Distributed Space-Time-Frequency Block Codes for Multiple-Access-Channel with Relaying

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***Abstract*— In this paper, we investigate diversity gain in coding for a 2-user multiple-access-channel (MAC) with cooperating transmitters–the MAC with relaying. We propose a simple distributed space-time-frequency block coding (D-STFBC) scheme and analyze the offered diversity gain. In particular, we show that full diversity (order 3) is possible if collaboration is well enough and rigorous signal processing is assumed both at the transmitters and the receiver. Bit-error-rate (BER) analysis and curves are provided for illustrative purposes.**

I. Introduction

The exploding demand for efficient high-quality and volume of digital wireless communications, along with high mobility, are driving recent developments in communication technologies for broadband wireless systems. Mitigating signal fading and co-channel interference (which are the main impairments that transmission might be subject to in real contexts) are among the main challenging problems. Diversity techniques [1] (be it spacial or due to multiple-antenna, time or frequency) play a central role in this trend. For example, it is by now of common knowledge that space-time (ST) codes can offer dramatic performance gain in multiple-input multiple-output (MIMO) systems [2]- [4]. Also, to cope with inter-symbol-interference (ISI) caused by multipath propagation, orthogonal frequency-division multiplexing (OFDM) has been utilized in conjunction with ST, space-frequency (SF) or the more involved space-time-frequency (STF) block codes [5]- [8]. More recently, cooperative diversity schemes in which multiple terminals in a network cooperate to realize spatial diversity gain in a distributed manner have been introduced [9]- [11]; and various coding techniques have been proposed (and analyzed). In particular, efficient block coding techniques based on either ST, SF or STF, have been reported [11]- [15]. It is demonstrated that, for channels with multiple relays, cooperative diversity with appropriately designed codes can offer full spatial diversity gain [13]- [15].

This work builds upon the classical 2-user multiple-access channel (MAC) with relaying, which nicely models two co-operating transmitters communicating with a common access point or next hop in a wireless network. We examine the problem of creating and exploiting additional degree of diversity gain by an appropriate design of transmitted signals (for each of the two users) in terms of space (through signalling), time (through two-fold repetition) and frequency (through circular shift in allocated OFDM subchannels (most like [16])). As a convenient by-product of this (space-time-frequency) code design third-order diversity is achieved, by creating and combining additional time/frequency degrees of freedom while maintaining full spatial (i.e., cooperative) diversity well established (by means of the well known Alamouti scheme [3]). Time diversity gain is brought up by having each user sending his information at two successive time transmit periods. Cooperative diversity is allowed by having each user sending both his own information and also other user's information (learned in the signalling period) at the second transmit period. The additional frequency diversity gain is brought up by allowing each information symbol to be carried over two (nearly) uncorrelated subchannels (i.e., carriers). We shall mention that it is the (here appropriate) combination of these three (type of) diversity mehods that achieves third-order diversity. We provide performance analysis of the proposed scheme through derivation of pair-wise error probability and illustrate the corresponding improvement through Monte-Carlo based simulations.

The remaining of this paper is organized as follows. Section II describes our system model for the 2-user MAC with relaying under consideration. Section III characterizes the performance of the proposed protocol in terms of offered diversity gain. Section IV illustrates the results and provides some comparisons from a number of perspectives, and Section V draws some concluding remarks.

Notation: All bold face letters indicate vectors (lower case) or matrices (upper case). For a vector $\mathbf{x}$, $x^{(i)}$ denotes its i^{th} element. For a matrix $\mathbf{X}$, diag($\mathbf{X}$) designates the vector formed by its diagonal elements and $X|_m^n$ designates the inner square matrix having m^{th} to n^{th} diagonal elements of $\mathbf{X}$ on its diagonal. Notations $\bar{(\cdot)}$, $(\cdot)^T$ and $(\cdot)^H$ denote conjugate, transpose, and conjugate transpose operations, respectively. We use $\mathbf{Q}$, $\mathbf{H}_{xy}$, and $\mathbf{\Lambda}_{xy} = \mathbf{Q}\mathbf{H}_{xy}\mathbf{Q}^*$ to denote the $N \times N$ normalized FFT matrix, the $N \times N$ circulant channel matrix (relative to link $X \rightarrow Y$) and the corresponding diagonal subchannel gain matrix.

II. System Model

Consider the three-terminal wireless network shown in Fig.1. Both terminals A and B send private information and also assist each other (through relaying) communicate with the destination terminal D. This forms a 2-user multiple-access channel (MAC) with relaying, which might correspond to 2

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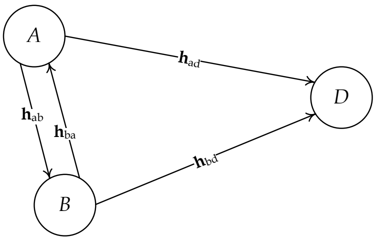

Fig. 1. Multiple-access-channel (MAC) with cooperating transmitters.

TABLE I

DESCRIPTION OF ONE PROTOCOL PERIOD WITH D-STFBC

Transmit period	Transmission	Transmitted Information	Number of Subchannels Transmit/Receive
1	$A \rightarrow B, D$	$\mathbf{x}_a$	N_a/N_b
	$B \rightarrow A, D$	$\mathbf{x}_b$	N_b/N_a
2	$A \rightarrow D$	$\mathbf{x}_a$,$\mathbf{x}_b$	$N/0$
	$B \rightarrow D$	$\mathbf{x}_a$,$\mathbf{x}_b$	$N/0$

cooperating transmitters trying to communicate with a common access point (e.g. base station). Our aim in the following is to devise (and analyze) an efficient communication protocol which allows the two transmitters to jointly communicate their information.

We divide the available time period for the whole transmission into two transmit periods and the available bandwidth into (a total of N) orthogonal subchannels (by use of OFDM). As summarized in Table I, during the first transmit period, we allocate N_a subchannels to terminal A (to transmit private information $\mathbf{x}_a$) and $N_b = N - N_a$ sub-channels to terminal B (to transmit private information $\mathbf{x}_b$). During this time, terminals A and B listen to each other's transmissions and interference is completely removed at the destination (terminal D). During the second transmit period, each of terminals A and B uses the whole available bandwidth (i.e., the N subchannels) and transmits both its own information and also the other terminal's information (which it learned from signalling over transmit period 1). For relaying, we assume each relay operating in the decode-and-forward (DF) mode. All terminals are equipped with single transmit and receive antennas and linear modulation techniques such as PSK and QAM might be used (see Section IV).

In our analysis, transmission suffers the effects of frequency selective block fading and additive noise. Let $\mathbf{h}_{xd} = [h_{xd}^{(0)}, \ldots, h_{xd}^{(L_{xd})}]$ be the channel impulse response for link $X \rightarrow D$, where $x =\in \{a, b\}$ and L_{xd} is the number of channel taps. The tap gains $\mathbf{h}_{xd}$ are assumed to be independent, zero-mean complex Gaussian with delay power profile given by $\mathbf{s}_{xd} = [\sigma_{xd}^2(1), \ldots, \sigma_{xd}^2(L_{xd})]$ (we assume without loss of generality that $\sum_{l=1}^{L_{xd}} \sigma_{xd}^2(l) = 1$).

Now, a key feature of our scheme is as follows: as distant (i.e., separate) subchannels are generally almost uncorrelated and adjacent subchannels are highly correlated (in the OFDM setting), it is possible to create additional frequency-time diversity by circularly shifting the transmitted codewords so that each symbol be carried over two different uncorrelated (or at least nearly uncorrelated) subchannels, while keeping full spatial diversity established by use of the Alamouti scheme [3]. Further details are provided below, where all derivations and input-outputs relations are related to one protocol period.

A. First transmit period

During this transmit period, terminal A transmits its information $\mathbf{x}_a$ over the first N_a subchannels and receives terminal B's information over the remaining $N_b = N - N_a$ subchannels. Terminal B acts similarly (see Table I). Let

$$\mathbf{x}_{a1} = [\mathbf{x}_a, \mathbf{0}_{1\times N_b}]^T, \tag{1a}$$

$$\mathbf{x}_{b1} = [\mathbf{0}_{1\times N_a}, \mathbf{x}_b]^T \tag{1b}$$

be the transmitted vectors, sent over one OFDM block during this transmit period. The observation vector $\mathbf{d}_1$ at the destination terminal D can be written as

$$\mathbf{d}_1 = \sqrt{E_a}\mathbf{H}_{ad}\mathbf{Q}^H\mathbf{x}_{a1} + \sqrt{E_b}\mathbf{H}_{bd}\mathbf{Q}^H\mathbf{x}_{b1} + \mathbf{n}_{d1} \tag{2}$$

where $\mathbf{n}_{d1}$ is additive white Gaussian noise (AWGN) vector with zero-mean and $N_0/2$ variance per dimension; and E_a and E_b denote the transmit symbol energies per subchannel available at terminals A and B. Hence, the FFT of the received vector at the destination terminal D is given by

$$\mathbf{r}_1 = \mathbf{Q}\mathbf{d}_1 = \sqrt{E_a}\mathbf{\Lambda}_{ad}\mathbf{x}_{a1} + \sqrt{E_b}\mathbf{\Lambda}_{bd}\mathbf{x}_{b1} + \mathbf{n}_{r1}, \tag{3}$$

where $\mathbf{n}_{r1} = \mathbf{Q}\mathbf{n}_{d1}$ is zero-mean AWGN vector with elements having $N_0/2$ variance per dimension. Emphasizing the effect of two parallel independent transmissions and making use of the structure of the transmitted vectors $\mathbf{x}_a$ and $\mathbf{x}_b$ in (1a) and (1b), we can rewrite (3) as

$$\mathbf{r}_1 = \begin{bmatrix} \sqrt{E_a}\mathbf{\Lambda}_{ad}|_{N_a}^{1} \\ \sqrt{E_b}\mathbf{\Lambda}_{bd}|_{N}^{N_a+1} \end{bmatrix} \mathbf{x} + \mathbf{n}_{r1} \tag{4}$$

where

$$\mathbf{x} = [\mathbf{x}_a, \mathbf{x}_b]^T, \tag{5}$$

carries information from both terminals A and B.

B. Second transmit period

As we indicated earlier, adjacent subchannels are highly correlated. It is precisely this high correlation which enables the use of the Alamouti scheme over adjacent subchannel pairs. During this transmit period, terminals A and B transmit vectors [1] $\mathbf{x}_{a2}$ and $\mathbf{x}_{b2}$, designed according to the Alamouti scheme as

$$\mathbf{x}_{a2} = [x^{(1)}, -\bar{x}^{(2)}, \ldots, x^{(N-1)}, -\bar{x}^{(N)}]^T, \tag{6}$$

$$\mathbf{x}_{b2} = [x^{(2)}, \bar{x}^{(1)}, \ldots, x^{(N)}, \bar{x}^{(N-1)}]^T, \tag{7}$$

where $x^(i)$, $i = 1, \cdots, N$, are elements of $\mathbf{x}$ given by (5).

From (1), (6) and (7), we see that each symbol $x^{(i)} \in \mathbf{x}$ is to be transmitted over either the same or adjacent subchannels (which are highly correlated as we already mentioned above) during first and second transmit periods. Hence, no (frequency) diversity advantage is exploited. For this same symbol $x^{(i)} \in$

[1] Recall that terminals A and B know $\mathbf{x}$ here since we assumed full decoding at the relays.

$\mathbf{x}$ (transmitted over i^{th} subchannel in transmit period 1), additional (frequency) diversity can be created and exploited by ensuring that $x^{(i)}$ is transmitted over a subchannel (say l) which is uncorrelated with subchannel i, during transmit period 2. Hence, a re-arrangement in frequency is needed and can be easily realized by circularly shifting the elements of $\mathbf{x}$ for transmission during transmit period 2. In the following, we will denote by $\boldsymbol{\Theta}$ such an operation. The gain $\lambda_{xy}^{(i)}$ of the i^{th} subchannel for link $X \rightarrow Y$ is given by

$$\lambda_{xy}^{(i)} = \sum_{k=0}^{L_{xy}} h_{xy}^{(k)} e^{-\jmath 2\pi ik/N} \tag{8}$$

and correlation $\rho_{xy}(\theta)$ between subchannels i and l can be written as

$$\rho_{xy}(\theta) = \sum_{k=0}^{L_{xy}} \sigma_{xy}^2(k) e^{j2\pi\theta k/N}, \tag{9}$$

where $\theta = i - l$. As it can be seen from (9), $\rho_{xy}(\theta)$ depends only on the channel delay power profile and the frequency separation θ. Thus, knowing the channel delay power profile, we can choose θ in such a way that $\rho_{xy}(\theta)$ is small enough, implying (near) uncorrelation between $\lambda_{xy}^{(i)}$ and $\lambda_{xy}^{(l)}$ channel gain pairs. We note here that the appropriate value of θ may be fixed for once or constantly fed-back to the transmitters A and B, based on channel measurement at the destination terminal D.

Processing signal $\mathbf{x}_2 = [\mathbf{x}_{a2}, \mathbf{x}_{b2}]^T$ as above and adopting the Alamouti scheme, observation $\mathbf{d}_2$ at the destination terminal D at the end of transmit period 2 becomes

$$\mathbf{d}_2 = \sqrt{E_a}\mathbf{H}_{ad}\mathbf{Q}^H\boldsymbol{\Theta}\mathbf{x}_{a2} + \sqrt{E_b}\mathbf{H}_{bd}\mathbf{Q}^H\boldsymbol{\Theta}\mathbf{x}_{b2} + \mathbf{n}_{d2}; \tag{10}$$

and after FFT operation is applied, we get

$$\mathbf{r}_2 = \sqrt{E_a}\boldsymbol{\Lambda}_{ad}\boldsymbol{\Theta}\mathbf{x}_{a2} + \sqrt{E_b}\boldsymbol{\Lambda}_{bd}\boldsymbol{\Theta}\mathbf{x}_{b2} + \mathbf{n}_{r2}, \tag{11}$$

where $\mathbf{n}_{r2} = \mathbf{Q}\mathbf{n}_{d2}$ is zero-mean AWGN vector with elements having $N_0/2$ variance per dimension. It is worthwhile to note that in (10) and (11), the used l^{th} subchannel for transmission of symbol $x^{(i)}$ is such that $l = 1 + (\theta + i) \bmod N$.

III. Performance Analysis

In this section, we start by analyzing different combining techniques (at the destination terminal D) of the signals sent over different subchannels and then, analyze the performance of the resulting scheme in terms of offered diversity gain. We will see that maximum diversity is exhibited here only when rigorous signal processing (based on a combination of both maximum-ratio-combining (MRC) and some kind of decision-feedback-equalisation (DFE) and referred to as *enhanced MRC, E-MRC*) is employed at the destination.

A. Combining techniques

Let $l = 1 + (\theta + i) \bmod N$. From (4) and (11), we obtain

$$r_1^{(i)} = \begin{cases} \sqrt{E_a}\lambda_{ad}^{(i)}x^{(i)} + n_1^{(i)}, & 1 \le i \le N_a \\ \sqrt{E_b}\lambda_{bd}^{(i)}x^{(i)} + n_1^{(i)}, & N_a < i \le N \end{cases} \tag{12}$$

and

$$r_2^{(l)} = \sqrt{E_a}\lambda_{ad}^{(l)}x^{(i)} + \sqrt{E_b}\lambda_{bd}^{(l)}x^{(i+1)} + n_2^{(l)}, \tag{13a}$$

$$\bar{r}_2^{(l+1)} = -\sqrt{E_a}\bar{\lambda}_{ad}^{(l+1)}\bar{x}^{(i+1)} + \sqrt{E_b}\bar{\lambda}_{bd}^{(l+1)}\bar{x}^{(i)} + n_2^{(l+1)}. \tag{13b}$$

The goal then is to appropriately combine (12), (13a) and (13b) to estimate $x^{(i)}$ and $x^{(i+1)}$. Let $y^{(i)}$ and $y^{(i+1)}$ be two intermediate decision variables in the estimation of $x^{(i)}$ and $x^{(i+1)}$, having the following (general) form

$$y^{(i)} = \alpha^{(i)}x^{(i)} + \beta^{(i)}x^{(i+1)} + \eta^{(i)}, \tag{14a}$$

$$y^{(i+1)} = \alpha^{(i+1)}x^{(i+1)} + \beta^{(i+1)}x^{(i)} + \eta^{(i+1)}, \tag{14b}$$

where $\eta^{(i)}$ and $\eta^{(i+1)}$ stand for effective noise components to be specified in the sequel. Clearly, in (14), the efficiency in estimating $x^{(i)}$ and $x^{(i+1)}$ depends on the strength of these effective noise terms, as well as on (the statistics of) coefficients $\alpha^{(k)}$, $\beta^{(k)}$, $k = i, i+1$ (as they contribute to the effective SNR acting on the decision variables). Of course, this depends on the employed combining technique (see below). At this stage, we only mention that if $\beta^{(i)} = \beta^{(i+1)} = 0$, $x^{(i)}$ and $x^{(i+1)}$ can be easily discriminated in (14). Hence, loosely speaking, non-zero coefficients $\beta^{(i)}$ and $\beta^{(i+1)}$ in (14) create some sort of inter-symbol-interference (ISI) term which can limit the efficiency of estimation. Cancellation of this ISI term will then improve global estimation efficiency. In the sequel, we discuss three combining techniques.

1) Zero-Forcing equalization (ZF D-STFBC): Assume $i \le N_a$ and let the following signals combination

$$y^{(i)} = \sqrt{E_a}\bar{\lambda}_{ad}^{(i)}r_1^{(i)} + \sqrt{E_a}\bar{\lambda}_{ad}^{(l+1)}r_2^{(l)} + \sqrt{E_b}\lambda_{bd}^{(l)}\bar{r}_2^{(l+1)}, \tag{15a}$$

$$y^{(i+1)} = \sqrt{E_a}\bar{\lambda}_{ad}^{(i+1)}r_1^{(i+1)} + \sqrt{E_b}\bar{\lambda}_{bd}^{(l+1)}r_2^{(l)} - \sqrt{E_a}\lambda_{ad}^{(l)}\bar{r}_2^{(l+1)}. \tag{15b}$$

After some calculation which is omitted for brevity, we get

$$\alpha^{(i)} = E_a|\lambda_{ad}^{(i)}|^2 + E_a\bar{\lambda}_{ad}^{(l+1)}\lambda_{ad}^{(l)} + E_b\lambda_{bd}^{(l)}\bar{\lambda}_{bd}^{(l+1)}, \tag{16a}$$

$$\alpha^{(i+1)} = E_a|\lambda_{ad}^{(i+1)}|^2 + E_a\bar{\lambda}_{ad}^{(l+1)}\lambda_{ad}^{(l)} + E_b\lambda_{bd}^{(l)}\bar{\lambda}_{bd}^{(l+1)} \tag{16b}$$

and $\beta^{(i)} = \beta^{(i+1)} = 0$. The effective noise components in (14) have then zero-means and (per-dimension) variances given by

$$\sigma_{(i)}^2 = \frac{N_0}{2}\left(E_a|\lambda_{ad}^{(i)}|^2 + E_a|\lambda_{ad}^{(l+1)}|^2 + E_b|\lambda_{bd}^{(l)}|^2\right), \tag{17a}$$

$$\sigma_{(i+1)}^2 = \frac{N_0}{2}\left(E_a|\lambda_{ad}^{(i+1)}|^2 + E_a|\lambda_{ad}^{(l)}|^2 + E_b|\lambda_{bd}^{(l+1)}|^2\right). \tag{17b}$$

2) Enhanced Maximum Ratio Combining (E-MRC D-STFBC): Again, assume $i \le N_a$. Using the maximum ratio combining (MRC) technique, we obtain (details are skipped)

$$y^{(i)} = \sqrt{E_a}\bar{\lambda}_{ad}^{(i)}r_1^{(i)} + \sqrt{E_a}\bar{\lambda}_{ad}^{(l)}r_2^{(l)} + \sqrt{E_b}\lambda_{bd}^{(l+1)}\bar{r}_2^{(l+1)} \tag{18a}$$

$$y^{(i+1)} = \sqrt{E_a}\bar{\lambda}_{ad}^{(i+1)}r_1^{(i+1)} + \sqrt{E_b}\bar{\lambda}_{bd}^{(l)}r_2^{(l)} - \sqrt{E_a}\lambda_{ad}^{(l+1)}\bar{r}_2^{(l+1)}. \tag{18b}$$

This leads to

$$\alpha^{(i)} = E_a|\lambda_{ad}^{(i)}|^2 + E_a|\lambda_{ad}^{(l)}|^2 + E_b|\lambda_{bd}^{(l+1)}|^2, \quad (19a)$$

$$\alpha^{(i+1)} = E_a|\lambda_{ad}^{(i+1)}|^2 + E_a|\lambda_{ad}^{(l+1)}|^2 + E_b|\lambda_{bd}^{(l)}|^2, \quad (19b)$$

and

$$\beta^{(i)} = \bar{\beta}^{(i+1)} = \sqrt{E_a E_b}\left(\bar{\lambda}_{ad}^{(l)}\lambda_{bd}^{(l)} - \bar{\lambda}_{ad}^{(l+1)}\lambda_{bd}^{(l+1)}\right). \quad (20)$$

The resulting noise components $\eta^{(i)}$ and $\eta^{(i+1)}$ in (14) have then zero means and (per-dimension) variances given by $\sigma_{(i)}^2 = \alpha^{(i)}\frac{N_0}{2}$ and $\sigma_{(i+1)}^2 = \alpha^{(i+1)}\frac{N_0}{2}$, where $\alpha^{(i)}$ and $\alpha^{(i+1)}$ are given by (19).

Now, observe that in comparison to the above ZF-approach, the noise terms here are more tractable (at least when we assume complex Gaussian distributed channel gains). An important shortcoming, however, comes from the fact that a steady error floor is possible here since $\beta^{(k)}$, $k = i, i+1$, in (20) are possibly non-zero (see discussion above). One way to alleviate the effect of this shortcoming is to apply a decision-feedback-equalization (DFE) technique at the destination terminal D. For instance, let $\hat{x}^{(i)}$ and $\hat{x}^{(i+1)}$ be the estimated symbols based on $y^{(i)}$ and $y^{(i+1)}$ (e.g., assuming ML decoding). Assuming perfect CSI, the destination terminal (which knows $\beta^{(i)}$ and $\beta^{(i+1)}$) can peel of the terms $\beta^{(k)}\hat{x}^{(k)}$, $k = i, i+1$, as

$$\tilde{y}^{(i)} = \alpha^{(i)}x^{(i)} + \beta^{(i)}x^{(i+1)} + \eta^{(i)} - \beta^{(i)}\hat{x}^{(i+1)} \quad (21a)$$

$$\tilde{y}^{(i+1)} = \alpha^{(i+1)}x^{(i+1)} + \bar{\beta}^{(i)}x^{(i)} + \eta^{(i+1)} - \bar{\beta}^{(i)}\hat{x}^{(i)}. \quad (21b)$$

For high SNR values, the receiver decodes perfectly (i.e., $\hat{x}^{(k)} \approx x^{(k)}$, $k = i, i+1$) and (21) becomes

$$\tilde{y}^{(i)} \sim \alpha^{(i)}x^{(i)} + \eta^{(i)} \quad (22a)$$

$$\tilde{y}^{(i+1)} \sim \alpha^{(i+1)}x^{(i+1)} + \eta^{(i+1)}, \quad (22b)$$

where the noise component $\eta^{(k)}$, $k = i, i+1$, is the same as above.

B. Diversity Analysis

We now turn to evaluate the diversity gain offered by the proposed scheme, through derivation of the corresponding pairwise error probability (PEP).

Let $\mathbf{e}$ denote a falsely decoded codeword. Then, assuming ML decoding with perfect CSI, the conditional PEP can be bounded as,

$$P(x^{(i)}, e^{(i)}|\mathrm{CSI}) \leq \exp\left(-\frac{d^2(x^{(i)}, e^{(i)})}{4N_0}\right), \quad (23)$$

where $d^2(x^{(i)}, e^{(i)})$ stands for Euclidean distance given by

$$d^2(x^{(i)}, e^{(i)}) = \alpha^{(i)}|x^{(i)} - e^{(i)}|^2. \quad (24)$$

Now, observe that $\lambda_{ad}^{(i)}$ and $\lambda_{ad}^{(l)}$ (in $\alpha^{(k)}$, $k = i, i+1$, in (23)) are (nearly) statistically uncorrelated. Hence, averaging (23) assuming independently Rayleigh distributed $|\lambda_{ad}^{(i)}|$, $|\lambda_{ad}^{(l)}|$ and $|\lambda_{bd}^{(l+1)}|$, we obtain the PEP expression for uncoded E-MRC D-STFBC

$$P_e(x^{(i)}, e^{(i)}) \leq \left(\frac{E_a}{4N_0}\delta_i^2 + 1\right)^{-2}\left(\frac{E_b}{4N_0}\delta_i^2 + 1\right)^{-1}, \quad (25)$$

where $\delta_i = |x^{(i)} - e^{(i)}|$. From (25), we clearly see that our uncoded E-MRC D-STFBC exhibits diversity order 3 if we assume power control such that $E_a = E_b$. Illustrative numerical results together with some comparisons and discussions are provided in Section IV.

IV. Numerical Results and Discussion

In this section, we concentrate on BER evaluation obtained with Monte-Carlo based simulations. However, we mention that while the tools described in this paper are powerful enough to consider general rate pairs (R_1, R_2) for terminals A and B of the MAC with relaying, we consider here the equal rate point, i.e., $R_1 = R_2 = R$, for purposes of exposition. The aim is to illustrate the offered diversity gain and also to emphasize the effects of delay power profile and to compare the above stated combining techniques.

A. Simulations set-up

We assume a quasi-static block Rayleigh fading channel and utilize 4-PSK modulation over an OFDM system with $N = 128$ subchannels. Moreover, we choose $L_{ad} = L_{bd} = 6$ and let $N_a = N_b = N/2$. In the sequel, we focus on transmission over two different channels (both being frequency selective channels with complex Gaussian distributed tap gains): the first, referred to as *Channel* 1 hereafter, has equal tap powers $\sigma_1^2(0) = \cdots = \sigma_1^2(5)$; and the second, referred to as *Channel* 2 hereafter, has exponentially decaying tap powers, i.e. $\sigma_2^2(k) = \varrho_k \exp(-k)$, $k = 0, \cdots, 5$. For comparison reasons, we fix $\sum_{l=0}^{5}\sigma_j^2(l) = 1$, $j = 1, 2$.

B. Simulation results and discussion

Fig.2 shows BER as function of average SNR per subchannel. Performance of both combining techniques discussed above (i.e., ZF-approach and E-MRC) are displayed, demonstrating (through the steeper slopes in Fig.2) that third-order diversity is achieved. Meanwhile, the curves also show the improvement brought by the circular shift (obtained here with the choice $\theta = N/2$ for *Channel* 1) in creating additional frequency diversity gain. Also, as expected, the curves illustrate the superiority of E-MRC technique (over the ZF-approach).

Fig.3 illustrates the effect of a residual (unavoidable) correlation in (9)–i.e., when the selected circular shift θ does not permit to nearly uncorrelate the different subchannels as it is required by our system design. Of course, as it is can be seen from (9), the degree by which circular shift meets the target uncorrelation depends of the considered channel delay power profile. Fig.3 shows that when this seeked uncorrelation is met only partially (e.g., for *Channel* 2 here as it is visible from Fig.4) this results in some BER degradation (but third-order diversity is still achieved).

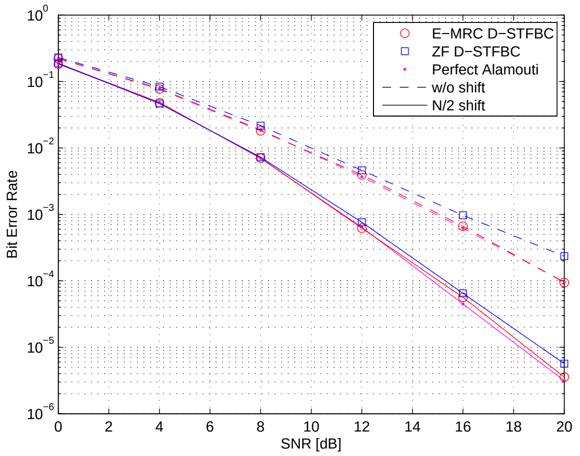

Fig. 2. Effect of the circular shift, $\theta = N/2$, over Channel 1.

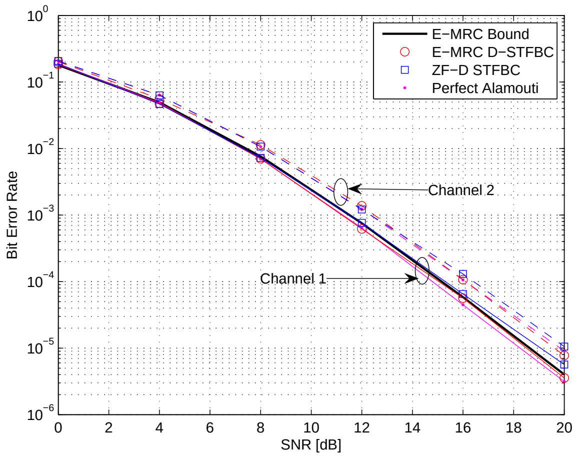

Fig. 3. Performance of the proposed scheme over different channels with $\theta = N/2$.

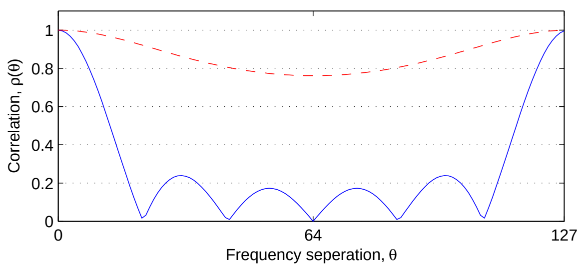

Fig. 4. Correlation $\rho_{xy}(\theta)$ given by (9) for *Channel* 1 (solid) and *Channel* 2 (dashed).

V. Conclusion Remarks

In this paper we proposed a simple space-time-frequency coding scheme for transmission over a 2-user multiple-access-channel (MAC) with cooperating transmitters. This scheme exhibits time-frequency diversity by carefully exploiting the correlation features of the channel (in the OFDM setting), in addition to full spatial (i.e., cooperative) diversity established by use of the well known Alamouti scheme. The appropriate combination of these cooperative-diversity, time-diversity and frequency-diversity methods yields third-order diversity and shows that transmission over the MAC with relaying with sufficiently low error rates is possible.

Furthermore, though intentionally kept in its simple form for illustration purposes (especially, the assumption of equal rate point), code design for the MAC with relaying here could gain in performance by further exploiting resource allocation (e.g., available powers, allocated subchannels) or precoding techniques and is likely to be applied in various practical situations where different transmitters cooperate to send information to the same destination.

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