Bounds on the Capacity of the Relay Channel with Noncausal State Information at Source

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Abstract— We consider a three-terminal state-dependent relay channel with the channel state available non-causally at only the source. Such a model may be of interest for node cooperation in the framework of cognition, i.e., collaborative signal transmission involving cognitive and non-cognitive radios. We study the capacity of this communication model. One principal problem in this setup is caused by the relay's not knowing the channel state. In the discrete memoryless (DM) case, we establish lower bounds on channel capacity. For the Gaussian case, we derive lower and upper bounds on the channel capacity. The upper bound is strictly better than the cut-set upper bound. We show that one of the developed lower bounds comes close to the upper bound, asymptotically, for certain ranges of rates.

I. INTRODUCTION

We consider a three-terminal state-dependent relay channel (RC) in which, as shown in Figure 1, the source wants to communicate a message W to the destination through the state-dependent RC in n uses of the channel, with the help of the relay. The channel outputs Y_2 and Y_3 for the relay and the destination, respectively, are controlled by the channel input X_1, the relay input X_2 and the channel state S, through a given memoryless probability law $W_{Y_2,Y_3|X_1,X_2,S}$. The channel state S is generated according to a given memoryless probability law Q_S. It is assumed that the channel state is known, noncausally, to only the source. The destination estimates the message sent by the source from the received channel output. In this paper we study the capacity of this communication system. We refer to this model as *state-dependent RC with informed source*.
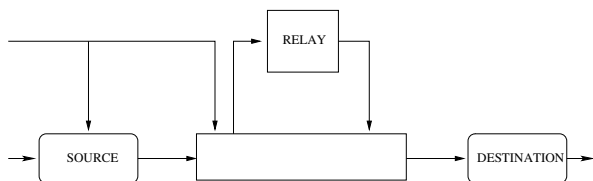

Fig. 1.   Relay channel with state information S^n available non-causally at only the source.

The state-dependent multiaccess channel (MAC) with only one informed encoder and degraded message sets is considered in [1], [2]; and the state-dependent relay channel (RC) with only informed relay is considered in [3]. For all these models, the authors develop non-trivial outer or upper bounds that permit to characterize the rate loss due to not knowing the state at the uninformed encoders. Key feature to the development of these outer or upper bounds is that, in all these models, the uninformed encoder not only does not know the channel state but, also, can learn no information about it.

The model for the RC with informed source that we study in this paper seemingly exhibits some similarities with the RC with informed relay considered in [3], and it also connects with the MAC with asymmetric CSI and degraded message sets considered in [1], [2], [4] and with the MAC with two states considered in [5]. However, establishing a non-trivial upper bound for the present model is more involved, comparatively. Partly, this is because, here, the uninformed encoder is also a receiver; and, so, it can potentially get some information about the channel states from directly observing its output. That is, at time i, the input $X_{2,i}$ of the relay can potentially depend on the channel states through $Y_2^{i-1} = (Y_{2,1}, \ldots, Y_{2,i-1})$. Further, since, for $j = 1, \ldots, i-1$, Y_2^j may depend on the channel states in a non-causal manner (through the source codeword $X_{1,j}(W, S^n)$), and not only through the current state S_i, so does the input of the relay, potentially.

Establishing good lower bounds for the present model is also a non-easy task, due to the asymmetry caused by knowing the state at only the source. In this paper, we establish two lower bounds on the capacity of the state-dependent RC with informed source, for both discrete memoryless (DM) and memoryless Gaussian cases. For the Gaussian case, we also establish a non-trivial upper bound that is strictly better than the max-flow min cut or cut-set upper bound. Our lower bounds exploit ideas that are inherently different; and, so, their comparison helps providing right guidance towards the appropriate design. We obtain the first lower bound by a coding scheme in which the source describes the known state to the relay and destination *ahead of time*, in addition to sending the information message. The relay performs collaborative binning against the learned state, through a combined binning and decode-and-forward (DF) scheme. We obtain the second lower bound by a coding scheme in which, rather than the channel state itself, the source describes to the

relay the appropriate input that the relay would send had the relay known the channel state. The relay then simply guesses this input and sends it in the appropriate block. The lower bound obtained with this scheme achieves close to optimal for some special cases.

We note that the lower bounds that we develop in this paper are tailored to primarily overcome the state asymmetry; and, so, they perform well in the situations in which classic DF for RC without state is suitable. Other interesting achievability results which perform well in other situations can be found in [6]–[8].

II. System Model and Definitions

As shown in Figure 1, we consider a state-dependent relay channel denoted by $W_{Y_2,Y_3|X_1,X_2,S}$ whose outputs $Y_2 \in \mathcal{Y}_2$ and $Y_3 \in \mathcal{Y}_3$ for the relay and the destination, respectively, are controlled by the channel inputs $X_1 \in \mathcal{X}_1$ from the source and $X_2 \in \mathcal{X}_2$ from the relay, along with a random state parameter $S \in \mathcal{S}$. It is assumed that the channel state S_i at time instant i is independently drawn from a given distribution Q_S and the channel states S^n are non-causally known at the source.

The source wants to transmit a message W to the destination with the help of the relay, in n channel uses. The message W is assumed to be uniformly distributed over the set $\mathcal{W} = \{1, \dots, M\}$. The information rate R is defined as $n^{-1} \log M$ bits per transmission.

An (M, n) code for the state-dependent relay channel with informed source consists of an encoding function at the source

$$\phi_1^n : \ \{1, \dots, M\} \times \mathcal{S}^n \to \mathcal{X}_1^n,$$

a sequence of encoding functions at the relay

$$\phi_{2,i} : \ \mathcal{Y}_{2,1}^{i-1} \to \mathcal{X}_2,$$

for $i = 1, 2, \dots, n$, and a decoding function at the destination

$$\psi^n : \ \mathcal{Y}_3^n \to \{1, \dots, M\}.$$

Let a (M, n) code be given. The sequences X_1^n and X_2^n from the source and the relay, respectively, are transmitted across a state-dependent relay channel modeled as a memoryless conditional probability distribution $W_{Y_2,Y_3|X_1,X_2,S}$. The joint probability mass function on $\mathcal{W} \times \mathcal{S}^n \times \mathcal{X}_1^n \times \mathcal{X}_2^n \times \mathcal{Y}_2^n \times \mathcal{Y}_3^n$ is given by

$$P(w, s^n, x_1^n, x_2^n, y_2^n, y_3^n) = P(w) \prod_{i=1}^{n} Q_S(s_i) P(x_{1,i}|w, s^n) P(x_{2,i}|y_2^{i-1})$$
$$\cdot W_{Y_2,Y_3|X_1,X_2,S}(y_{2,i}, y_{3,i}|x_{1,i}, x_{2,i}, s_i). \quad (1)$$

The destination estimates the message sent by the source from the channel output Y_3^n. The average probability of error is defined as $P_e^n = \mathbb{E}_S\left[\Pr\left(\psi^n(Y_3^n) \neq W|S^n = s^n\right)\right]$.

An (ϵ, n, R) code for the state-dependent RC with informed source is an $(2^{nR}, n)$−code $(\phi_1^n, \phi_2^n, \psi^n)$ having average probability of error P_e^n not exceeding ϵ.

A rate R is said to be achievable if there exists a sequence of (ϵ_n, n, R)−codes with $\lim_{n \to \infty} \epsilon_n = 0$. The capacity $\mathcal{C}$ of the state-dependent RC with informed source is defined as the supremum of the set of achievable rates.

Due to space limitation, the results of this paper are either outlined only or mentioned without proofs. Detailed proofs can be found in [9].

III. The Discrete Memoryless RC with Informed Source

In this section, we assume that the alphabets $\mathcal{S}$, $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}_2$, $\mathcal{Y}_3$ in the model are all discrete and finite.

Theorem 1: The capacity of the discrete memoryless state-dependent relay channel with informed source is lower bounded by

$$R^{\text{lo}} = \max \min \{ I(U; Y_2|V, \hat{S}_R) - I(U; S, \hat{S}_D|V, \hat{S}_R),$$
$$I(U, V; Y_3|\hat{S}_D) - I(U, V; S, \hat{S}_R|\hat{S}_D) \} \quad (2)$$

subject to the constraints

$$I(S; \hat{S}_R) \leq I(U_R; Y_2, \hat{S}_R|U, V) - I(U_R; S, \hat{S}_R, \hat{S}_D|U, V) \quad (3a)$$

$$I(S; \hat{S}_D) \leq I(U_D; Y_3, \hat{S}_D|U, V) - I(U_D; S, \hat{S}_R, \hat{S}_D|U, V)$$
$$+ [I(U; Y_3, \hat{S}_D|V) - I(U; S, \hat{S}_R, \hat{S}_D|V)]_- \quad (3b)$$

$$I(S; \hat{S}_R, \hat{S}_D) + I(\hat{S}_R; \hat{S}_D) \leq I(U_R; Y_2, \hat{S}_R|U, V) - I(U_R; S, \hat{S}_R, \hat{S}_D|U, V)$$
$$+ I(U_D; Y_3, \hat{S}_D|U, V) - I(U_D; S, \hat{S}_R, \hat{S}_D|U, V)$$
$$+ [I(U; Y_3, \hat{S}_D|V) - I(U; S, \hat{S}_R, \hat{S}_D|V)]_-$$
$$- I(U_R; U_D|U, V, S, \hat{S}_R, \hat{S}_D) \quad (3c)$$

where $[x]_- \triangleq \min(x, 0)$, and the maximization is over all joint measures on $\mathcal{S} \times \hat{\mathcal{S}}_R \times \hat{\mathcal{S}}_D \times \mathcal{U}_R \times \mathcal{U}_D \times \mathcal{U} \times \mathcal{V} \times \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{Y}_2 \times \mathcal{Y}_3$ of the form

$$P_{S,\hat{S}_R,\hat{S}_D,U_R,U_D,U,V,X_1,X_2,Y_2,Y_3} = Q_S P_{\hat{S}_R,\hat{S}_D|S} P_{V|\hat{S}_R} P_{U|V,S,\hat{S}_R,\hat{S}_D} P_{U_R,U_D|V,U,S,\hat{S}_R,\hat{S}_D}$$
$$P_{X_1|U_R,U_D,U,V,S,\hat{S}_R,\hat{S}_D} P_{X_2|V,\hat{S}_R} W_{Y_2,Y_3|X_1,X_2,S}. \quad (4)$$

and satisfying

$$I(V; Y_3, \hat{S}_D) - I(V; \hat{S}_R) > 0. \quad (5)$$

Remark 1: The intuition for the coding scheme which we use to establish the lower bound in Theorem 1 is as follows. Had the relay known the state, the source and the relay could implement collaborative binning against that state for transmission to the destination [10]. Since the source knows the state of the channel non-causally, it can transmit a description of it to the relay *ahead of time*. The relay recovers the state (with a certain distortion), and then utilizes it in the relevant block through a collaborative binning scheme. The hope is that the benefit that the source can get from being assisted by a more capable relay largely compensates the loss caused by the source's spending some of its resources to make the relay learn the state.

In general, it may also turn to be useful to send a dedicated description of the state to the destination. The destination utilizes the recovered state as side information at the receiver. The coding scheme that we employ to establish the lower bound in Theorem 1 uses block Markov encoding. In each block i, in addition to its message, the source also sends a two-layer description of the state $\mathbf{s}[i + 2]$ to the relay and destination; a description $\hat{\mathbf{s}}_R[\iota_{Ri}]$ of $\mathbf{s}[i + 2]$ intended to be recovered at the relay and a description $\hat{\mathbf{s}}_D[\iota_{Di}]$ of $\mathbf{s}[i + 2]$ intended to be recovered at the destination. The two layers are possibly correlated. (In Remark 2, we will comment on the delay of two blocks needed here). The relay guesses the source's message w_i and the individual state description

$\hat{\mathbf{s}}_R[\iota_{Ri}]$ dedicated to it from the source transmission and the previously recovered state description $\hat{\mathbf{s}}_R[\iota_{Ri-2}]$. It will then utilize the new state estimate as non-causal state at the encoder for collaborative source-relay binning in block $i+2$, through a combined decode-and-forward and Gelf'and-Pinsker binning. The destination guesses the source's message w_{i-1} sent cooperatively by the source and relay and the individual state description $\hat{\mathbf{s}}_D[\iota_{Di-1}]$ which is dedicated to it from its output $(\mathbf{y}_3[i-1], \mathbf{y}_3[i])$ and the previously recovered state $(\hat{\mathbf{s}}_D[\iota_{Di-3}], \hat{\mathbf{s}}_D[\iota_{Di-2}])$, using a window-decoding method.

Remark 2: The source sends the descriptions intended to the relay and destination *two blocks* ahead of time. That is, at the beginning of block i the source describes the state vector $\mathbf{s}[i+2]$ to the relay and destination. While one block delay is sufficient to describe the state to the relay, a minimum of two blocks is necessary for the state reconstruction at the destination because of the used window decoding technique.

We can generalize Theorem 1 by allowing the source to also send a common description of the state which is intended to be recovered at both the relay and the destination (see [9]).

The following theorem provides a lower bound on the capacity of the state-dependent general discrete memoryless RC with informed source.

Theorem 2: The capacity of the discrete memoryless state-dependent relay channel with informed source is lower bounded by

$$R^{\text{lo}} = \max \min \{I(U, U_R; Y_3) - I(U, U_R; S),$$
$$I(U, U_R; Y_2, \hat{X}) - I(U, U_R; S) - I(X; \hat{X})\} \quad (6)$$

subject to the constraint

$$I(X; \hat{X}) < I(U_R; Y_2, \hat{X}|U) - I(U_R; S|U) + [I(U; Y_2, \hat{X}) - I(U; S)]_- \quad (7)$$

where $[x]_- = \min(x, 0)$, and the maximization is over all joint measures on $\mathcal{S} \times \mathcal{U} \times \mathcal{U}_R \times \mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X} \times \hat{\mathcal{X}} \times \mathcal{Y}_2 \times \mathcal{Y}_3$ of the form

$$P_{S,U,U_R,X_1,X_2,X,\hat{X},Y_2,Y_3}$$
$$= Q_S P_{U|S} P_{U_R|U,S} P_{X_1|U_R,U,S} P_{X|U,S} P_{\hat{X}|X} \mathbb{1}_{X_2=\hat{X}} W_{Y_2,Y_3|X_1,X_2,S}. \quad (8)$$

Remark 3: The rationale for the coding scheme which we use to obtain the lower bound in Theorem 2 is as follows. With DF relaying, had the relay known the state then in each block the relay generates its input using the source transmission in the previous block and the state that controls the channel in the current block, as in [10]. For our model, the source knows what cooperative information, i.e., part of the message, the relay would send in each block. It also knows the state sequence that corrupts the transmission in that block. It can then generate the appropriate relay input vector that the relay would send had the relay known the state. The source can send this vector to the relay *ahead of time*, and if the relay can estimate it to high accuracy, then collaborative source-relay binning in the sense of [10] is readily realized for transmission from the source and relay to the destination.

Outline of Proof: A detailed proof of Theorem 2 can be found in [9]. An outline proof is as follows. A block Markov encoding with $B+1$ blocks is used. Let us denote by $\mathbf{x}[k]$ the relay input carrying message $w_k \in [1, 2^{nR}]$ that the relay would send in block k had the relay known the state $\mathbf{s}[k]$,

assuming DF relaying, with $k = 2, \ldots, B+1$. Let us now consider transmission in two adjacent blocks i and $i+1$. In the beginning of block i, the source sends information message w_i of the current block, and, in addition, describes to the relay the input $\mathbf{x}[i+1]$ that the relay would send in the next block $i+1$ had the relay known the state $\mathbf{s}[i+1]$. Let $\hat{\mathbf{x}}[m_i]$ be a description of $\mathbf{x}[i+1]$. The source generates its input $\mathbf{x}_1[i]$ using two auxiliary codewords that are superimposed, a codeword $\mathbf{u}_R[i]$ that carries the index m_i on top of a codeword $\mathbf{u}[i]$ that carries message w_i. Both codewords are selected using binning against the state $\mathbf{s}[i]$ that controls transmission in the current block i. The vector $\mathbf{x}[i+1]$, however, is the input that the relay would send in the next block $i+1$ had the relay known the state $\mathbf{s}[i+1]$, and so is generated at the source using binning against the state $\mathbf{s}[i+1]$. The description of vector $\mathbf{x}[i+1]$, which is sent to the relay in block i, is intended to combine coherently with the source transmission in block $i+1$. In the beginning of block i, the relay knows m_{i-1} from the source transmission in previous block $i-1$, and sends $\mathbf{x}_2[i] = \hat{\mathbf{x}}[m_{i-1}]$.

Remark 4: In the scheme we described briefly in Remark 3, the relay needs only estimate the code vector $\mathbf{x}[i]$ sent by the source in block $i-1$, and transmit the obtained estimate in the next block i. For instance, the relay does not need know the state sequence that actually controls the channel. Thus, transmission from the source terminal to the relay terminal can be regarded as that of an analog source which, in block i, produces a sequence $\mathbf{x}[i+1]$. This source has to be transmitted by the source terminal over a state-dependent channel and reconstructed at the relay terminal. The reconstruction error at the relay terminal influences the rate at which information can be decoded reliably at the destination by acting as an additional noise term.

IV. The Gaussian RC with Informed Source

In this section, we consider a full-duplex state-dependent RC informed source in which the channel states and the noise are additive and Gaussian. In this model, the channel state can model an additive Gaussian interference which is assumed to be known (non-causally) to only the source.

General Gaussian Model: For the general Gaussian model, the channel outputs $Y_{2,i}$ and $Y_{3,i}$ at time instant i for the relay and the destination, respectively, are related to the channel input $X_{1,i}$ from the source and $X_{2,i}$ from the relay, and the channel state S_i, by

$$Y_{2,i} = X_{1,i} + S_i + Z_{2,i} \quad (9a)$$
$$Y_{3,i} = X_{1,i} + X_{2,i} + S_i + Z_{3,i}. \quad (9b)$$

The channel state S_i is zero mean Gaussian random variable with variance Q; and only the source knows the state sequence S^n (non-causally). The noises $Z_{2,i}$ and $Z_{3,i}$ are zero mean Gaussian random variables with variances N_2 and N_3, respectively; and are mutually independent and independent from the state sequence S^n and the channel inputs (X_1^n, X_2^n).

We consider the following individual power constraints on the average transmitted power at the source and the relay

$$\sum_{i=1}^{n} X_{1,i}^2 \le nP_1, \qquad \sum_{i=1}^{n} X_{2,i}^2 \le nP_2. \quad (10)$$

Some Special Cases: We shall also consider the following subclass of Gaussian RC with informed source, the frequency-division (FD) Gaussian RC with informed source and orthogonal components where $X_{1,i} = (X_{1R,i}, X_{1D,i})$, $Y_{3,i} = (Y_{3,i}^{(1)}, Y_{3,i}^{(2)})$ and

$$Y_{2,i} = X_{1R,i} + S_i + Z_{2,i} \tag{11a}$$

$$Y_{3,i}^{(1)} = X_{1D,i} + S_i + Z_{3,i}^{(1)} \tag{11b}$$

$$Y_{3,i}^{(2)} = X_{2,i} + S_i + Z_{3,i}^{(2)}, \tag{11c}$$

where the noises $Z_{3,i}^{(1)}$ and $Z_{3,i}^{(2)}$ are zero mean Gaussian random variables with variances N_3, and are mutually independent and independent from the state sequence S^n and the channel inputs (X_1^n, X_2^n).

A FD Gaussian RC with informed source and orthogonal components in which the state S_i does not affect transmission from the relay to the destination will be said to be *degenerate*. Its input-output relation is given by (11) with (11c) substituted by $Y_{3,i}^{(2)} = X_{2,i} + Z_{3,i}^{(2)}$.

The reader may refer to [9], where we also consider the following other interesting special cases.

1) *Deaf helper problem:* The relay does not hear the source, and receives $Y_{2,i} = S_i + Z_{2,i}$.
2) Gaussian RC with orthogonal state-independent link between the source and the relay. The source communicates with the relay over a noise-free bit-pipe of given capacity.

A. Upper Bound on the Capacity

Theorem 3: The capacity of the state-dependent general Gaussian RC with informed source is upper-bounded by

$$R_G^{up} = \max \min \Big\{ \frac{1}{2} \log\Big(1 + P_1(1 - \rho_{12}^2)\big(\frac{1}{N_2} + \frac{1}{N_3}\big)\Big),$$

$$\frac{1}{2} \log\Big(1 + \frac{(\sqrt{P_2} + \rho_{12}\sqrt{P_1})^2}{P_1(1 - \rho_{12}^2 - \varrho_{1s}^2) + (\sqrt{\Delta_Q} + \varrho_{1s}\sqrt{P_1})^2 + N_3}\Big)$$

$$+ \frac{1}{2} \log\Big(1 + \frac{P_1(1 - \rho_{12}^2 - \varrho_{1s}^2)}{N_3}\Big)\Big\}, \tag{12}$$

where $\Delta_Q = QN_2/((\sqrt{Q} + \sqrt{P_1})^2 + N_2)$ and the maximization is over parameters $\rho_{12} \in [0,1]$, $\varrho_{1s} \in [-1,0]$ such that

$$\rho_{12}^2 + \varrho_{1s}^2 \leq 1. \tag{13}$$

Outline of Proof: We only sketch the important steps, due to lack of space. A detailed proof of Theorem 3 can be found in [9]. The proof of the bound given by the first term of the minimization in (12) trivially follows by revealing the state S^n to the relay and the destination. The proof of the bound given by the second term of the minimization in (12) is as follows. First, we show that there is an inevitable residual uncertainty at the relay about the state sequence S^n after observing the channel outputs $Y_2^{i-1} = (Y_{2,1}, \ldots, Y_{2,i-1})$. Then, considering transmission from the source and relay to the destination, we upper bound the sum rate that can be conveyed to the destination on the multiaccess part of the channel by accounting for the rate penalty that is caused by not knowing the state fully at the relay. In doing so, we assume that the message is revealed to the relay by a genie.

Remark 5: The established upper bound improves upon the cut-set upper bound through the second term of the minimization. The second term of the minimization is strictly tighter than that of the cut-set upper bound because it accounts for not knowing a part Δ_S^n of the state of power Δ_Q at the relay.

B. Lower Bound on the Capacity

Definition 1: Let

$$\tilde{Q}_S(t, Q, D) := (1 - t)^2 Q - t(t - 2)D$$

$$R(\alpha, P, Q, N) := \frac{1}{2} \log\Big(\frac{P(P + Q + N)}{PQ(1 - \alpha)^2 + N(P + \alpha^2 Q)}\Big)$$

for non-negative t, D, P, Q, N, and $\alpha \in \mathcal{A}(P, Q, N) := \{x \in \mathbb{R} : R(x, P, Q, N) \geq 0\}$.

The following theorem provides a lower bound on the capacity of the state-dependent general Gaussian RC with informed source.

Theorem 4: The capacity of the state-dependent Gaussian RC with informed source is lower-bounded by

$$R_G^{lo} = \max \min \Big\{ R(\alpha, \beta\bar{\gamma}P_1, \tilde{Q}, N_2 + \gamma P_1),$$

$$R(\alpha, \beta\bar{\gamma}P_1, \tilde{Q}, N_3 + \gamma P_1) + \frac{1}{2} \log\Big(1 + \frac{(\sqrt{\beta\bar{\gamma}P_1} + \sqrt{P_2})^2}{N_3 + D + \gamma P_1 + \beta\bar{\gamma}P_1}\Big)\Big\}, \tag{14}$$

where

$$D = Q\frac{N_2}{N_2 + \gamma P_1}, \quad \tilde{Q} = \tilde{Q}_S(\alpha_2, Q, D) \tag{15}$$

$$\alpha_2 = \frac{(\sqrt{\beta\bar{\gamma}P_1} + \sqrt{P_2})^2}{(\sqrt{\beta\bar{\gamma}P_1} + \sqrt{P_2})^2 + \beta\bar{\gamma}P_1 + (N_3 + D + \gamma P_1)} \tag{16}$$

and the maximization is over $\beta \in [0,1]$, $\gamma \in [0,1]$ and $\alpha \in \mathcal{A}(\beta\bar{\gamma}P_1, \tilde{Q}, N_2 + \gamma P_1) \cap \mathcal{A}(\beta\bar{\gamma}P_1, \tilde{Q}, N_3 + \gamma P_1)$.

Outline of Proof of Theorem 4: A detailed proof of Theorem 4 can be found in [9]. An outline of proof is as follows. The result in Theorem 1 for the DM case can be extended to memoryless channels with discrete time and continuous alphabets using standard techniques [11, Chapter 7]. For the state-dependent Gaussian relay channel (9), we evaluate the rate (2) with the following choice of input distribution. We choose $\hat{S}_D = \varnothing$, $U_D = \varnothing$. Furthermore, we consider the test channel $\hat{S}_R = aS + \tilde{S}_R$, where $a := 1 - D/Q$ and $\tilde{S}_R$ is a Gaussian random variable with zero mean and variance $\sigma_{\tilde{S}_R}^2 = D(1 - D/Q)$, independent from S. The random variable X_2 is Gaussian with zero mean and variance P_2, independent of S and $\hat{S}_R$. The random variable X_1 is composed of two parts, $X_1 = X_{1R} + X$, where X_{1R} is Gaussian with zero mean and variance γP_1, for some $\gamma \in [0,1]$, is independent of S, $\hat{S}_R$, X_2; and $X = \sqrt{\beta\bar{\gamma}P_1/P_2}X_2 + X'$, where X' is Gaussian with zero mean and variance $\beta\bar{\gamma}P_1$, for some $\beta \in [0,1]$, and is independent of X_{1R}, X_2 and $(S, \hat{S}_R)$. The auxiliary random variables are chosen as

$$V = \Big(\sqrt{\frac{\beta\bar{\gamma}P_1}{P_2}} + 1\Big)X_2 + \alpha_2\hat{S}_R \tag{17a}$$

$$U = X' + \alpha(S - \alpha_2\hat{S}_R) \tag{17b}$$

$$U_R = X_{1R} + \alpha_R(1 - \alpha)S \tag{17c}$$

where

$$\alpha_2 = \frac{(\sqrt{\bar{\beta}\bar{\gamma}}P_1 + \sqrt{P_2})^2}{(\sqrt{\bar{\beta}\bar{\gamma}}P_1 + \sqrt{P_2})^2 + \beta\bar{\gamma}P_1 + (N_3 + D + \gamma P_1)} \quad (18a)$$

$$\alpha_R = \frac{\gamma P_1}{\gamma P_1 + N_2}, \quad D := Q\frac{N_2}{N_2 + \gamma P_1}. \quad (18b)$$

Remark 6: In [9], we also establish another lower bound on the capacity of the state-dependent Gaussian RC with informed source. This lower bound is obtained by a coding scheme in which the source sends to the relay in block i a quantized version of the input the relay would send in block $i + 1$ had the relay known the state of the channel in that block, in the spirit of Theorem 2. This result is not included in this paper due to space limitation. However, it is depicted for some numerical examples in Section IV-D, for comparison reasons; and is shown to be asymptotically optimal for very small noise at the relay. The reader may refer to [9] for the details of this scheme.

C. Analysis of Some Special Cases

***Corollary** 1:* The capacity of the degenerate frequency-division Gaussian RC with informed source and orthogonal components is given by

$$\mathcal{C}_{\text{G-DegParOrth}} = \max_{0 \le \gamma \le 1} \min\left\{\frac{1}{2}\log(1 + \frac{\gamma P_1}{N_2}), \frac{1}{2}\log(1 + \frac{P_2}{N_3})\right\}$$
$$+ \frac{1}{2}\log(1 + \frac{(1 - \gamma)P_1}{N_3}). \quad (19)$$

1) If $N_2 \longrightarrow \infty$, the upper bound of Theorem 3 tends to

$$\mathcal{C}_{\text{G}} = \frac{1}{2}\log(1 + \frac{P_1}{N_3}), \quad (20)$$

which is achieved by standard DPC at the source.

2) *Arbitrarily strong channel state:* In the asymptotic case $Q \to \infty$, the lower bound of Theorem 4 tends to
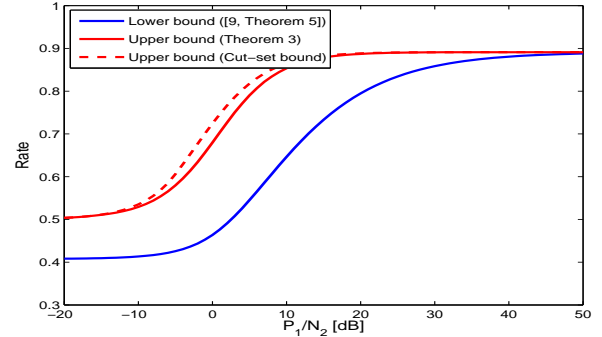
$$R_{\text{G}}^{\text{lo}} = \frac{1}{2}\log\left(1 + \frac{P_1}{\max(N_2, N_3)}\right). \quad (21)$$
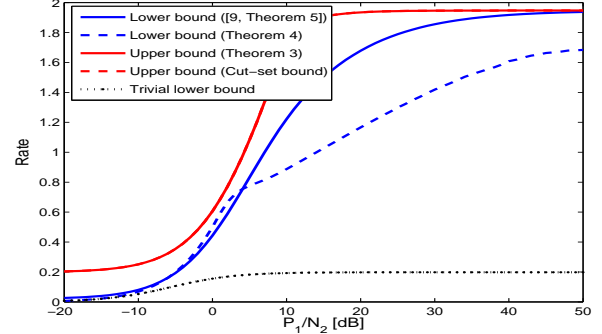
D. Numerical Examples and Discussion

Figure 2 illustrates the upper bound of Theorem 3, the lower bound of Theorem 4 and the lower bound of [9, Theorem 5] (as mentioned in Remark 6) for the model (9) as functions of SNR = P_1/N_2 (in decibels). Also shown for comparison are the cut-set upper bound and the trivial lower bound obtained by considering the channel state as unknown noise and implementing classic DF at the relay. The figure shows that the lower bound of [9, Theorem 5] is asymptotically optimal in SNR. Note that it outperforms the lower bound (14) of Theorem 4 for almost all SNR values. Also, the upper bound (12) is strictly better than the cut-set upper bound, as we indicated in the proof of Theorem 3.

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(a)

(b)

Fig. 2. Lower and upper bounds on the capacity of the state-dependent General Gaussian RC with informed source versus the SNR in the link source-to-relay, for two examples of numerical values (a) $P_1 = N_3 = 10$ dB, $P_2 = 5$ dB, $Q = 30$ dB, and (b) $P_2 = 20$ dB, $P_1 = Q = N_3 = 10$ dB.

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