optimization problem (B) is a mixed integer linear programming problem; and, so, it is not easy to solve it optimally. In what follows, we solve this optimization problem iteratively, by finding appropriate powers $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ and indicators $\{\mathbf{a}[k]\}_{k=1}^{K}$ alternately. We note that the selection of $\{\mathbf{a}[k]\}_{k=1}^{K}$ determine the decoding orders at the relay and destination, and the relay operation mode (i.e., helping none, only one, or both sources simultaneously).

Let us, with a slight abuse of notation, denote by $R_{\text{sum}}^{\text{OFDM}}[\iota]$ the value of the sum-rate at some iteration $\iota \geq 0$. To compute $R_{\text{sum}}^{\text{OFDM}}$ as given by (28) iteratively, we develop the following algorithm, to which we refer to as "Algorithm B" in reference to the optimization problem (B).

**Algorithm B** Iterative algorithm for computing $R_{\text{sum}}^{\text{OFDM}}$ as given by (28)

1: Initialization: set $\iota = 1$
2: Set $\{\boldsymbol{\beta}[k] = \boldsymbol{\beta}^{(\iota-1)}[k]\}_{k=1}^{K}$ in (47), and solve the obtained problem as we will describe in Section IV-B2 given below. Denote by $\{\mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ the found $\{\mathbf{a}[k]\}_{k=1}^{K}$
3: Set $\{\mathbf{a}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ in (47), and solve the obtained problem using "Algorithm B-1" given below. Denote by $\{\boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ the found $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$
4: Increment the iteration index as $\iota = \iota + 1$, and go back to Step 2
5: Terminate if $|R_{\text{sum}}^{\text{OFDM}}[\iota] - R_{\text{sum}}^{\text{OFDM}}[\iota - 1]| \leq \epsilon$

As described in "Algorithm B", we compute the power values given by $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ and the indicator values given by $\{\mathbf{a}[k]\}_{k=1}^{K}$, alternately. More specifically, at iteration $\iota \geq 1$, the algorithm computes appropriate indicator values $\{\mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$ that correspond to a maximum of (47) with the choice of the power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ set to their values obtained from the previous iteration, i.e., $\{\boldsymbol{\beta}[k] = \boldsymbol{\beta}^{(\iota-1)}[k]\}_{k=1}^{K}$ (for the initialization, set $\{\boldsymbol{\beta}^{(0)}[k]\}_{k=1}^{K}$ to an appropriate value). This sub-problem is an integer linear programming (ILP) problem [36] and we solve it by selecting the largest sum-rate ($R_l[k]$) on each subcarrier $k$. Next, the power values $\{\boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ can be computed in order to maximize (47) with the choice of $\{\mathbf{a}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$. This sub-problem can be formulated as a Complementary geometric programming problem. We solve it through a geometric programming and successive convex optimization approach (see "Algorithm B-1" below). The iterative algorithm ("Algorithm B") terminates if the following condition holds: $|R_{\text{sum}}^{\text{OFDM}}[\iota] - R_{\text{sum}}^{\text{OFDM}}[\iota - 1]|$ is smaller than a prescribed small strictly positive constant $\epsilon$ — in this case, the optimized value of the sum-rate is $R_{\text{sum}}^{\text{OFDM}}[\iota]$, and is attained using the power values $\{\tilde{\boldsymbol{\beta}}[k] = \boldsymbol{\beta}^{(\iota)}[k]\}_{k=1}^{K}$ and indicator values $\{\tilde{\mathbf{a}}[k] = \mathbf{a}^{(\iota)}[k]\}_{k=1}^{K}$.

In the following two sections, we study the aforementioned two sub-problems of problem (B), and describe the algorithms that we propose to solve them.

*2) Indicator Allocation:* In this section, we focus on the problem of finding the indicator values $\{\mathbf{a}[k]\}_{k=1}^{K}$ for a given choice of power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$. Investigating the objective function in (47), it can be stated as

$$\max \quad \sum_{k=1}^{K} \sum_{l=1}^{7} a_l[k] R_l[k], \tag{50a}$$

$$\text{s. t.} \quad \|\mathbf{a}[k]\|^2 \leq 1, \text{ for } 1 \leq k \leq K \tag{50b}$$

$$a_l[k] \in \{0,1\},\ l \in \{1,2,..,7\},\ \text{for } 1 \leq k \leq K. \tag{50c}$$

It can be easily seen from (50a), that the optimum value of $\mathbf{a}[k]$, at a subcarrier $k$, can be obtained by investigating the sum-rate $R_l[k]$. The indicator $\mathbf{a}[k]$ is calculated in such a way that the largest sum-rate $R_l[k]$ is selected, and it is given by

$$a_l[k] = \begin{cases} 1, & l = \arg\max_{1 \leq l \leq 7} R_l[k] \\ 0, & \text{otherwise.} \end{cases}$$

Hence, the largest sum-rate at subcarrier $k$ is

$$\tilde{R}[k] = \max_{1 \leq l \leq 7} R_l[k]. \tag{51}$$

*3) Power Allocation:* In this section, we focus on the problem of calculating $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ for a given choice of $\{\mathbf{a}[k]\}_{k=1}^{K}$. Investigating the objective function in (47), it can be stated as

$$\max \quad \sum_{k=1}^{K} \tilde{R}[k], \tag{52a}$$

$$\text{s. t.} \quad \sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \leq 1, \tag{52b}$$

$$\beta_i^2[k] \geq 0,\ i \in \{a, b, ar, br\},\ \text{for } 1 \leq k \leq K \tag{52c}$$

where $\tilde{R}[k]$ is given in (51). It can be easily seen that this problem can be equivalently stated as

$$
\begin{aligned}
&\min \prod_{k=1}^{K} \Delta_a[k]\Delta_b[k], && \text{(53a)}\\
&\text{s. t.} \sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \le 1, && \text{(53b)}\\
&\quad \Delta_a[k] \ge f_1(\boldsymbol{\beta}[k]),\ \Delta_a[k] \ge f_2(\boldsymbol{\beta}[k]) \ \text{if } f_1(\boldsymbol{\beta}[k]) \ne f_2(\boldsymbol{\beta}[k]) && \text{(53c)}\\
&\quad \Delta_a[k] = f_1(\boldsymbol{\beta}[k]) \qquad \text{if } f_1(\boldsymbol{\beta}[k]) = f_2(\boldsymbol{\beta}[k]) && \text{(53d)}\\
&\quad \Delta_b[k] \ge f_3(\boldsymbol{\beta}[k]),\ \Delta_b[k] \ge f_4(\boldsymbol{\beta}[k]) \ \text{if } f_3(\boldsymbol{\beta}[k]) \ne f_4(\boldsymbol{\beta}[k]) && \text{(53e)}\\
&\quad \Delta_b[k] = f_3(\boldsymbol{\beta}[k]) \qquad \text{if } f_3(\boldsymbol{\beta}[k]) = f_4(\boldsymbol{\beta}[k]) && \text{(53f)}\\
&\quad \beta_i^2[k] \ge 0,\ i \in \{a, b, ar, br\}, && \text{(53g)}\\
&\quad \Delta_a[k] \ge 0,\ \Delta_b[k] \ge 0 \in \mathbb{R},\ \text{for } 1 \le k \le K && \text{(53h)}
\end{aligned}
$$

where $\Delta_a[k]$ and $\Delta_b[k]$ are simultaneously extra optimization variables and the objective function in (53a). Also, it is easy to see that the power values $\{\boldsymbol{\beta}[k]\}_{k=1}^{K}$ that achieve the minimum value of $\prod_{k=1}^{K} \Delta_a[k]\Delta_b[k]$ also achieve the maximum value of the objective function in (47). The functions $f_l(\boldsymbol{\beta}[k])$ are given in Table II for the cases described in section III-B. For brevity, we did not include the functions of the remaining three cases (case 5, case 6 and case 7) in Table II, as these can be easily obtained by swapping the indices a and b in the functions of case 2, case 3 and case 4, respectively.

*Remark 3:* we note that $\Delta_a[k]$ acts as an upper bound for the two functions $f_1(\boldsymbol{\beta}[k])$ and $f_2(\boldsymbol{\beta}[k])$, and whenever the two functions $f_1(\boldsymbol{\beta}[k])$ and $f_2(\boldsymbol{\beta}[k])$ are identical, for example Case 1 and 2 given in Table II, $\Delta_a[k]$ is not anymore an upper bound. Thus, the inequalities in (53c) should be considered as equality as given in (53d). Similarly, $\Delta_b[k]$ acts as an upper bound for the two functions $f_3(\boldsymbol{\beta}[k])$ and $f_4(\boldsymbol{\beta}[k])$, and whenever the two functions $f_3(\boldsymbol{\beta}[k])$ and $f_4(\boldsymbol{\beta}[k])$ are identical, for example Case 1 given in Table II, $\Delta_b[k]$ is not anymore an upper bound. Thus, the inequalities in (53e) should be considered as equality as given in (53f).

The optimization problem (53a) is non-linear and non-convex. Therefore, we consider geometric programming (GP) which is a special form of convex optimization for which efficient algorithms have been developed [33], [37]. There are two forms of GP: the standard form and the convex form. In its standard form, a GP optimization problem is generally written as [33]

$$
\begin{aligned}
&\text{minimize} && f_0(\boldsymbol{\beta}[k], \Delta_a[k], \Delta_b[k]) && \text{(54a)}\\
&\text{subject to} && f_j(\boldsymbol{\beta}[k], \Delta_a[k], \Delta_b[k]) \le 1, \quad j = 1, \cdots, J, && \text{(54b)}\\
& && g_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_a[k], \Delta_b[k]) = 1, \quad l = 1, \cdots, L, && \text{(54c)}
\end{aligned}
$$

where the functions $f_0$ and $f_j, j = 1, \cdots, J$, are posynomials and the functions $g_l, l = 1, \cdots, L$, are monomials in $\boldsymbol{\beta}[k]$, $\Delta_a[k]$ and $\Delta_b[k]$. In its standard form, (54) is not a convex optimization problem. However, a careful application of an appropriate logarithmic transformation of the involved variables and constants generally turns the problem (54) into one that is equivalent and convex. That is, (54) is a GP nonlinear, nonconvex optimization problem that can be transformed into a nonlinear, convex optimization problem.

We can rewrite the optimization problem (53a) in a way such that we have ratios of posynomial functions, given by

$$
\begin{aligned}
&\min \prod_{k=1}^{K} \Delta_a[k]\Delta_b[k], && \text{(55a)}\\
&\text{s. t.} \sum_{k=1}^{K} \|\boldsymbol{\beta}[k]\|^2 \le 1, && \text{(55b)}\\
&\quad \frac{p_1(\boldsymbol{\beta}[k], \Delta_a[k])}{g_1(\boldsymbol{\beta}[k], \Delta_a[k])} \le 1 \ \text{and} \\
&\quad \frac{p_2(\boldsymbol{\beta}[k], \Delta_a[k])}{g_2(\boldsymbol{\beta}[k], \Delta_a[k])} \le 1 \ \text{if } f_1(\boldsymbol{\beta}[k]) \ne f_2(\boldsymbol{\beta}[k]), && \text{(55c)}\\
&\quad \frac{p_1(\boldsymbol{\beta}[k], \Delta_a[k])}{g_1(\boldsymbol{\beta}[k], \Delta_a[k])} = 1 \ \text{if } f_1(\boldsymbol{\beta}[k]) = f_2(\boldsymbol{\beta}[k]), && \text{(55d)}\\
&\quad \frac{p_3(\boldsymbol{\beta}[k], \Delta_b[k])}{g_3(\boldsymbol{\beta}[k], \Delta_b[k])} \le 1 \ \text{and} \\
&\quad \frac{p_4(\boldsymbol{\beta}[k], \Delta_b[k])}{g_4(\boldsymbol{\beta}[k], \Delta_b[k])} \le 1 \ \text{if } f_3(\boldsymbol{\beta}[k]) \ne f_4(\boldsymbol{\beta}[k]), && \text{(55e)}\\
&\quad \frac{p_3(\boldsymbol{\beta}[k], \Delta_b[k])}{g_3(\boldsymbol{\beta}[k], \Delta_b[k])} = 1 \ \text{if } f_3(\boldsymbol{\beta}[k]) = f_4(\boldsymbol{\beta}[k]), && \text{(55f)}\\
&\quad \beta_i^2[k] \ge 0,\ i \in \{a, b, ar, br\}, && \text{(55g)}\\
&\quad \Delta_a[k] \ge 0,\ \Delta_b[k] \ge 0 \in \mathbb{R},\ \text{for } 1 \le k \le K && \text{(55h)}
\end{aligned}
$$

In our case, the constraints in (55a) contain functions that are non posynomial as a ratio of two posynomials is not a posynomial. Minimizing or upper bounding a ratio between two posynomials belongs to a non-convex class of problems known as Complementary GP [33]. One can transform a Complementary GP problem into a GP problem using series of approximations. In order to get posynomial functions, we approximate the denominator of the functions $g_l(\boldsymbol{\beta}[k], \Delta_i[k])$, $l \in \{1, 2, 3, 4\}$, $i \in \{a, b\}$, with monomials, by using *lemma 1* [37].

*Remark 4:* we should note that whenever the two functions $f_1(\boldsymbol{\beta}[k])$ and $f_2(\boldsymbol{\beta}[k])$ are identical, the constraints (55d) should contain functions that are monomial— recall that a ratio between posynomial and monomial is in general non monomial. In order to get monomial functions, we approximate both the numerator $p_1(\boldsymbol{\beta}[k], \Delta_a[k])$ and denominator $g_1(\boldsymbol{\beta}[k], \Delta_a[k])$ with monomial functions, by using *lemma 1*. Similarly, whenever the two functions $f_3(\boldsymbol{\beta}[k])$ and $f_4(\boldsymbol{\beta}[k])$ are identical, the constraints (55f) should contain functions that are monomial. In order to get monomial functions, we approximate both the numerator $p_3(\boldsymbol{\beta}[k], \Delta_b[k])$ and denominator $g_3(\boldsymbol{\beta}[k], \Delta_b[k])$ with monomial functions, by using *lemma 1*.

*Lemma 1:* Let $g_l(\boldsymbol{\beta}[k], \Delta_i[k]) = \sum_j u_j(\boldsymbol{\beta}[k], \Delta_i[k])$ be a posynomial. Then

$$
g_l(\boldsymbol{\beta}[k], \Delta_i[k]) \ge \tilde{g}_l(\boldsymbol{\beta}[k], \Delta_i[k]) = \prod_j \left( \frac{u_j(\boldsymbol{\beta}[k], \Delta_i[k])}{\gamma_j} \right)^{\gamma_j}. \quad (56)
$$

Here, $\gamma_j = u_j(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) / g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$, $\forall j$, for any fixed positive $\boldsymbol{\beta}^{(0)}[k]$ and $\Delta_i^{(0)}[k]$ then $\tilde{g}_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) = g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$, and $\tilde{g}(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$ is the best local monomial approximation to $g(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$ near $\boldsymbol{\beta}^{(0)}[k]$ and $\Delta_i^{(0)}[k]$. □

Let $\tilde{g}_l(\boldsymbol{\beta}[k], \Delta_i[k])$ be the monomial approximation of the function $g_l(\boldsymbol{\beta}[k], \Delta_i[k])$ obtained using Lemma 1. Using this monomial approximation, the ratios of posynomials involved in the constraint (55a) can be upper bounded by posynomials. The optimal solution of the problem obtained using the convex approximations

| | $(f_1(\boldsymbol{\beta}[k]))^{-1}$ | $(f_2(\boldsymbol{\beta}[k]))^{-1}$ | $(f_3(\boldsymbol{\beta}[k]))^{-1}$ | $(f_4(\boldsymbol{\beta}[k]))^{-1}$ |
|---|---|---|---|---|
| Case 1 | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}{N}$ | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\lvert h_{\mathrm{bd}}[k]\rvert^2 P_t}{N+\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}$ | $1+\frac{\beta_b^2[k]\lvert h_{\mathrm{bd}}[k]\rvert^2 P_t}{N+\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}$ |
| Case 2 | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}{N}$ | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P_t}{N+\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P_t}$ | $1+\frac{\beta_b^2[k]\lvert h_{\mathrm{bd}}[k]\rvert^2 P_t}{N+\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}+\frac{\beta_{br}^2[k]\lvert h_{\mathrm{rd}}[k]\rvert^2 P_t}{N}$ |
| Case 3 | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P_t}{N}$ | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}{N}+\frac{\beta_{ar}^2[k]\lvert h_{\mathrm{rd}}[k]\rvert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P_t}{N+\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P_t}$ | $1+\mathrm{snr}_{\mathrm{b}}[k]$ |
| Case 4 | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ar}}[k]\rvert^2 P_t}{N+\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P_t}$ | $1+\frac{\beta_a^2[k]\lvert h_{\mathrm{ad}}[k]\rvert^2 P_t}{N}+\frac{\beta_{ar}^2[k]\lvert h_{\mathrm{rd}}[k]\rvert^2 P_t}{N}$ | $1+\frac{\beta_b^2[k]\lvert h_{\mathrm{br}}[k]\rvert^2 P_t}{N}$ | $1+\mathrm{snr}_{\mathrm{b}}[k]$ |

TABLE II
USEFUL FUNCTIONS FOR THE ANALYSIS OF THE CASES DESCRIBED IN SECTION III-B.

is also optimal for the original problem, i.e., satisfies the KKT conditions of the original problem (55a), if the applied approximations satisfy the following three properties [38], [37]:

1) $g_l(\boldsymbol{\beta}[k], \Delta_i[k]) \leq \tilde{g}_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_i[k])$ for all $\boldsymbol{\beta}[k]$ and $\Delta_i[k]$ where $\tilde{g}_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_i[k])$ is the approximation of $g_l(\boldsymbol{\beta}[\mathrm{k}], \Delta_i[k])$.
2) $g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) = \tilde{g}_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$ where $\boldsymbol{\beta}^{(0)}[k]$ and $\Delta_i^{(0)}[k]$ are the optimal solution of the approximated problem in the previous iteration.
3) $\triangledown g_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k]) = \triangledown \tilde{g}_l(\boldsymbol{\beta}^{(0)}[k], \Delta_i^{(0)}[k])$, where $\triangledown g_l(\cdot)$ stands for the gradient of function $g_l(\cdot)$.

In summary, applying the aforementioned transformations, we transformed the original optimization problem (53a) first into a Complementary GP problem and then into a GP problem by applying the convex approximations (56). Finally, the obtained GP problem can be solved easily for instance using an interior point approach. More specifically, the problem of finding the appropriate $\{\boldsymbol{\beta}[k]\}_{k=1}^K$ can be solved using "Algorithm B-1" hereinafter.

**Algorithm B-1** Power allocation policy for $R_{\mathrm{sum}}^{\mathrm{OFDM}}$ as given by (28)

1: Set $\{\boldsymbol{\beta}^{(0)}[k]\}_{k=1}^K$ to an initial value. Compute $\{\Delta_a^{(0)}[k]\}_{k=1}^K$ and $\{\Delta_b^{(0)}[k]\}_{k=1}^K$ for the value of $\{\boldsymbol{\beta}^{(0)}[k]\}_{k=1}^K$ and set $\iota_1 = 1$ and $k = 1$
2: **While** $k \leq K$ **do**
3: **If** $f_1(\boldsymbol{\beta}[k]) \neq f_2(\boldsymbol{\beta}[k])$ **then**
4: Approximate $g_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{g}_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ and $g_2(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{g}_2(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_a^{(\iota_1-1)}[k]$ using (56)
5: **else**
6: Approximate $p_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{p}_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ and $g_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ with $\tilde{g}_1(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_a^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_a^{(\iota_1-1)}[k]$ using (56)
7: **end if**
8: **If** $f_3(\boldsymbol{\beta}[k]) \neq f_4(\boldsymbol{\beta}[k])$ **then**
9: Approximate $g_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{g}_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ and $g_4(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{g}_4(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_b^{(\iota_1-1)}[k]$ using (56)
10: **else**
11: Approximate $p_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{p}_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ and $g_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ with $\tilde{g}_3(\boldsymbol{\beta}^{(\iota_1)}[k], \Delta_b^{(\iota_1)}[k])$ around $\boldsymbol{\beta}^{(\iota_1-1)}[k]$ and $\Delta_b^{(\iota_1-1)}[k]$ using (56)
12: **end if**
13: Increment the subcarrier $k$ as $k = k + 1$
14: **end while**
15: Solve the resulting approximated GP problem using an interior point approach. Denote the found solutions as $\{\boldsymbol{\beta}^{(\iota_1)}[k]\}_{k=1}^K$, $\{\Delta_a^{(\iota_1)}[k]\}_{k=1}^K$, and $\{\Delta_b^{(\iota_1)}[k]\}_{k=1}^K$.
16: Increment the iteration index as $\iota_1 = \iota_1 + 1$ and go back to Step 2 using $\{\boldsymbol{\beta}[k]\}_{k=1}^K$, $\{\Delta_a[k]\}_{k=1}^K$ and $\{\Delta_b[k]\}_{k=1}^K$ of step 15
17: Terminate if $\|\boldsymbol{\beta}^{(\iota_1)}[k] - \boldsymbol{\beta}^{(\iota_1-1)}[k]\| \leq \epsilon$, for $1 \leq k \leq K$

## V. Numerical Examples

Throughout this section, we set the number of subcarriers to $K = 128$ and we model the channel coefficients as independent and randomly generated variables. The channel impulse response (CIR) is modeled as a delay line with length $L = 32$ taps. The taps are generated from i.i.d circular complex Gaussian distributions with zero mean and the variance is chosen according to the strength of the corresponding link. More specifically, the link from source **A** to the relay has a variance $\sigma_{\mathrm{ar}}^2$; that from source **B** to the relay has a variance $\sigma_{\mathrm{br}}^2$; and that from the relay to the destination has a variance $\sigma_{\mathrm{rd}}^2$. Similar assumptions and notations are used for the direct links from the sources to the destination. The channel state information (CSI) $\{h_{\mathrm{ar}}, h_{\mathrm{br}}\}$, $\{h_{\mathrm{ad}}, h_{\mathrm{bd}}\}$, and $\{h_{\mathrm{rd}}\}$ are computed by taking $K$-points Fast Fourier Transform of the CIR. Furthermore, we assume that, at every time instant, all the nodes know, or can estimate with high accuracy, the values taken by the channel coefficients at that time, i.e., full CSI. Also, we set $P_t = 30$ dBW.

In order to illustrate the theoretical analysis and the effectiveness of the OFDM transmission scheme of Proposition 2 given in (28), we compare it with the OFDMA transmission scheme of Proposition 1 given in (7).

It can be easily seen, from the theoretical analysis of OFDM and OFDMA, that the complexity of finding the optimum power values for the OFDM is larger than that for the OFDMA. In Figure 2 we observe that the maximum sum-rate for the OFDM transmission scheme can be obtained with only 180 iterations.

As discussed before, OFDMA-based scheme allows only one source to transmit on each subcarrier. On the contrary, OFDM-based scheme does not have such a restriction and thus it has higher spectral efficiency than OFDMA.

Figure 3 depicts the evolution of the sum-rate obtained using the OFDMA transmission scheme, i.e., the sum-rate $R_{\mathrm{sum}}^{\mathrm{OFDMA}}$ of Proposition 1; and the sum-rate obtained using the OFDM transmission scheme, i.e., the sum-rate $R_{\mathrm{sum}}^{\mathrm{OFDM}}$ of Proposition 2, as functions of the signal-to-noise ratio SNR= $10\log(P_t/N)$ (in decibels). Note that the curves correspond to numerical values of channel coefficients chosen such that $\sigma_{\mathrm{ar}}^2 = \sigma_{\mathrm{br}}^2 = 26$ dBW, $\sigma_{\mathrm{rd}}^2 = 20$ dBW, and $\sigma_{\mathrm{ad}}^2 = \sigma_{\mathrm{bd}}^2 = 0$ dBW. The figure also shows the rate obtained using the OFDMA transmission scheme of source **A**, i.e., $R_{\mathrm{A}}^{\mathrm{OFDMA}}$, and source **B**, i.e., $R_{\mathrm{B}}^{\mathrm{OFDMA}}$ and the rate obtained using the OFDM transmission scheme of source **A**, i.e., $R_{\mathrm{A}}^{\mathrm{OFDM}}$, and source **B**, i.e., $R_{\mathrm{B}}^{\mathrm{OFDM}}$.

For the example shown in Figure 3, we observe that the transmission scheme of Proposition 2 gives larger sum-rate than the transmission scheme of Proposition 1 at SNR values higher than 5 dB, and gives almost the same sum-rate at low SNR, between 0 and

5 dB. Also, we observe that the rate of source **A** and source **B** using the OFDM transmission scheme give larger value than the rate of source **A** and source **B** using the OFDMA transmission scheme respectively.

Figure 4 depicts the same curves for other combinations of channel coefficients, chosen such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 20$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 0$ dBW. In this case, we observe that the curves show behaviors that are generally similar to those of Figure 3. Also, note that the gap between the sum-rate of Proposition 1 and the sum-rate of Proposition 2 decreases comparing with the results of Figure 3. This is precisely due to that the sources-to-relay links chosen for the example shown in Figure 4 are of weaker quality than the sources-to-relay links chosen for the example shown in Figure 3.

Figure 5 shows similar curves for other combinations of channel coefficients, chosen that such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 20$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 26$ dBW. Note that the gap between the sum-rate of Proposition 1 and the sum-rate of Proposition 2 increases at low SNR, between 0 and 10 dB. This is precisely due to that the sources-to-destination links chosen for the example shown in Figure 5 are of better quality than the sources-to-destination links chosen for the example shown in Figure 4.

As discussed in Section IV-B, for OFDM transmission scheme it is not possible to a priori select on each subcarrier the case which outperforms the others. In order to observe which of the cases are being considered in the optimization for the OFDM transmission scheme, Figure 6 shows the probability of occurrence for the different cases given in Table I. Note that the histogram corresponds to 8000 carriers, SNR = 15 dB and numerical values of channel coefficients chosen such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 26$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 0$ dBW.

For the example shown in Figure 6, we observe that whenever the sources-to-relay links and relay-destination links are of better quality than the sources-to-destination links, it is more probable that the relay helps both sources, i.e., cases 3, 4, 6, and 7.

Figure 7 shows the same histogram for other combinations of channel coefficients, chosen such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 20$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 20$ dBW. Comparing with Figure 6, we observe that when the sources-to-relay, sources-to-destination, and relay-to-destination links have the same strength, the probability that the relay helps both sources (cases 3, 4, 6, and 7) decreases, the probability that the relay helps one source (cases 2 and 5) increases, and the probability that the relay is idle (case 1) increases.

Figure 8 shows the same histogram for other combinations of channel coefficients, chosen such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 20$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 26$ dBW. Comparing with Figure 7, we observe that when the sources-to-destination links have better quality than the other links, the probability that the relay helps both sources (cases 3, 4, 6, and 7) decreases, the probability that the relay helps one source (cases 2 and 5) increases, and the probability that the relay is idle (case 1) increases.

Figure 9 shows the rate obtained using the OFDMA transmission scheme of source **A** and source **B** on each subcarrier $k$, and the sum-rate on each subcarrier $k$. Figure 10 shows the rate obtained using the OFDM transmission scheme of source **A** and source **B** on each subcarrier $k$, and the sum-rate on each subcarrier $k$. Note that, for Figures 9 and 10, the curves corresponds to SNR = 20 dB and numerical values of channel coefficients chosen such that $\sigma_{\text{ar}}^2 = \sigma_{\text{br}}^2 = 20$ dBW, $\sigma_{\text{rd}}^2 = 20$ dBW, and $\sigma_{\text{ad}}^2 = \sigma_{\text{bd}}^2 = 7$ dBW. We observe that on average the OFDM scheme gives larger sum-rate on each subcarrier than the OFDMA scheme, and on some subcarriers, for example $k = 20$, both schemes give the same sum-rate.

## VI. Conclusion

We consider communication over a multicarrier two-source multiaccess channel in which the transmission is aided by a half-duplex relay node. We study and analyze the performance of two transmission schemes in which the relay implements decode-and-forward strategy. We propose two multicarrier transmission schemes, based respectively on OFDMA and OFDM. In the first scheme, each subcarrier can only be used by at most one source at a time. In the second scheme, each subcarrier can be used by both sources simultaneously. For both schemes, we derive the allowed sum-rate. Also, we study the problem of allocating the resources (i.e., powers and subcarriers), selecting the relay operation modes and decoding orders at the relay and destination (for OFDM transmission) optimally in a way to maximize the obtained sum-rate. For the OFDMA-based transmission, we develop a duality-based algorithm that finds a globally optimum solution. For the OFDM-based transmission, we propose an iterative coordinate-descent algorithm that finds a suboptimum solution. For both schemes, we illustrate our results through some numerical examples. In particular, our analysis shows that by allowing the sources to possibly transmit on the same subcarrier simultaneously, one can afford a larger sum-rate, i.e., the OFDM-based transmission scheme offers a substantial sum-rate gain over the one that is based on OFDMA.

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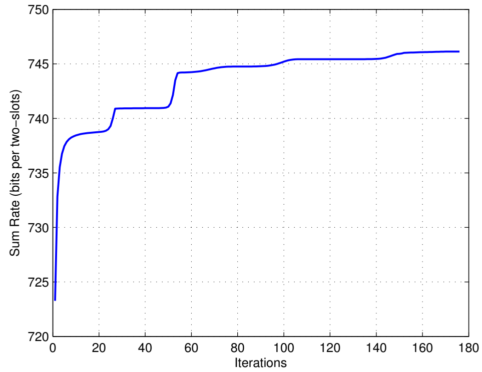


Fig. 2. Convergence of "Algorithm B". Numerical values are $K = 128$, SNR = 20 dB, $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = \sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 14$ dBW.

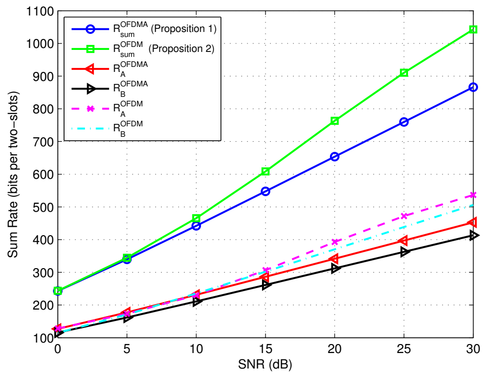


Fig. 3. Sum-rate comparison. Numerical values are $K = 128$, $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = 26$ dBW, $\sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 0$ dBW.

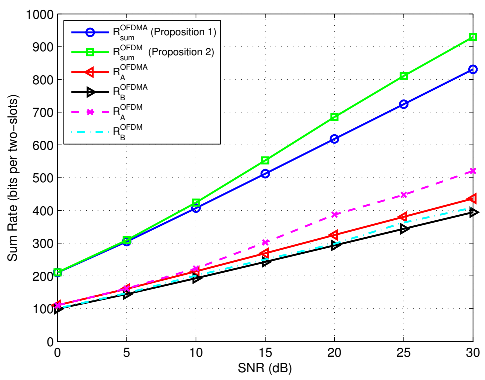


Fig. 4. Sum-rate comparison. Numerical values are $K = 128$, $\sigma^2_{\text{ar}} = \sigma^2_{\text{br}} = 20$ dBW, $\sigma^2_{\text{rd}} = 20$ dBW, and $\sigma^2_{\text{ad}} = \sigma^2_{\text{bd}} = 0$ dBW.

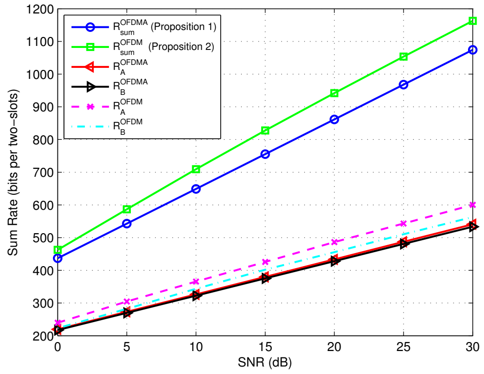


Fig. 5. Sum-rate comparison. Numerical values are $K$ = 128, $\sigma^2_{\rm ar} = \sigma^2_{\rm br}$ = 20 dBW, $\sigma^2_{\rm rd}$ = 20 dBW, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd}$ = 26 dBW.

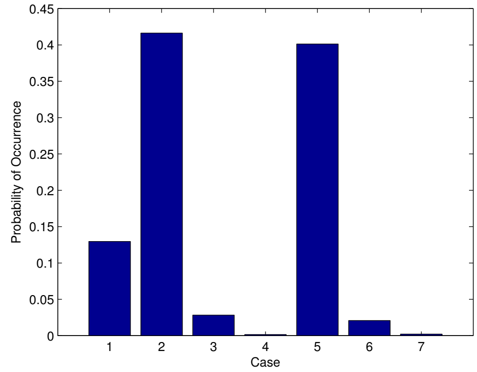


Fig. 8. Selection of relay operation modes and decoding orders at the relay and destination for the cases given in Table I. Numerical values are 8000 carriers, $SNR$ = 15 dB, $\sigma^2_{\rm ar} = \sigma^2_{\rm br}$ = 20 dBW, $\sigma^2_{\rm rd}$ = 20 dBW, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd}$ = 26 dBW.

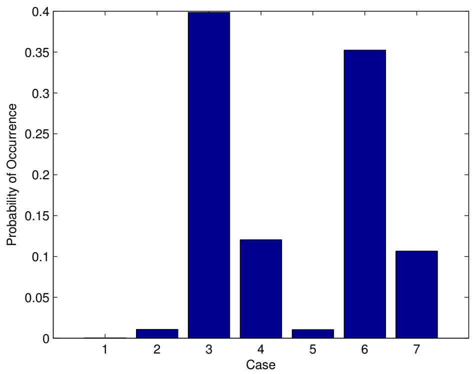


Fig. 6. Selection of relay operation modes and decoding orders at the relay and destination for the cases given in Table I. Numerical values are 8000 carriers, $SNR$ = 15 dB, $\sigma^2_{\rm ar} = \sigma^2_{\rm br}$ = 26 dBW, $\sigma^2_{\rm rd}$ = 20 dBW, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd}$ = 0 dBW.

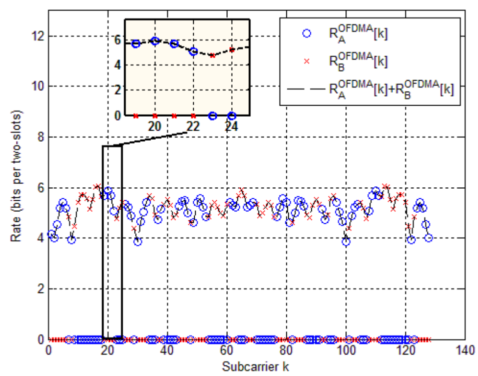


Fig. 9. Rate values per subcarrier for the OFDMA multicarrier transmission. Numerical values are $K$ = 128, $SNR$ = 20 dB, $\sigma^2_{\rm ar} = \sigma^2_{\rm br}$ = 20 dBW, $\sigma^2_{\rm rd}$ = 20 dBW, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd}$ = 7 dBW.

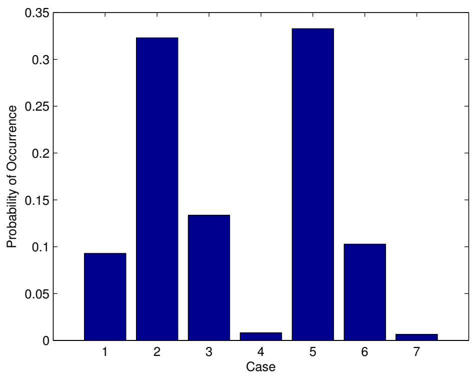


Fig. 7. Selection of relay operation modes and decoding orders at the relay and destination for the cases given in Table I. Numerical values are 8000 carriers, $SNR$ = 15 dB, $\sigma^2_{\rm ar} = \sigma^2_{\rm br}$ = 20 dBW, $\sigma^2_{\rm rd}$ = 20 dBW, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd}$ = 20 dBW.

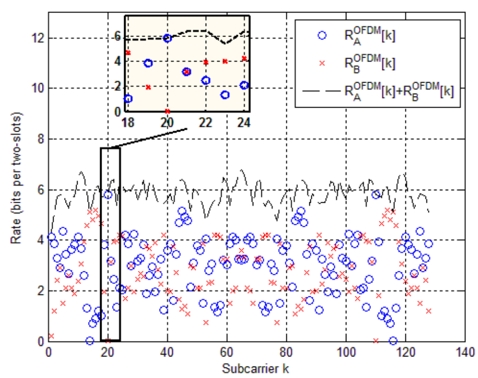


Fig. 10. Rate values per subcarrier for the OFDM multicarrier transmission. Numerical values are $K$ = 128, $SNR$ = 20 dB, $\sigma^2_{\rm ar} = \sigma^2_{\rm br}$ = 20 dBW, $\sigma^2_{\rm rd}$ = 20 dBW, and $\sigma^2_{\rm ad} = \sigma^2_{\rm bd}$ = 7 dBW.