On the Capacity of a Class of Relay Channels with Orthogonal Components and Noncausal State Information at Source

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Abstract— **We study the capacity of a class of state-controlled relay channels with orthogonal channels from the source to the relay and from the source and relay to the destination. The channel states are assumed to be known, non-causally, to only the source. This model is useful for relaying in the context of cognition and certain interference-aware networks. For the discrete memoryless case, we establish lower bounds on the channel capacity. For the memoryless Gaussian case, we establish lower and upper bounds on the channel capacity. The upper bound is strictly better than the cut-set upper bound, and is tight for certain special cases.**

I. Introduction

A state-controlled discrete memoryless relay channel (RC) consists of: a source input $X_1 \in \mathcal{X}_1$, a relay input $X_2 \in \mathcal{X}_2$, a relay output $Y_2 \in \mathcal{Y}_2$, a destination output $Y_3 \in \mathcal{Y}_3$, and a random parameter $S \in \mathcal{S}$ that controls the channel, through a given memoryless probability law $W_{Y_2,Y_3|X_1,X_2,S}$. In this paper, we assume that only the source knows the states of the channel, non-causally.

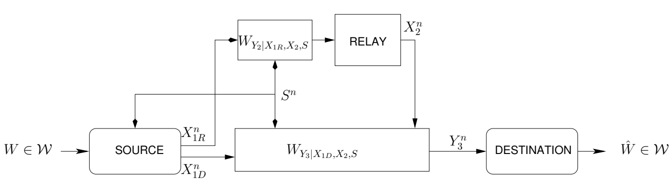


Fig. 1. Relay channel with orthogonal components from the source to the relay and from the source and relay to destination, and state information S^n available non-causally at only the source.

The source wants to transmit a message W to the destination with the help of the relay, in n channel uses. The message W is assumed to be uniformly distributed over the set $\mathcal{W} = \{1, \ldots, M\}$. The information rate R is defined as $n^{-1} \log M$ bits per transmission.
An (M, n) code for the state-dependent relay channel with informed source consists of an encoding function at the source

$$\phi_1^n : \ \{1, \ldots, M\} \times \mathcal{S}^n \to \mathcal{X}_1^n,$$

a sequence of encoding functions at the relay

$$\phi_{2,i} : \ \mathcal{Y}_{2,1}^{i-1} \to \mathcal{X}_2,$$

for $i = 1, 2, \ldots, n$, and a decoding function at the destination

$$\psi^n : \ \mathcal{Y}_3^n \to \{1, \ldots, M\}.$$

Let a (M, n) code be given. The sequences X_1^n and X_2^n from the source and the relay, respectively, are transmitted across a state-dependent relay channel modeled as a memoryless conditional probability distribution $W_{Y_2,Y_3|X_1,X_2,S}$. The joint probability mass function on $\mathcal{W} \times \mathcal{S}^n \times \mathcal{X}_1^n \times \mathcal{X}_2^n \times \mathcal{Y}_2^n \times \mathcal{Y}_3^n$ is given by

$$\begin{aligned} P(w, s^n, x_1^n, x_2^n, y_2^n, y_3^n) =& P(w) \prod_{i=1}^{n} Q_S(s_i) P(x_{1,i}|w, s^n) P(x_{2,i}|y_2^{i-1}) \\ & \cdot W_{Y_2,Y_3|X_1,X_2,S}(y_{2,i}, y_{3,i}|x_{1,i}, x_{2,i}, s_i). \qquad (1) \end{aligned}$$

The destination estimates the message sent by the source from the channel output Y_3^n. The average probability of error is defined as $P_e^n = \mathbb{E}_S\Big[\Pr\Big(\psi^n(Y_3^n) \neq W | S^n = s^n\Big)\Big]$.

An (ϵ, n, R) code for the state-dependent RC with informed source is an $(2^{nR}, n)-$code $(\phi_1^n, \phi_2^n, \psi^n)$ having average probability of error P_e^n not exceeding ϵ.

A rate R is said to be achievable if there exists a sequence of $(\epsilon_n, n, R)-$codes with $\lim_{n\to\infty} \epsilon_n = 0$. The capacity $\mathcal{C}$ of the state-dependent RC with informed source is defined as the supremum of the set of achievable rates.

A. Studied Model

In this paper, we study the following class of discrete-memoryless relay channels with states at the source and orthogonal components from the source to the relay and from the source and relay to the destination.

Definition 1: A discrete memoryless state-dependent relay channel with informed source $W_{Y_2,Y_3|X_1,X_2,S}$ is said to have orthogonal components if the source alphabet $\mathcal{X}_1 = \mathcal{X}_{1R} \times \mathcal{X}_{1D}$ and the joint probability mass function on

$\mathcal{W}\times\mathcal{S}^n\times\mathcal{X}_1^n\times\mathcal{X}_2^n\times\mathcal{Y}_2^n\times\mathcal{Y}_3^n$ can be expressed as

$$\begin{aligned}&P(w,s^n,x_1^n,x_2^n,y_2^n,y_3^n)\\&=P(w)\prod_{i=1}^{n}Q_S(s_i)P(x_{1R,i}|w,s^n)P(x_{1D,i}|w,s^n)P(x_{2,i}|y_2^{i-1})\\&\cdot W_{Y_2|X_{1R},X_2,S}(y_{2,i}|x_{1R,i},x_{2,i},s_i)W_{Y_3|X_{1D},X_2,S}(y_{3,i}|x_{1D,i},x_{2,i},s_i).\end{aligned}\quad(2)$$

The studied model is shown in Figure 1.

B. Related Work

A growing body of work studies multi-user state-dependent models. A comprehensive overview about recent advances in this regard can be found in [1], and many other works. Key to the investigation of a state-dependent model is whether the parameters controlling the channel are known to *all* or *only some* of the users in the communication model. If the parameters of the channel are known to only some of the users, the problem exhibits some *asymmetry* which makes its investigation more difficult in general. Also, in this case one has to expect some rate penalty due to the lack of knowledge of the state at the uninformed encoders, relative to the case in which all encoders would be informed.

The state-dependent multiaccess channel (MAC) with only one informed encoder and degraded message sets is considered in [2], [3]. The MAC with two correlated states each known at one encoder is considered in [4], [5]. The state-dependent relay channel (RC) with only informed relay is considered in [6]–[8].For all these models, the authors develop non-trivial outer or upper bounds that permit to characterize the rate loss due to not knowing the state at the uninformed encoders. Key feature to the development of these outer or upper bounding techniques is that, in all these models, the uninformed encoder not only does not know the channel state but can learn no information about it.

The state-dependent relay channel with only informed source seemingly exhibits some similarities with the aforementioned models. However, it departures from them in that establishing a non-trivial upper bound on the channel capacity is more involved, comparatively. Mostly, this is because, in this model, the uninformed encoder (the relay) is also a receiver, and so, it can potentially get some information about the channel states from directly observing the past received sequence from the informed encoder. That is, at time i, the input $X_{2,i}$ of the relay can potentially depend on the channel states through $Y_2^{i-1}=(Y_{2,1},\ldots,Y_{2,i-1})$.

The state-dependent relay channel with informed source has been studied in many recent works. In [8]–[12] (and the references therein), the authors establish lower bounds on the capacity. In the very recent works [13], [14], the authors establish lower as well as well non-trivial upper bounds on the capacity of the state-dependent relay channel with informed source. The derived bounds agree in certain special cases, and so fully characterize the capacity for these cases.

C. Main Contributions

In this work, we study a class of state-dependent relay channels with orthogonal components from the source to the relay and from the source and relay to the destination and states known (noncausally) only at the source. We focus on discrete memoryless (DM) as well as Gaussian models. For both cases, we derive lower bounds on the channel capacity; and for the Gaussian case, we also derive an upper bound on the capacity. The upper bound in the Gaussian case is tighter than that obtained by assuming that the channel state is also available at the relay and the destination, i.e., the max-flow min-cut or cut-set upper bound, and it helps characterizing the rate loss due to the asymmetry caused by having the channel state available at the source but not the relay.

A key feature of the studied model is that, assuming decode-and-forward relaying, the input of the relay should be generated using binning against the state that controls the channel in order to combat its effect and, at the same time, combine coherently with the source transmission. We develop two lower bounds on the capacity by using coding schemes which achieve this goal differently. In the first coding scheme, the source describes the channel state to the relay, through a combined classic rate distortion, binning and decode-and-forward scheme. The relay guesses an estimate of the transmitted information message and of the channel state and then utilizes the state estimate to perform cooperative binning with the source for sending the information message on the multiaccess part of the channel.

In the second coding scheme, the source describes to the relay the appropriate input that the relay would send had the relay known the channel state. The relay then simply guesses this input and sends it in the next block. We show that the lower bound obtained with this scheme achieves close to optimal for some special cases

The results in this paper are extended to general state-dependent relay channels with states at the source in [13], [14].

II. Discrete Memoryless Case

In this section, we assume that the alphabets $\mathcal{S}$, $\mathcal{X}_1$, $\mathcal{X}_2$, $\mathcal{Y}_2$, $\mathcal{Y}_3$ in the model are all discrete and finite.

A. Lower Bound on Channel Capacity: State Description

The following theorem provides a lower bound on the capacity of the state-dependent discrete memoryless RC with orthogonal components and informed source.

Theorem 1: The capacity of the discrete memoryless state-dependent relay channel with orthogonal components and informed source is lower bounded by

$$\begin{aligned}R^{\mathrm{lo}}=\max\ \min\ \Big\{&I(U_R,U_1;Y_2|V,\hat{S}_R)-I(U_R,U_1;S|V,\hat{S}_R)-I(S;\hat{S}_R),\\&I(V;Y_3)-I(V;\hat{S}_R)\Big\}+[I(U_2;Y_3|V)-I(U_2;S,\hat{S}_R|V)]\end{aligned}\quad(3)$$

subject to the constraint

$$\begin{aligned}I(S;\hat{S}_R)\le{}&I(U_R;Y_2|V,U_1,\hat{S}_R)-I(U_R;S|V,U_1,\hat{S}_R)\\&+[I(U_1;Y_2|V,\hat{S}_R)-I(U_1;S|V,\hat{S}_R)]_-\end{aligned}\quad(4a)$$

where $[x]\triangleq\min(x,0)$, and the maximization is over all joint measures on $\mathcal{S}\times\hat{\mathcal{S}}_R\times\mathcal{U}_R\times\mathcal{U}_1\times\mathcal{U}_2\times\mathcal{V}\times\mathcal{X}_1\times\mathcal{X}_2\times\mathcal{Y}_2\times\mathcal{Y}_3$ of the form

$$\begin{aligned}&P_{S,\hat{S}_R,U_R,U_1,U_2,V,X_1,X_2,Y_2,Y_3}\\&\quad=Q_SP_{\hat{S}_R|S}P_{V|\hat{S}_R}P_{U_1,U_R|V,S,\hat{S}_R}P_{U_2,|V,S,\hat{S}_R}P_{X_{1R}|V,U_1,U_R,S,\hat{S}_R}\\&\quad\cdot P_{X_{1D}|V,U_2,S,\hat{S}_R}W_{Y_2|X_{1R},X_2,S}W_{Y_3|X_{1D},X_2,S}\end{aligned}\quad(5)$$

and satisfying

$$I(U_2;Y_3|V) - I(U_2;S,\hat{S}_R|V) > 0. \tag{6}$$

Remark 1: The intuition for the coding scheme which we use to establish the lower bound in Theorem 1 is as follows. Had the relay known the state, the source and the relay could implement collaborative binning against that state for transmission to the destination [15]. Since the source knows the state of the channel non-causally, it can transmit a description of it to the relay *ahead of time*. The relay recovers the state (with a certain distortion), and then utilizes it in the relevant block through a collaborative binning scheme. The hope is that the benefit that the source can get from being assisted by a more capable relay largely compensates the loss caused by the source's spending some of its resources to make the relay learn the state.

Remark 2: The coding scheme that we employ to establish the lower bound in Theorem 1 uses a combination of generalized block Markov encoding [16], Gel'fand-Pinsker binning [17], and classic rate distortion theory [18].

The message W to be transmitted is split into two independent parts, $W = (W_r, W_d)$, where message W_r will be sent cooperatively with the relay at rate R_r, and message W_d will be sent directly to the destination at rate R_d. The total rate is $R = R_r + R_d$. Transmission is performed in B blocks. In each block i, the source sends message w_{ri} and a description of the state sequence $\mathbf{s}[i+1]$ on the channel to the relay. Also, it sends cooperative information w_{ri-1} and additional information w_{di} on the multiaccess channel that connects the source and relay to the destination. Let $\hat{\mathbf{s}}_R[\iota_{Ri}]$ be the description of $\mathbf{s}[i+1]$ intended to be recovered at the relay. The relay guesses the source's message w_{ri} and recovers the description $\hat{\mathbf{s}}_R[\iota_{Ri}]$. It will then utilize the state estimate $\hat{\mathbf{s}}_R[\iota_{Ri}]$ as non-causal state at the encoder for sending the cooperative information w_{ri-1} to the destination in block $i+1$, through a collaborative source-relay binning. The destination jointly decodes the source's message w_{ri-1} sent cooperatively by the source and relay and the additional information w_{di} sent directly by the source from its output $\mathbf{y}_3[i]$.

B. Lower Bound on Channel Capacity: Transmitting Analog Input

The following theorem provides a lower bound on the capacity of the state-dependent discrete memoryless RC with orthogonal components and informed source.

***Theorem** 2:* The capacity of the discrete memoryless state-dependent relay channel with orthogonal components $W_{Y_2|X_{1R},X_2,S}W_{Y_3|X_{1D},X_2,S}$ with informed source is lower bounded by

$$R = \max\ I(U;Y_3) - I(U;S) \tag{7}$$

subject to the constraint

$$I(X;\hat{X}) \leq I(V;Y_2) - I(V;S) \tag{8}$$

where the maximization is over all joint measures on $\mathcal{S} \times \mathcal{V} \times \mathcal{U} \times \mathcal{X}_{1R} \times \mathcal{X}_{1D} \times \mathcal{X}_2 \times \mathcal{X} \times \hat{\mathcal{X}} \times \mathcal{Y}_2 \times \mathcal{Y}_3$ of the form

$$\begin{aligned} &P_{S,V,U,X_{1R},X_{1D},X_2,X,\hat{X},Y_2,Y_3} \\ &= Q_S P_{V,X_{1R}|S} P_{U,X|S} P_{\hat{X}|X} \mathbb{1}_{X_2=\hat{X}} \mathbb{1}_{X_{1D}=X} W_{Y_2|X_{1R},X_2,S} W_{Y_3|X_{1D},X_2,S}. \end{aligned} \tag{9}$$

Remark 3: The rationale for the coding scheme which we use to obtain the lower bound in Theorem 2 is as follows. With DF relaying, had the relay known the state then in each block the relay generates its input using the source transmission in the previous block and the state that controls the channel in the current block, as in [15]. For our model, the source knows what cooperative information, i.e., part of the message, the relay would send in each block. It also knows the state sequence that corrupts the transmission in that block. It can then generate the appropriate relay input vector that the relay would send had the relay known the state. The source can send this vector to the relay *ahead of time*, and if the relay can estimate it to high accuracy, then collaborative source-relay binning in the sense of [15] is readily realized for transmission from the source and relay to the destination.

Outline of Proof: A block Markov encoding with $B+1$ blocks is used. Let us denote by $\mathbf{x}[k]$ the relay input carrying message $w_k \in [1, 2^{nR}]$ that the relay would send in block k had the relay known the state $\mathbf{s}[k]$, assuming DF relaying, with $k = 2, \ldots, B+1$. That is, for $k = 2, \ldots, B+1$, the vector $\mathbf{x}[k]$ is generated as a deterministic function of $\mathbf{s}[k]$ and some auxiliary random vector $\mathbf{u}[k]$ which represents the associated Gel'fand-Pinsker auxiliary random vector that carries message w_k. Let $\hat{\mathbf{x}}[m_i]$ be a description of vector $\mathbf{x}[i+1]$. In the beginning of block i, the source sends the index m_i on the channel to the relay, and message w_i on the multiaccess channel connecting the source and relay to the destination. Let $\mathbf{v}[i]$ be the Gel'fand-Pinsker auxiliary random vector that the source utilizes to implement binning against the state $\mathbf{s}[i]$ for transmission of message m_i on the channel to the relay, and $\mathbf{x}_{1R}[i]$ the corresponding input, generated as a deterministic function of $\mathbf{s}[i]$ and $\mathbf{v}[i]$ as in classical Gel'fand-Pinsker binning. In the beginning of block i, the source sends $x_1[i] = (\mathbf{x}_{1R}[i], \mathbf{x}_{1D}[i])$, with $\mathbf{x}_{1D}[i] := \mathbf{x}[i]$. In the beginning of block i, the relay knows m_{i-1} from the source transmission in previous block $i-1$, and sends $\mathbf{x}_2[i] = \hat{\mathbf{x}}[m_{i-1}]$. (Observe that since $\hat{\mathbf{x}}[m_{i-1}]$ is a description of $\mathbf{x}[i]$, the source and relay inputs on the multiacess part of the channel will then combine coherently).

III. Memoryless Gaussian Case

In this section, we consider a state-dependent RC with orthogonal components and informed source in which the channel states and the noise are additive and Gaussian. In this model, the channel state can model an additive Gaussian interference which is assumed to be known (non-causally) to only the source.

The channel outputs $Y_{2,i}$ and $Y_{3,i}$ at time instant i for the relay and the destination, respectively, are related to the channel input $X_{1,i} = (X_{1R,i}, X_{1D,i})$ from the source and $X_{2,i}$ from the relay, and the channel state S_i, by

$$Y_{2,i} = X_{1R,i} + S_i + Z_{2,i} \tag{10a}$$

$$Y_{3,i} = X_{1D,i} + X_{2,i} + S_i + Z_{3,i}. \tag{10b}$$

The channel state S_i is zero mean Gaussian random variable with variance Q; and only the source knows the state sequence S^n (non-causally). The noises $Z_{2,i}$ and $Z_{3,i}$ are zero mean Gaussian random variables with variances N_2 and N_3, respectively; and are mutually independent and independent from the state sequence S^n and the channel inputs (X_{1R}^n, X_{1D}^n) and X_2^n.

We shall also consider the following subclass of Gaussian RC with orthogonal components and informed source, the parallel Gaussian RC with orthogonal components and informed source with $Y_{3,i} = (Y_{3,i}^{(1)}, Y_{3,i}^{(2)})$ and

$$Y_{2,i} = X_{1R,i} + S_i + Z_{2,i} \tag{11a}$$
$$Y_{3,i}^{(1)} = X_{1D,i} + S_i + Z_{3,i}^{(1)} \tag{11b}$$
$$Y_{3,i}^{(2)} = X_{2,i} + S_i + Z_{3,i}^{(2)}, \tag{11c}$$

where the noises $Z_{3,i}^{(1)}$ and $Z_{3,i}^{(2)}$ are zero mean Gaussian random variables with variances N_3, and are mutually independent and independent from the state sequence S^n and the channel inputs (X_1^n, X_2^n).

A parallel Gaussian RC with informed source and orthogonal components in which the state S_i does not affect transmission from the relay to the destination will be said to be *degenerate*. Its input-output relation is given by (11) with (11c) substituted by $Y_{3,i}^{(2)} = X_{2,i} + Z_{3,i}^{(2)}$.

We consider the following individual power constraints on the average transmitted power at the source and the relay

$$\sum_{i=1}^{n} X_{1,i}^2 \leq nP_1, \qquad \sum_{i=1}^{n} X_{2,i}^2 \leq nP_2. \tag{12}$$

The definition of a code for this channel is the same as that given in the discrete case, with the additional constraint that the channel inputs should satisfy the power constraints (12).

A. Upper Bound on the Capacity

The following theorem provides an upper bound on the capacity of the state-dependent Gaussian RC with orthogonal components and informed source.

***Theorem** 3:* The capacity of the state-dependent Gaussian RC orthogonal components and informed source is upper-bounded by

$$\begin{aligned} R_{\mathrm{G}}^{\mathrm{up}} = \max\min\Big\{ & \frac{1}{2}\log\Big(1+\frac{\gamma P_1}{N_2}\Big) + \frac{1}{2}\log\Big(1+\frac{\bar{\gamma}P_1(1-\rho_{12}^2)}{N_3}\Big), \\ & \frac{1}{2}\log\Big(1+\frac{(\sqrt{P_2}+\rho_{12}\sqrt{\bar{\gamma}P_1})^2}{\bar{\gamma}P_1(1-\rho_{12}^2-\varrho_{1s}^2)+(\sqrt{\Delta_Q}+\varrho_{1s}\sqrt{\bar{\gamma}P_1})^2+N_3}\Big) \\ & + \frac{1}{2}\log\Big(1+\frac{\bar{\gamma}P_1(1-\rho_{12}^2-\varrho_{1s}^2)}{N_3}\Big)\Big\}, \end{aligned} \tag{13}$$

where $\Delta_Q = QN_2/((\sqrt{Q}+\sqrt{\gamma P_1})^2 + N_2)$ and the maximization is over parameters $\gamma \in [0,1]$, $\rho_{12} \in [0,1]$, $\varrho_{1s} \in [-1,0]$ such that

$$\rho_{12}^2 + \varrho_{1s}^2 \leq 1. \tag{14}$$

Remark 4: In the upper bound in Theorem 3, for given ρ_{12}, the maximization over ϱ_{1s} can in fact be restricted to either $\varrho_{1s} = -\sqrt{1-\rho_{12}^2}$, $\varrho_{1s} = 0$, or a real root ϱ_{1s} of the third-order polynomial $F(\varrho_{1s})$ that satisfies $\varrho_{1s} \in [-\sqrt{1-\rho_{12}^2}, 0]$, where

$$F(\varrho_{1s}) = A\varrho_{1s}^3 + B\varrho_{1s}^2 + C\varrho + D \tag{15}$$

with

$$A = 4(\bar{\gamma}P_1\Delta_Q)^{\frac{3}{2}} \tag{16a}$$
$$B = \bar{\gamma}P_1\Delta_Q\Big[(\sqrt{P_2}+\rho_{12}\sqrt{\bar{\gamma}P_1})^2 + 4(\bar{\gamma}P_1(1-\rho_{12}^2)+\Delta_Q+N_3)\Big] \tag{16b}$$
$$\begin{aligned} C = (\bar{\gamma}P_1\Delta_Q)^{\frac{1}{2}}&\Big[(\sqrt{P_2}+\rho_{12}\sqrt{\bar{\gamma}P_1})^2 + \bar{\gamma}P_1(1-\rho_{12}^2)+\Delta_Q+N_3\Big] \\ &\cdot\Big[\bar{\gamma}P_1(1-\rho_{12}^2)+\Delta_Q+N_3\Big] \end{aligned} \tag{16c}$$
$$D = \Delta_Q\Big[\bar{\gamma}P_1(1-\rho_{12}^2)+N_3\Big]\Big[\sqrt{P_2}+\rho_{12}\sqrt{\bar{\gamma}P_1}\Big]^2. \tag{16d}$$

Outline of Proof: The proof of Theorem 3 follows along the lines of that of [13, Theorem 3]. Thus, we only sketch the important steps, for brevity. The proof of the bound given by the first term of the minimization in (13) trivially follows by revealing the state S^n to the relay and the destination. The proof of the bound given by the second term of the minimization in (13) is as follows. First, we show that there is an inevitable residual uncertainty at the relay about the state sequence S^n after observing the channel outputs $Y_2^{i-1} = (Y_{2,1}, \ldots, Y_{2,i-1})$. Then, considering transmission from the source and relay to the destination, we upper bound the sum rate that can be conveyed to the destination on the multiaccess part of the channel by accounting for the rate penalty that is caused by not knowing the state fully at the relay. In doing so, we assume that the message is revealed to the relay by a genie.

B. Lower Bounds on the Capacity

The following theorem provides a lower bound on the capacity of the state-dependent Gaussian RC with orthogonal components and informed source.

***Theorem** 4:* The capacity of the state-dependent Gaussian RC with orthogonal components and informed source is lower-bounded by

$$R_{\mathrm{G}}^{\mathrm{lo}} = \max\frac{1}{2}\log\Big(1+\frac{(\sqrt{\bar{\gamma}P_1}+\sqrt{P_2-D})^2}{N_3+D}\Big), \tag{17}$$

where

$$D := P_2\frac{N_2}{N_2+\gamma P_1} \tag{18}$$

and the maximization is over $\gamma \in [0,1]$.

The following three remarks are useful for a better understanding of the coding scheme which we use to achieve the lower bound in Theorem 4. A detailed description of this coding scheme will follow.

Remark 5: In (10), if the source and the relay both know the state S^n, then channel capacity can be achieved through a cooperation binning scheme that consists in an appropriate combination of generalized block-Markov coding [16] and binning [17]. Similar to the case of degraded Gaussian RC with informed source and informed relay studied in [15], it can be shown that the capacity expression in this case is same as if there were no state at all [16, Section III] or the state were known at all terminals. Because source and relay both know the state in this case, they can cooperate to mitigate its effect completely; and this achieves the max-flow min-cut upper bound for the RC with orthogonal components model.

Remark 6: The rationale for the coding scheme which we use to obtain the lower bound (17) is as follows. Ideally, in each block the relay should send an appropriate code vector which is precoded against the state that corrupts the transmission in that block, in the manner described in Remark 5. This is relevant as it would allow sending at maximal rate on the multiaccess part of the channel through joint source-relay binning.
For our model, the source knows what cooperative information, i.e., part of the message, the relay should send in each block. It also knows the state sequence that corrupts the transmission in that block. It can then generate the appropriate relay input vector that the relay should send had the relay known the state. The source can send this vector to the relay *ahead of time*, and if the relay can estimate it to high accuracy, then appropriate source-relay cooperation similar to that in the scheme in Remark 5 is readily obtained for transmission from source and relay to the destination.

Proof of Theorem 4: The result of Theorem 2 for the DM case can be extended to memoryless channels with discrete time and continuous alphabets using standard techniques [19, Chapter 7]. For the state-dependent Gaussian relay channel with orthogonal components (10), we evaluate the rate (7) with the following choice of input distribution. The input X is Gaussian with zero mean and variance P_2, and is independent of S. The source input is such that X_{1R} is Gaussian with zero mean and variance γP_1, for some $\gamma \in [0,1]$, is independent of S and X_2; and $X_{1D} = \sqrt{\bar{\gamma}P_1/P_2}X$. Furthermore, we choose $\hat{X}$ according to the test channel $X = \hat{X}+Z$, where $\hat{X}$ is a Gaussian random variable with zero mean and variance $P_2 - D$ and Z is a Gaussian random variable with zero mean and variance D independent of $\hat{X}$. The relay input is $X_2 = \sqrt{P_2/(P_2-D)}\hat{X}$. The auxiliary random variables are chosen as

$$V = X_{1R} + \alpha_{1R} S \tag{19}$$

$$U = \Big(\sqrt{\frac{\bar{\gamma}P_1}{P_2}} + \sqrt{\frac{P_2-D}{P_2}}\Big)X + \alpha S, \tag{20}$$

with

$$\alpha_{1R} = \frac{\gamma P_1}{\gamma P_1 + N_2} \tag{21}$$

$$\alpha = \frac{(\sqrt{\bar{\gamma}P_1} + \sqrt{P_2-D})^2}{(\sqrt{\bar{\gamma}P_1} + \sqrt{P_2-D})^2 + N_3 + D} \tag{22}$$

and

$$D := P_2 \frac{N_2}{N_2 + \gamma P_1}. \tag{23}$$

Through straightforward algebra which is omitted for brevity, it can be shown that (7) achieves (17).

Remark 7: In classic Gaussian RC with orthogonal components, i.e., Gaussian channels without states, the capacity achieving coding scheme is such that, on the multiacess part of the channel, the source sends additional new information on top of the cooperative information that is also sent by the relay, i.e., not only the cooperative information. That is, the source input is composed of two parts, one part which is proportional to the relay input and carries cooperative information and another part which is independent from the relay input and carries additional information. This is relevant in this case as the rate of the cooperative information is limited by the information that the relay decodes reliably from Y_2, and so allowing the source to send some additional information is beneficial in general. In our case, however, the source needs not send additional information as the additional degree of freedom to the source over the relay is (already) captured by the distortion between their inputs. That is, the input $X_{1D} = X$ from the source carries more information than does the input $X_2 = \hat{X}$ from the relay.

C. Extreme Cases

We now summarize the behavior of the lower and upper bounds in some special and extreme cases.

***Corollary** 1:* The capacity of the degenerate parallel Gaussian RC with informed source and orthogonal components is given by

$$\mathcal{C}_{\text{G-DegPar}} = \max_{0\le\gamma\le 1} \min\Big\{\frac{1}{2}\log(1+\frac{\gamma P_1}{N_2}), \frac{1}{2}\log(1+\frac{P_2}{N_3})\Big\} + \frac{1}{2}\log(1+\frac{(1-\gamma)P_1}{N_3}). \tag{24}$$

1. If $N_2 \longrightarrow 0$, e.g, the relay is located spatially very close to the source, the lower and upper bounds tend asymptotically to the same value

$$\mathcal{C}_{\text{G}} = \frac{1}{2}\log\Big(1+\frac{(\sqrt{P_1}+\sqrt{P_2})^2}{N_3}\Big) - o(1) \tag{25}$$

where $o(1) \longrightarrow 0$ as $N_2 \longrightarrow 0$.
Equation (25) reflects the rationale for our coding scheme for the lower bound which is tailored to be asymptotically optimal whenever the relay can learn the input which it should send with negligible distortion. In this case, the rate (25) can be interpreted as the information between two transmit antennas which both know the channel state and one receive antenna.

2. If $N_2 \longrightarrow \infty$, the link to the relay is broken or too noisy and the distortion is equal to its maximum value P_2, resulting in

$$R_{\text{G}}^{\text{up}} = \frac{1}{2}\log(1+\frac{P_1}{N_3})$$
$$R_{\text{G}}^{\text{lo}} = \frac{1}{2}\log(1+\frac{P_1}{N_3+P_2}). \tag{26}$$

Equation (26) reflects a limitation of our coding scheme for the lower bound if the relay fails to reconstruct the input described by the source. In this case, the input from the relay acts as additional noise at the destination, thus causing the cooperative transmission to perform less good than simple direct transmission. The achievable rate (26) is, however, still better than had the state been merely treated as unknown noise if $P_2 \le Q$.

3. If $P_2 = 0$, the lower and upper bounds meet and yield the capacity of transmission to the destination through only

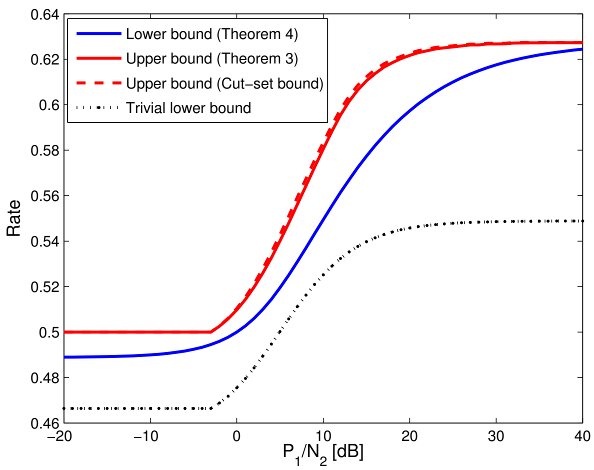
the direct link,

$$\mathcal{C}_G = \frac{1}{2}\log(1 + \frac{P_1}{N_3}). \tag{27}$$

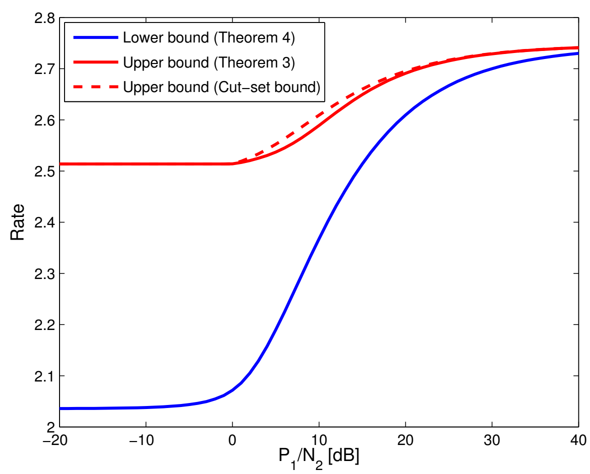
4. If $P_1 \longrightarrow \infty$, the lower bound reaches the upper bound asymptotically in the power at the source if $P_2 \ll P_1$, yielding

$$\mathcal{C}_G = \frac{1}{2}\log(1 + \frac{P_1}{N_3}). \tag{28}$$

D. Numerical Examples



(a)



(b)

Fig. 2. Lower and upper bounds on the capacity of the state-dependent Gaussian RC with orthogonal components and informed source versus the SNR in the link source-to-relay, for two examples of numerical values (a) P_2 = 20 dB, P_2 = 5 dB, Q = 10 dB, N_3 = 20 dB, and (b) P_1 = 20 dB, $P_2 = N_3$ = 5 dB, Q = 80 dB.

Figure 2 illustrates the lower bound (17) and the upper bound (13) as functions of the signal-to-noise-ratio (SNR) at the relay, i.e., SNR = P_1/N_2 (in decibels), for a degraded channel. Also shown for comparison are the cut-set upper bound given by the channel capacity had the state been known also at the relay and the trivial lower bound obtained by considering the channel state as unknown noise a generalized block-Markov coding schemes [16].

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