Lower Bounds on the Capacity Regions of the Relay Channel and the Cooperative Relay-Broadcast Channel with Non-causal Side Information.

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***Abstract*— In this work, coding for the relay channel and the cooperative relay broadcast channel controlled by random parameters are studied. In the first channel, the relay channel (RC), information is transferred from the transmitter to the receiver through a multiplicity of nodes which all ”simply” act as relays. In the second channel, the cooperative relay broadcast channel (RBC), each intermediate node also acts as a receiver, i.e., it decodes a ”private message”. For each of these two channels, we consider the situation where side information (SI) on the random parameters is non-causally provided to the transmitter and all the intermediate nodes but not the final receiver, and derive an achievable rate region based on the relays using the decode-and-forward scheme. In the special case where the channels are degraded Gaussian and the side information (SI) is additive i.i.d. Gaussian, we show that 1) the rate regions are tight and provide the corresponding capacity regions and 2) the state S^n does not affect these capacity regions, even though the final receiver has no knowledge of the state. For the degraded Gaussian RC, the results in this paper can be seen as an extension of those by Kim *et al.* to the case of more than one relay.**

I. Introduction

Channels that depend on random parameters received considerable attention over the last years, due to a wide range of possible applications. Two examples of such channels are the broadcast channel (BC) with random parameters [1], [2] and the multiple access channel (MAC) with random parameters [3]. Two key points in the study of such channels are: 1) whether the parameters controlling the channel (also called state or side-information (SI)) are known causally or non-causally and 2) whether they are known to all, only some, or none of the nodes (including the transmitter and the receiver). As pointed out in [2], problems with SI known either at the receiver or at the transmitter and the receiver can be handled using (variants of) standard state-independent coding techniques.

In this paper, we consider two other channels: 1) The T-node relay channel (RC) with state information (SI) non causally known to the transmitter and the relays but not to the destination and, 2) the T-node cooperative relay broadcast channel (RBC) with SI known to the transmitter and all the intermediate receivers (which also act as relays) but not to the final receiver. Relaying and cooperative relaying of information model problems where one or more relays help each other communicate. This may occur in a variety of applications such as a multihop wireless network and a sensor network. In a multi-hop wireless network, each mobile node operates not only as a host but also as a *router*, forwarding information for other mobile nodes in the network that may not be within direct wireless transmission range. In a sensor network, due to limited power, nodes with sufficient available power assist other nodes transfer information. The SI may be any information (about the channel) that the transmitter and all the intermediate nodes know but not the final receiver. For instance, this may model any (measured/genie-provided) information about channel coefficients in fading transmissions. Another example is the case where the transmitter and the intermediate nodes know some interfering signal that the final receiver (located far away from this interfering source) is not aware about.

For the T-node RC with SI non-causally available at the transmitter and the relays but not the destination, we first derive an achievable rate region based on a combination of sliding-window decoding [4]–[6] and Gelfand and Pinsker's binning [7]. Then, we specialize this result to the case where the channel is degraded Gaussian and the state is additive i.i.d. Gaussian.[1] In this case, we show that the presence of the state does not affect the channel capacity. This result may be viewed as an extension of previous work by Kim *et. al.* [8, Theorem 3] to the case of more than one relay.

For the T-node cooperative RBC with SI non-causally available at the transmitter and all the intermediate receivers but not the final receiver, we first consider the case where the receivers partially cooperate and derive an achievable rate region based on a combination of window-sliding [4], [5], superposition-coding [9] and Gelfand and Pinsker's binning [7]. In the Gaussian case, capacity region is provided for the degraded-AWGN Cooperative RBC with additive i.i.d. Gaussian state, in both situations of partially cooperating and fully cooperating receivers. Again, we show that the state does not affect the capacity region, meaning that the channel has the same capacity region as if the state were zero or were

[1] An additive state may model any additive interference which is non-causally known to the transmitter and the relays but not the destination.

non-zero but known also to the final receiver.

The remaining of this paper is organized as follows. In section II, we derive an achievable rate region for the T-node RC with non-causal SI available at the transmitter and the relays, but not the destination. Section III specializes to the T-node degraded Gaussian RC with non-causal i.i.d. additive Gaussian SI. In section IV, we derive an achievable rate region for the partially cooperative RBC with non-causal SI available at the transmitter and all but the final receiver. Section V specializes the results of Section IV to the degraded-AWGN partially/fully cooperative T-node RBC with non-causal additive i.i.d. Gaussian SI. Final concluding remarks are stated in section VI.

II. Achievable rate region for the T-node relay channel with non-causal state

Consider the network model depicted in Fig.1. The T-node relay network has a source terminal (node 1), $T-2$ relays denoted sequentially as $2, 3, \cdots, T-1$ in arbitrary order and a destination terminal (node T). Assume each node $i \in \{1, 2, \cdots, T-1\}$ sends $X_i(t)$ at time t, and each node $k \in \{2, 3, \cdots, T\}$ receives $Y_k(t)$ at time t, where the finite sets $\mathscr{X}_i$ and $\mathscr{Y}_k$ are the corresponding output and input alphabets for the corresponding nodes, respectively. The destination terminal has input $Y \triangleq Y_T$. The state S is assumed to be random, taking values in a finite set $\mathscr{S}$ and non-causally known to all nodes but the destination. The source output $X_1(t)$, $t \leq n$, is function of the message $W \in \mathscr{W}$ and the state sequence $S^n \in \mathscr{S}^n$, and the $X_i(t)$, $i \neq 1$, are functions of node i's past inputs $Y_i^{t-1} \triangleq Y_i(1), Y_i(2), \cdots, Y_i(t-1)$ and the state sequence $S^n \in \mathscr{S}^n$. The channel is supposed to be memoryless and is described by the following conditional probability mass function:

$$p(y_2, y_3, \cdots, y_{T-1}, y | x_1, x_2, \cdots, x_{T-1}, s) \qquad (1)$$

for all $s^n \in \mathscr{S}^n$, all

$$(x_1, \cdots, x_{T-1}) \in \mathscr{X}_1 \times \cdots \times \mathscr{X}_{T-1}$$

and all

$$(y_2, \cdots, y_{T-1}, y) \in \mathscr{Y}_2 \times \cdots \times \mathscr{Y}_{T-1} \times \mathscr{Y}.$$

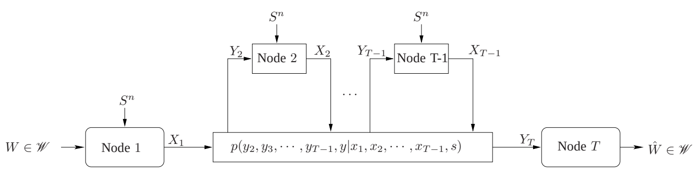

Fig. 1. Relay network with SI non-causally known at the source terminal and all the intermediate terminals (relays) but not the destination.

In the following section, we investigate the impact of the SI S^n on the reliability and the throughputs of the wireless network. We provide an achievable rate region for the T-node RC with non-causal SI. For the single-node RC, this rate region is contained in the one recently proposed in [8, Lemma 3] for general RC (see Remark 1 below), and is equally optimal for the special case of degraded Gaussian RC (see Remark 2 below). Results are extended to the case of multiple relay nodes in Section II-B.

A. One relay

Assume the one-step problem in which there is only one relay node, i.e., $T = 3$. We have the following result:

Theorem 2.1: (Inner bound on the capacity of one-node RC with state) For a discrete memoryless one-node relay channel $p(y_2, y|x_1, x_2, s)$ with state information S^n non-causally available at the transmitter and the relay but not the destination, the following rate is achievable:

$$R = \max_{p(u_1,u_2,x_1,x_2|s)} \min \Big\{ I(U_1;Y_2|SU_2), I(U_1U_2;Y) - I(U_1U_2;S) \Big\}, \quad (2)$$

where the maximum is over all auxiliary random variables U_1 and U_2 with finite cardinality bounds and all joint distributions of the form $p(s)p(u_1, u_2, x_1, x_2|s)p(y_2|x_1, x_2, s)p(y|y_2, x_2)$.

Proof: The proof is based on a random code construction which combines sliding-window decoding [4], [5] and Gelfand and Pinsker's binning [7]. Similar proofs based (only) on sliding-window have already appeared in [6], [10]. Here, binning is added to take into account the state. For brevity, we only outline the random code construction and the encoding. The decoding and analysis of probability of error are similar to [6], [10]. Fix $\gamma > 0$. Let

$$\begin{aligned} J_1 &\triangleq \exp(nI(U_1;S|U_2) + n\gamma) \\ J_2 &\triangleq \exp(nI(U_2;S) + n\gamma) \\ M_1 &\triangleq \exp(nI(U_1;Y_2|SU_2) - 2n\gamma) \\ M_2 &\triangleq \exp(nI(U_1U_2;Y) - nI(U_1U_2;S) - 2n\gamma) \\ M &\triangleq \min(M_1, M_2). \end{aligned} \qquad (3)$$

We consider transmission over B blocks each with length n. At each of the first $B-1$ blocks, a message W_i is sent, where i denotes the index of the block. We generate two statistically independent codebooks (codebooks 1 and 2) to be used for blocks with odd and even indices, respectively, in similar way to [6], [10]. Then, we generate an auxiliary collection $\mathbf{a}$ of i.i.d. u_2^n-vectors $\mathbf{a} = \{u_2^n(j_2, w'), \quad j_2 \in \{1, 2, \cdots, J_2\}, w' \in \{1, 2, \cdots, M\}\}$. For each $u_2^n \triangleq u_2^n(j_2, w')$, we generate a collection of u_1^n-vectors $\mathbf{b}(u_2^n = \{u_{1\,j_1,w}^n(u_2^n), j_1 \in \{1, \cdots, J_1\}, w \in \{1, 2, \cdots, M\}\}$ with appropriate distribution. At the beginning of block i, if w_i is to be transmitted and w_{i-1} is the message being sent in previous block $i-1$, we select u_2^n and u_1^n such that both $(u_2^n(j_2, w_{i-1}), s^n)$ and $(u_{1\,j_1,w_i}^n(u_2^n), u_2^n, s^n)$ are typical (see [11] for definition). Then, the relay sends $x_2^n = x_2^n(s^n, w_{i-1})$ such that the tuple (x_2^n, u_2^n, s^n) is jointly typical and the source sends $x_1^n = x_1^n(s^n, w_{i-1}, w_i)$ such that the tuple

$(x_2^n, u_1^n(u_2^n), s^n)$ is jointly typical. Decoding uses the same techniques as in [6], [10] and it can be shown that all the error events have small probabilities for sufficiently large n. ■

Remark 1: In [8, Lemma 3], Kim *et. al* provided an achievable rate region for the case of just one relay. This rate region uses $I(U_1; Y_2|SX_2)$ instead of $I(U_1; Y_2|SU_2)$ in (2) and is hence generally larger (than (2)). To see that $I(U_1; Y_2|SX_2) \geq I(U_1; Y_2|SU_2)$, observe that

$$\begin{aligned} I(U_1U_2SX_2; Y_2) &= I(X_2S; Y_2) + I(U_1; Y_2|SX_2) + I(U_2; Y_2|SX_2U_1) \\ &= I(U_2S; Y_2) + I(U_1; Y_2|SU_2) + I(X_2; Y_2|SU_1U_2). \end{aligned}$$

Then, note that $I(X_2; Y_2|SU_1U_2) = 0$ (since $p_{X_2|U_2S} = 0, 1$) and $I(U_2; Y_2|SX_2U_1) = 0$ (since $(U_1, U_2) \ominus (X_1, X_2, S) \ominus (Y_2, Y)$ forms a Markov chain under the specified distribution in (2.1)). Finally, use $I(U_2S; Y_2) \geq I(X_2S; Y_2)$ to get $I(U_1; Y_2|SU_2) \leq I(U_1; Y_2|SX_2)$.

Remark 2: In the Gaussian case, X_2 is a linear combination of U_2 and S (see [12]) and hence, $I(U_1; Y_2|SX_2) = (U_1; Y_2|SU_2)$. So, for the special case of one-node Gaussian RC, the rate region (2) is equal to the one in [8, Lemma 3].

B. Multiple relays

We now consider the problem of multiple-relay channel with SI non-causally available at the transmitter and all the relays but not the destination. The result in Theorem 2.1 straightforwardly extends to the case of T nodes. Let $\pi(\cdot)$ be a permutation on $1, 2, \cdots, T$ with $\pi(1) = 1$ and $\pi(T) = T$, and let $\pi(i : j) = \{\pi(i), \pi(i+1), \cdots, \pi(j)\}$.

Theorem 2.2: (Inner bound on the capacity of the T-node RC with state) For a discrete memoryless T-node relay channel $p(y_{2:T-1}, y|x_{1:T-1}, s)$ with state information S^n non-causally available at the transmitter, all the $T-2$ relays but not the destination, the following rate is achievable:

$$\begin{aligned} R = \max_{\pi(\cdot)} \max_{p(\cdot|\cdot)} \min_{1\leq t\leq T-2} \Big\{ & I(U_{\pi(1:t)}; Y_{\pi(t+1)}|SU_{\pi(t+1:T-1)}), \\ & I(U_{\pi(1:T-1)}; Y) - I(U_{\pi(1:T-1)}; S) \Big\} \end{aligned} \tag{5}$$

where the second maximization is over all auxiliary random variables $U_1, \cdots, U_{T-1}$ with finite cardinality bounds and all joint distributions satisfying $p(y_{\pi(t)}|x_{\pi(1:T-1)}, y_{\pi(2:t-1)}, s) = p(y_{\pi(t)}|x_{\pi(t-1)}, x_{\pi(t)}, y_{\pi(t-1)}, s)$ for $t = 2, \cdots, T-1$ and $p(y|x_{\pi(1:T-1)}, y_{\pi(2:T-1)}, s) = p(y|y_{\pi(T-1)}, x_{\pi(T-1)})$.

Proof: In general, permutations maximize the rate, for this can be viewed as a tacit search for the correct coding order. For fixed permutation $\pi(\cdot)$, the proof corresponds to a straightforward generalization of that in Section II-A to the case of $T-2$ relays. ■

Remark 3: Theorem 2.2 has an intuitive interpretation as for the impact of the state S^n on the different throughputs: From the point of view of communicating to relays 2 through $T-1$, the additive state S^n (e.g., an interfering source) has no effect, since this state is known to the transmitter and all the relays (thus, conditioning on S in the first term in the RHS of (5)). Now, from the point of view of communicating to the final destination which does not know the state S^n, cooperation between the source terminal and the $T-2$ relays transforms the original T-node relay channel into a fictitious channel with SI non-causally known to the fictitious transmitter–the $T-1$ auxiliary inputs $U_1, \cdots, U_{T-1}$, but not to the receiver Y.

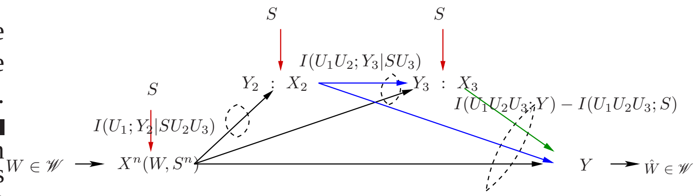

Fig. 2. The information transfer for two relays, using regular encoding/sliding window decoding combined with binning.

Example 1: For a four-node RC with SI non-causally known everywhere, but not to the destination, Theorem 2.2 shows that the rate

$$\begin{aligned} R = \max\min \Big\{ & I(U_1; Y_2|SU_2U_3), I(U_1U_2; Y_3|SU_3) \\ & I(U_1U_2U_3; Y) - I(U_1U_2U_3; S) \Big\} \end{aligned} \tag{6}$$

is achievable. A diagram of information transfer is depicted in Fig.2 where the incoming edges are labeled by the mutual information expressions in (6).

III. Capacity region of degraded Gaussian $T-$node relay channel with additive state

In this section, we prove explicitly the capacity of the multiple relay degraded Gaussian channel with additive SI non-causally known to the transmitter and all the intermediate relays, but not the destination. It turns out that the achievable rate (5) is the capacity of the degraded multi-relay channel with non-causal state, which is attained with an appropriate choice of the input distribution. In Section III-B, we use an inductive argument to determine the capacity region. This result can be viewed as an extension of the work [8] to the case of more than one relay.

A. Channel model

Consider the channel depicted in Fig.1. We now assume that the signal received at node k, $2 \leq k \leq T$, is corrupted by an i.i.d. Gaussian noise $Z_k \sim \mathcal{N}(0, N_k)$, resulting from the accumulation of the noise at the different beforehand stages. We also assume that the channel is physically degraded, meaning that there exist independently generated Gaussian random variables $Z'_k \sim \mathcal{N}(0, N'_k)$ such that

$$\begin{aligned} y_{k,i} &= y_{k-1,i} + x_{k,i} + z'_{k,i}, \quad 3 \leq k \leq T \\ y_{2,i} &= x_{1,i} + s_i + z_{2,i}, \end{aligned} \tag{7}$$

where $z_k(t) = z_{k-1}(t) + z'_k(t)$. Let the transmitter has power by P_1 and relay k, $k = 2, \cdots, T-1$, has power P_k. We make the additional assumption that the state S^n is additive Gaussian and is independent of $(Z_2, \cdots, Z_T)$. The goal is to evaluate the capacity of this channel for any given set of $P_1, \cdots, P_{T-1}$ and $N_2, \cdots, N_T$.

B. Multiple relays

For a specified choice of $\beta_{i,j}$ with $1 \leq i \leq j \leq T-1$ satisfying

$$\sum_{j=i}^{T-1} \beta_{i,j} = 1, \ \forall \ 1 \leq i \leq T-1, \tag{8}$$

and for $k \in \{1, \cdots, T-1\}$, define

$$R_k(\underline{\beta}) = C\left(\frac{\sum_{j=1}^{k}\left(\sum_{i=1}^{j}\sqrt{\beta_{i,j}P_i}\right)^2}{N_{k+1}}\right), \tag{9}$$

where we use $\underline{\beta}$ as a shorthand for $\{\beta_{i,j}\}_{1\leq i\leq j\leq T-1}$ and $C(x) \triangleq 0.5\log(1+x)$.

When there is no additive state, the physically degraded T-node RC has capacity given by [13]

$$C_T = \max_{\{\beta_{i,j}\}} \ \min \ \left\{R_1(\underline{\beta}), \cdots, R_{T-1}(\underline{\beta})\right\}. \tag{10}$$

When the state S^n is available everywhere—at the transmitter, the relays 2 through $T-1$ and the receiver, these nodes can simply subtract S^n to reduce the channel to the case without additive state and attain the same region as in (10).

Now, we turn to the case where only the receiver does not know the state S^n. Here is the main result of this section.

Theorem 3.1: (Capacity of the T-node degraded Gaussian RC with non-causal state) The capacity of the T-node degraded Gaussian relay channel (7) with state information non-causally available at the transmitter and the relays but not the destination is given by the standard capacity (10).

Note that Theorem 3.1 means that in degraded Gaussian relay networks, an additive Gaussian interference non-causally known to all nodes but the destination has no impact on the capacity of this network. Thus, it suffices for the network to know the interference at the transmitter and the relays (but not the destination) to cancel its effect. In this case, capacity (10) is attained with an appropriate choice of auxiliary random variables $U_1, U_2, \cdots, U_{T-1}$ in the achievable rate (5) (see the proof).

Proof: (Proof of Theorem 3.1) Proceeding similarly to Costa's approach [12], we need only prove the achievability of the region. We prove this achievability by induction. We use [8, Theorem 3] as the initial step in the induction. For the induction step, assume that the theorem holds for a T-node degraded GRC with state (i.e., $T-2$ relays). Fix some appropriate choice of $\{\beta_{i,j}\}$. Let $C_T(\underline{\beta})$ be the rate achievable in this channel and with this choice of $\beta_{i,j}$'s and assume that the rate is achievable using a codebook choice $\widetilde{\mathscr{C}}(T)$ such that the output of transmitter k is given by a random variable $\widetilde{X}_k \sim \mathcal{N}(0, P_k)$ and that the appropriate choice of the auxiliary random variable is $\widetilde{U}_k$ (U_k in Theorem 2).

Now consider adding another relay (node $T+1$) at the end of the last stage. One way to do this is to turn the final receiver (node T) into a relay, provide it with the state S^n and add a new receiver (node $T+1$) which does not know the state S^n, after this relay. We shall show that there exists a good choice of a codebook $\mathscr{C}(T+1)$ such that the theorem also holds for the newly formed $T+1$-node GRC with state. For instance, we will provide expressions for the optimal output X_T of node T and the corresponding optimal auxiliary random variable U_T.

We consider B blocks, each of n symbols. A sequence of $B-T+1$ messages $w_i \in \{1, \cdots, 2^{nR}\}$, $i = 1, \cdots, B-T+1$, will be sent over the channel in nB transmissions. Similarly to the approach in [14] and [13], we assume that at time t_i– the beginning of transmission block i, relay k has successfully decoded messages $w_1, w_2, \cdots, w_{i-k+1}$ (in particular, at time t_i, all relays up to and including relay T have successfully decoded message w_{i-T+1}). This assumption should be thought of as part of the induction hypothesis. Following the approach in [14], each node k, $k \in \{1, \cdots, T-1\}$, allocates a part $\beta_{k,k}P_k$ of its power to assist node T transmit, by sending *refinement information* on top of the information that node k would have transmitted if there were no added relay. Let $X_T \sim \mathcal{N}(0, \beta_{T,T}P_T)$ be a random codebook to be used to assist node T transmit. At time t_i , node k sends

$$X_k = \sqrt{1-\beta_{k,T}}\widetilde{X}_k + \sqrt{\frac{\beta_{k,T}P_k}{\beta_{T,T}P_T}}X_T. \tag{11}$$

Thus, from the point of view of communicating to the newly added relay T, the ensemble formed by the transmitter and all nodes k, $k = 2, \cdots, T-1$, can be viewed as a single fictitious node which knows S^n and which , at time t_i, sends $\bar{X}^{T-1} = \bar{X}_1^{T-1} + \bar{X}_2^{T-1}$, where

$$\bar{X}_1^{T-1} = \sum_{k=1}^{T-1} \sqrt{1-\beta_{k,T}}\widetilde{X}_k, \tag{12a}$$

$$\bar{X}_2^{T-1} = (\sum_{k=1}^{T-1}\sqrt{\beta_{k,T}P_k})\frac{X_T}{\sqrt{\beta_{T,T}P_T}}. \tag{12b}$$

Now, in the T-node channel formed by all nodes but the final receiver, the relays 2 through T can successfully remove the contribution from X_T to the received signal (since they are all assumed to have successfully decoded message w_{i-T+1} at time t_i). Thus, the rate $C_T(\underline{\beta})$ as defined by (10) is achievable. Then, since relay T also knows w_{i-T+1} at time t_i, the channel to the final receiver can be viewed as a fictitious two-user multiple access channel (MAC) with two independent inputs — the information $\bar{X}_1^{T-1}$ with power $\bar{P}_1^{T-1} = \sum_{j=1}^{T-1}\left(\sum_{i=1}^{j}\sqrt{\beta_{i,j}P_i}\right)^2$ and the cooperative information $\bar{X}_2^{T-1} + X_T$ with power $\bar{P}_2^{T} = (\sum_{i=1}^{T}\sqrt{\beta_{i,T}P_i})^2$, i.e.,

$$Y = Y_{T+1} = \bar{X}_1^{T-1} + (\bar{X}_2^{T-1} + X_T) + S + Z_{T+1}. \tag{13}$$

The SI S^n is non-causally known to the two users, but not the receiver. Using [8, Theorem 2], optimal inputs for this channel can be generated as

$$U^{T-1} \sim \mathcal{N}(\alpha^{T-1}S, \bar{P}_1^{T-1}), \quad \bar{X}_1^{T-1} = U^{T-1} - \alpha^{T-1}S, \tag{14a}$$

$$U_T \sim \mathcal{N}(\alpha_T S, \bar{P}_2^T), \quad X_T = \sqrt{\beta_{T,T}\frac{P_T}{\bar{P}_2^T}}(U_T - \alpha_T S), \tag{14b}$$

$$\alpha^{T-1} = \frac{\bar{P}_1^{T-1}}{\bar{P}_1^{T-1} + \bar{P}_2^T + N_{T+1}}, \; \alpha_T = \frac{\bar{P}_2^T}{\bar{P}_1^{T-1} + \bar{P}_2^T + N_{T+1}}. \tag{14c}$$

This allows to attain the MAC sum rate

$$\begin{aligned} R_{\text{MAC}}^{\text{sum}} &= C\Big(\frac{\bar{P}_1^{T-1} + \bar{P}_2^T}{N_{T+1}}\Big) \\ &= C\Big(\frac{\sum_{j=1}^T \big(\sum_{i=1}^j \sqrt{\beta_{i,j}P_i}\big)^2}{N_{T+1}}\Big) \\ &\triangleq R_T(\underline{\beta}). \end{aligned} \tag{15}$$

Finally, the rate

$$\min\ (C_T(\underline{\beta}), R_T(\underline{\beta})) = \min\ \{R_1(\underline{\beta}), \cdots, R_T(\underline{\beta})\} \triangleq C_{T+1}(\underline{\beta})$$

is achievable, since we can communicate reliably at this rate to all receivers. This completes the proof of achievability. For the converse, note that one can do no better since this is the capacity of the degraded channel with no state at all. ■

Remark 4: In the proof of Theorem 3.1, degradedness is only needed for the converse. If the channel is Gaussian but not degraded a lower bound on capacity can be obtained by evaluating the region (5) with the choice of U_i's given by (14).

Remark 5: The question of the impact of the state on the throughputs when the channel is only Gaussian (not necessarily degraded) is trickier to be dealt with. While the rate obtained with the Gaussian codebooks (14) (see Remark 4) is independent of the state, this is only an achievable region and one can not claim that the state has no effect (though one is tempted to), since capacity of the Gaussian RC with no state is not known yet.

IV. Achievable rate region for the Partially Cooperative RBC with non-causal state

We now turn to the cooperative RBC. In the case when there is no state, such channel has previously been considered by others, most notably by Liang and Veeravalli [10], but also by Kramer et. al. [6] and Reznik et. al. [15]. The aim there was to show that the original capacity of the BC is enlarged due to relaying and user cooperation. Here, we consider the same channel with, this time, SI non-causally known to all nodes but the final receiver. Fig.3 illustrates the setup for a T-node cooperative RBC with nodes 2 through $T-1$ acting not only as relays but also as receivers (of private messages). We assume that message W_i, $i = 0, 2, \cdots, T$ is transmitted at rate R_i (message W_0 is common and message W_i, $i \neq 0$, is dedicated to receiver i). Each node k , $k = 2, \cdots, T-2$, receives $Y_k(t)$ at time t and tries to decode the pair $(W_k, W_0) \in \mathcal{W}_k \times \mathcal{W}_0$. We use the following definitions (introduced for the first time in [10]): The channel is *partially* cooperative if every node k assists only those nodes that are "further away" decode their messages. The channel is *fully* cooperative if, in addition, node k assists node j, where $j = 2, \cdots, k-1$.

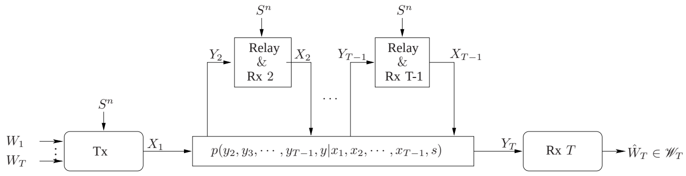

Fig. 3. General T-node Cooperative RB network with SI non-causally known at the source terminal, all the intermediate receivers but not the final receiver.

A. One-node Partially Cooperative RBC

Assume the one-step problem in which there is only one relay node, i.e., $T = 3$. Here, we have a common message W_0 at rate R_0 which is decoded by the relay (node 2) and the final receiver (node 3), and private messages W_2 and W_3 at rates R_2 and R_3 that are decoded by nodes 2 and 3, respectively. The following result holds:

Theorem 4.1: (Inner bound on the capacity of one-node Partially Cooperative RBC with state) For a discrete memoryless one-node Partially Cooperative Relay Broadcast Channel $p(y_2, y|x_1, x_2, s)$ with state information S^n non-causally available at the transmitter and the relay but not the destination, the following rate is achievable:

$$\begin{aligned} R_2 &< I(X_1; Y_2|SU_1X_2) \\ R_0 + R_3 &< \min\ \Big\{I(U_1; Y_2|SU_2), I(U_1U_2; Y) - I(U_1U_2; S)\Big\}, \end{aligned} \tag{16}$$

where the maximum is over all auxiliary random variables U_1 and U_2 with finite cardinality bounds and all joint distributions of the form $p(s)p(u_1, u_2, x_1, x_2|s)p(y_2|x_1, x_2, s)p(y|y_2, x_2)$.

Remark 6: The intuition behind (16) is as follows: The source terminal employs superposition coding to transmit message W_2 intended to the relay on top of that, W_3, intended to the destination. For the transmission of W_3 through the relay, the situation is equivalent to that in Section II and rate $\min\ \Big\{I(U_1; Y_2|SU_2), I(U_1U_2; Y) - I(U_1U_2; S)\Big\}$ is achievable, as showed above. Then, how much information W_2 can be transferred to the relay? exactly as much as the information contained in X_1 and which is not intended to carry message W_3, i.e., $I(X_1; Y_2|SU_1X_2)$. Conditioning on S and X_2 is there because the relay knows the state and X_2.

Now, we turn to the proof of Theorem 4.1. Note that it suffices to show the result for the case without common message W_0. This is because, one can view part of the rate R_3 to be the common rate R_0, since the relay also decodes message W_3 (see proof below).

Proof: (Proof of Theorem 4.1) The proof is similar in nature to that in Section II-A and is omitted for brevity. We only outline the main steps. Generate two random codebooks U_1 and U_2 to transmit message W_3 through the relay to the final receiver (in a similar way to that in II-A as this is basically a relaying task, i.e., by a combination of sliding window and binning). Then for each u_2^n, for each $u_1^n(u_2^n)$, use superposition coding to generate 2^{nR_2} i.i.d. x_1^n and index them as $x_1^n(u_2^n, u_1^n, w_{2,i})$. These x_1^n's are intended to carry message W_2 (on top of message W_3). Potential encoding errors (of W_2 and W_3) and potential decoding errors of message W_3 at both the relay and the destination can be shown to be small for sufficiently large n, by similar arguments to those in Section II-A. Two additional potential decoding error events at the relay (related to decoding message W_2) can be shown to be small for sufficiently large n, using standard joint typicality decoding arguments. ■

V. Capacity region of D-AWGN $T-$node partially/fully cooperative relay broadcast channel with state

In this section, we consider a partially/fully cooperative RBC with additive i.i.d. Gaussian state where the channel outputs are corrupted by degraded Gaussian noise terms. We refer to this channel as the D-AWGN cooperative RBC with state. In sections V-A and V-B, we focus on the case of partially cooperating receivers, meaning that there exist independently generated Gaussian random variables $Z_k \sim \mathcal{N}(0, N_k)$ and $Z'_k \sim \mathcal{N}(0, N'_k)$ such that

$$y_{k,i} = y_{k-1,i} + x_{k,i} + z'_{k,i}, \quad 3 \le k \le T$$
$$y_{2,i} = x_{1,i} + s_i + z_{2,i}, \tag{17}$$

where $z_{k,i} = z_{k-1,i} + z'_{k,i}$ and Z_{k-1} and Z'_k are statistically independent. Let $\mathbb{E}[X_k^2] = P_k$, $k = 1, \cdots, T-1$. The goal is to determine the capacity region of this channel for any given set of $P_1, \cdots, P_{T-1}$ and $N_2, \cdots, N_T$. It turns out that, in this case, (16) is the capacity region. In section V-C, we shortly discuss the case of fully cooperating receivers.

A. D-AWGN Partially Cooperative RBC

Assume the one-step problem in which there is only one relay node, i.e., $T = 3$. Extension to the T-node case is undertaken below. Note that when there is no additive state, capacity region is given by the region with the rate tuples (R_0, R_1, R_2) satisfying [10]

$$R_2 \; < \; C\Big(\frac{\alpha P_1}{N_2}\Big) \tag{18a}$$

$$R_0 + R_3 < \max_{\beta} \min \Big\{ C\big(\frac{\beta\bar{\alpha}P_1}{\alpha P_1 + N_2}\big), \; C\Big(\frac{P_2 + \bar{\alpha}P_1 + 2\sqrt{\bar{\beta}\bar{\alpha}P_1P_2}}{\alpha P_1 + N_3}\Big)\Big\}, \tag{18b}$$

for some $\alpha \in [0,1]$, where $\bar{\alpha} = 1-\alpha$ and $\bar{\beta} = 1 - \beta$. When the state S^n is available everywhere —at the transmitter, receiver 2 and the final receiver, these nodes can simply subtract S^n to reduce the channel to the case without additive state and attain the same region as in (18). Now, we turn to the case where only the final receiver does not know the state S^n. Here is the main result of this section.

Theorem 5.1: (Capacity of single-node D-AWGN Partially Cooperative RBC with State) The capacity region of the D-AWGN Partially Cooperative Relay Broadcast Channel with state information non-causally available at the transmitter and the relay but not the final receiver is given by the standard capacity (18).

Proof: Proceeding similarly to Costa's approach [12], we need only prove the achievability of the region. The proof of achievability follows by evaluating the achievable region (16) with the input distribution given by (20) and (21).

Alternative proof: A (more intuitive) alternative proof is as follows. We decompose the input signal X_1 into two parts, X'_1 with power αP_1 (stands for the information carried by X_1 and intended for the relay), and U with power $\bar{\alpha}P_1$ (stands for the information carried by X_1 through the relay and intended for the final receiver), i.e., $X_1 = X'_1 + U$. Next, we decompose the signal U into two parts, $U^{(1)}$ of power $\beta\bar{\alpha}P_1$ and $U^{(2)}$ of power $\bar{\beta}\bar{\alpha}P_1$ and carrying, respectively, *fresh* and *refinement* information for the transmission of W_3. Then, assuming the relay decoded the previously sent message $W_{3,i-1}$ correctly, the channel to the final receiver

$$\begin{aligned} Y &= X_1 + X_2 + S + Z_3 \\ &= (U^{(2)} + X_2) + U^{(1)} + S + (X'_1 + Z_3), \end{aligned} \tag{19}$$

can be viewed as a MAC with independent inputs–the cooperative transmission $(U^{(2)} + X_2)$ with power $\bar{P} := (\sqrt{\bar{\beta}\bar{\alpha}P_1} + \sqrt{P_2})^2$ by nodes 1 and 2 and the independent transmission $U^{(1)}$ of the fresh information with power $\bar{P}_2 := \beta\bar{\alpha}P_1$. This MAC has SI non-causally known to the two-fictitious users but not to the receiver and transmission is corrupted by total Gaussian noise $(X'_1 + Z_3)$ of power $\alpha P_1 + N_3$. Using [8, Theorem 2], optimal inputs for this channel can be generated as

$$U_2 \sim \mathcal{N}(\alpha_2 S, \bar{P}_2), \quad U^{(1)} = U_2 - \alpha_2 S, \tag{20a}$$
$$U_1 \sim \mathcal{N}(\alpha_1 S, \bar{P}), \quad U^{(2)} + X_2 = U_1 - \alpha_1 S, \tag{20b}$$
$$\alpha_2 = \frac{\bar{P}_2}{\bar{P}_2 + \bar{P} + (\alpha P_1 + N_3)}, \; \alpha_1 = \frac{\bar{P}}{\bar{P}_2 + \bar{P} + (\alpha P_1 + N_3)}.$$

Thus, the input signals are given by

$$X_2 = (1-\lambda)(U_1 - \alpha_1 S), \;\; \lambda = \frac{\sqrt{P_2}}{\sqrt{\bar{P}}} \tag{21a}$$
$$X_1 = \lambda(U_1 - \alpha_1 S) + (U_2 - \alpha_2 S) + X'_1. \tag{21b}$$

The second term in the RHS of (18b) can be attained as the sum rate over this MAC. The first term in the RHS of (18b) can be attained since the relay can peel of S and $U^{(1)}$ before decoding the refinement information contained in $U^{(2)}$. The RHS of (18a) can be attained since the relay can peel of S, $U^{(1)}$ and $U^{(2)}$ to make the channel Y_2 equivalent to $Y'_2 = X'_1 + Z_2$. ■

B. Multiple receivers

The two-receiver case extends in a rather straightforward manner to the T-node D-AWGN Partially Cooperative RBC with state where each receiver can act as a relay for the receivers that are "farther away". More specifically, let $T-1$ receivers each experiencing Gaussian noise with variance N_k and indexed such that $N_2 \leq \cdots \leq N_T$. Define the set $\{\beta_{i,j}\}$ with $1 \leq i \leq j \leq T-1$ such that $\sum_{j=i}^{T-1} \beta_{i,j} = 1$ and the set $\{\alpha_{i,j,k}\}$ with $1 \leq i \leq j \leq k \leq T-1$ such that $\sum_{j=i}^{T-1} \alpha_{i,j,k} = \beta_{i,k}$. And for $1 \leq l < k \leq T$, define

$$R_{l,k}(\underline{\alpha}, \underline{\beta}) = C\left(\frac{\sum_{j=1}^{l}\left(\sum_{i=1}^{j}\sqrt{\alpha_{i,j,k}P_i}\right)^2}{N_{l+1} + \sum_{i=1}^{l} P_i \sum_{j=l+1}^{k-1} \beta_{i,j}}\right), \quad (22)$$

where we use $\underline{\beta}$ and $\underline{\alpha}$ for $\{\beta_{i,j}\}$ and $\{\alpha_{i,j,k}\}$. When there is no additive state (or the state is available everywhere), capacity region is given by [15]

$$C_T = \bigcup_{\underline{\alpha}, \underline{\beta}} \left\{R_2, \cdots, R_T : R_k \leq \min_{1 \leq l < k} R_{l,k}(\underline{\alpha}, \underline{\beta})\right\}. \quad (23)$$

By using an inductive argument similar to that used above to prove Theorem 3.1 (here, Theorem 5.1 serves as the initial step), we can show that capacity is also given by (23), even when the receiver does not know the state S^n.

C. On Fully Cooperative RBC with state

To derive an achievable region for the Fully Cooperative RBC with state, we need to choose relaying schemes for users to assist each other. A natural choice is one where "better" receivers use decode-and-forward and those receivers that are further away (to which we loosely refer to as "degraded" receivers) use compress-and-forward. When, in addition, we assume that there is some SI non-causally known to the receivers, the random code construction (for the general case) becomes complex, especially if we have more than 3 nodes. There is one case for which code construction is particularly easy: the D-AWGN Fully Cooperative RBC with state. In this case, the feedback from those degraded receivers does not increase the capacity region [10]. Hence, the D-AWGN Fully Cooperative RBC with sate has the same capacity (23) as the D-AWGN Partially Cooperative RBC with sate. If, however, the channel is Gaussian but not necessarily degraded, full cooperation may enlarge the achievable rate region.

VI. Concluding Remarks

In this paper, we derived an achievable rate region for two channels controlled by random parameters: the T-node relay channel with state information non-causally known to the source terminal and all the relays but not the destination, and the partially cooperative relay-broadcast channel with state information non-causally known to the source terminal and all the intermediate receivers (which also act as relays in this case) but not the final receiver. These achievable rate regions are derived based on the relays using the decode-and-forward scheme. Next, we considered the special case where the two considered channels are degraded Gaussian and the state is additive (i.e., interference-like) i.i.d. Gaussian and show that the derived rate regions are then the corresponding capacity regions. An immediate consequence is that, in this case, the state has no impact on the capacity region of the considered channels, even though the final receiver does not know it. For the degraded Gaussian relay channel, the results in this paper can be viewed as an extension of those by Kim *et. al.* to the case of more than one relay.

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References

[1] Y. Steinberg, "On the broadcast channel with random parameters," in *Int. Symps. on IT, ISIT*, June 2002, p. 225.

[2] ——, "Coding for the degraded broadcast channel with random parameters, with causal and noncausal side information," *IEEE Transactions on Information Theory*, vol. IT-51, pp. 2867–2877, August 2005.

[3] Y. Cemal and Y. Steinberg, "The multiple-access channel with partial state information at the encoders," *IEEE Transactions on Information Theory*, vol. IT-51, pp. 3992–4003, November 2005.

[4] R. C. King, *Multiple access channels with generalized feedback.* Stanford, CA: Ph.D. dissertation, Stanford Univ., May 1978.

[5] A. B. Carleial, "Multiple-access channels with different generalized feedback signals," *IEEE Transactions on Information Theory*, vol. IT-28, pp. 841–850, November 1982.

[6] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Transactions on Information Theory*, vol. IT-51, pp. 3037–3063, September 2005.

[7] S. I. Gel'fand and M. S. Pinsker, "Coding for channel with random parameters," *Problems of Control and IT.*, vol. 9, pp. 19–31, 1980.

[8] Y.-H. Kim, A. Sutivong, and S. Sigurjonsson, "Multiple user writing on dirty paper," in *Proc. ISIT 2004*, Chicago-USA, June 2004, p. 534.

[9] T. M. Cover, "Broadcast channels," *IEEE Transactions on Information Theory*, vol. IT-18, pp. 2–14, Junuary 1972.

[10] Y. Liang and V. V. Veeravalli, "Cooperative relay broadcast channels," *Submitted to IEEE Transactions on Information Theory; available at http://www.arxiv.org/PS-cache/cs/pdf/0605/0605016.pdf*, July 2005.

[11] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: John Willey & Sons INC., 1991.

[12] M. H. M. Costa, "Writing on dirty paper," *IEEE Trans. on IT*, vol. IT-29, pp. 439–441, may 1983.

[13] A. Reznik, S. Kulkarni, and S. Verdù, "Degraded gaussian multirelay channel: Capacity and optimal power allocation," *IEEE Transactions on Information Theory*, vol. 50, pp. 3037–3046, 2004.

[14] T. M. Cover and A. A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Transactions on Information Theory*, vol. IT-25, pp. 572–584, September 1979.

[15] A. Reznik, S. Kulkarni, and S. Verdù, "Broadcast-relay channel: capacity region bounds," in *IEEE Int. Symp. on Info. Theory (ISIT)*, 2005, pp. 820–824.