# Lattice-based Wyner-Ziv Coding for Parallel Gaussian Two-Way relay Channels

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***Abstract*—Parallel Two-way relay channel models a cooperative communication scenario where a relay helps two terminals to exchange their messages over independent Gaussian channels. For the single channel case, we have shown previously that lattice-based physical layer network coding achieves the same rate as compress-and-forward scheme with a random coding strategy. A direct extension of this lattice-based scheme to parallel Gaussian channel is to repeat the same strategy for each sub-channel. However this approach is not scalable with the number of sub-channels since the complexity of the scheme becomes prohibitive when a large number of sub-channels is employed. In this contribution, we investigate a lattice-based physical layer network coding scheme where the relay jointly processes all the sub-channels together. We characterize the rate region allowed by our coding scheme and assess the performance penalty compared to the separate channel processing approach.**

## I. Introduction

Two-way relay channel (TWRC) is encountered in many wireless communication scenarios: ad-hoc networks, range extension for cellular and local networks, satellite transmissions. . .

While network level routing is the standard option to this problem, it has been shown that network coding (NC) strategies provide better performance by leveraging the side information that is available in each node. In fact, NC allows to improve the rates by combining raw bits or packets at the network layer. Is has been shown that the capacity can be improved when NC is applied in the physical layer. It takes advantage of the linear superposition properties of the wireless channel in order to turn interference nuisance into useful signal [1].

One can distinguish four main strategies that have been investigated for physical layer network coding: Amplify and Forward (AF), Decode and Forward (DF), Compress and Forward (CF) and XOR emulation. The three first schemes are direct application of the corresponding strategy in single relaying scenarios [2], [3], [4], [5], [6]. The last scheme consists in emulating the XOR operation at the bit level by using specific signal mapping in each node [1]. In [7] the authors suggested to implement this functionality using nested lattices. The relay decodes a modulo-lattice sum of the transmitted codewords in order to emulate the XOR operation at the bit level. In a former contribution [8], we investigated the use of nested lattice to implement compress and forward scheme. The coding strategy is based on Wyner-Ziv (WZ) source coding proposed in [9], and achieves the sames rates as random coding techniques. The proposed approach offers a good compromise between the complexity of the processing borne by the relay and the noise amplification.

All the aforementioned schemes have been devised for two way relaying over a single channel. A significant number of modern communication scenarios are modelled by a parallel Gaussian channel: multi-carrier systems (OFDM, OFDMA, SCFDMA), multi-stream transmission with multiple antennas, spread spectrum with multiple sequences to name a few. For these systems, two way relaying offers a competitive solution for range extension. Two-way relaying over parallel Gaussian channel have been addressed in [10] and [11] for AF scheme, in [12] and [13] for DF scheme and in [14] and [15] for XOR emulation approach by duplicating the single channel strategy for all sub-channels. However, CF strategy has been only extended to parallel Gaussian channel for one-way relaying in [16] and to the best of the authors' knowledge, was not extended to TWRC.

In this paper, we design a new relaying scheme for parallel TWRC based on the CF strategy that we have proposed in [8]. We consider a joint processing strategy at the relay, where the signals received over all the channels are compressed together. The proposed scheme offers a reduced complexity compared to the separate processing of each sub-channel. We derive the achievable rate region for both, joint and separate source compression and assess the gap between both strategies.

The rest of the paper is organized as follows. In section II, we introduce our system model. In section III and IV, we propose the joint lattice-based Wyner-Ziv Coding scheme and we derive its achievable rate region. In section V, we present the optimization problem that solves the optimal power and distortion allocation for our scheme. Then we present some numerical results. Finally, section VI concludes the paper.

**Notations** Random variables (r.v.) are indicated by capital letters where the realizations are written in small letters. Vector of r.v. or a sequence of realizations are indicated by bold fonts.

## II. System Model

We consider a two way relaying scenario where $K$ parallel Gaussian channels are employed. Two nodes $\mathrm{T}_1$ and $\mathrm{T}_2$ exchange two individual messages $m_1$ and $m_2$, with the help

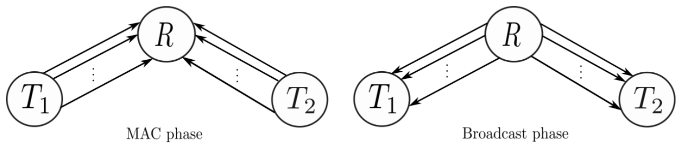


Fig. 1. The two-phase transmission of parallel Gaussian TWRC

of a relay $R$ as shown in Fig.1. The relay operates in half-duplex mode. The communication takes $n$ channel uses that are split into two phases: a multiple access (MAC) phase during which each node sends its message to the relay and a broadcast (BC) phase where the relay sends a signal which helps each node to decode its destined message. The MAC and BC phases are of lengths $n_1 = \alpha n$ and $n_2 = (1-\alpha)n$ , $\alpha \in [0,1]$ respectively. The channel coefficient between node $\mathrm{T}_i$ and the relay in sub-channel $k$ (respectively the relay and node $\mathrm{T}_i$) is denoted $h_{i\to r,k}$ (respectively $h_{r\to i,k}$). Without loss of generality, channel reciprocity is assumed for each sub-channel, i.e. $h_{i\to r,k} = h_{r\to i,k} = h_{i,k}$. During the MAC phase, node $\mathrm{T}_i$, $i = 1,2$, draws a message $m_i$ from a set $\mathcal{M}_i = \{1, 2, \cdots, 2^{nR_{i\bar{i}}}\}$ according to a uniform distribution. Each message $m_i$ is encoded to a codeword $\mathbf{x}_i(m_i) = [\mathbf{x}_{i,1}(m_i) \cdots \mathbf{x}_{i,K}(m_i)]$ which is spread over $K$ sub-channels and $n_1$ channel uses. $\mathbf{x}_{i,k}(m_i)$ is a $n_1$ sized row vector codeword which is sent by node $\mathrm{T}_i$ in the sub-channel $k$. Transmission at node $\mathrm{T}_i$ is subject to an individual power constraint $P_i$ such as:

$$\sum_{k=1}^{K} P_{i,k} \leq P_i \tag{1}$$

with $P_{i,k}$ is the allocated power in sub-channel $k$.
The messages are transmitted simultaneously through the memoryless Gaussian sub-channels and the relay $R$ receives in each sub-channel $k$ a signal $\mathbf{Y}_{r,k}$ given by

$$\mathbf{Y}_{r,k} = h_{1,k}\mathbf{X}_{1,k} + h_{2,k}\mathbf{X}_{2,k} + \mathbf{Z}_{r,k} \tag{2}$$

where $Z_{r,k} \sim \mathcal{N}(0, \sigma_{r,k}^2)$ is an i.i.d additive white Gaussian noise (AWGN).

Let $\mathbf{Y}_r = [\mathbf{Y}_{r,1} \cdots \mathbf{Y}_{r,K}]$ be the row-wise concatenation of all signals received by the relay during the MAC phase. During the BC phase, the relay generates a codeword $\mathbf{x}_r(\mathbf{y}_r) = [\mathbf{x}_{r,1}(\mathbf{y}_r) \cdots \mathbf{x}_{r,K}(\mathbf{y}_r)]$ which is spread over $n_2$ channel uses and $K$ sub-channels. The signals $\mathbf{X}_{r,k}$ are transmitted through broadcast memoryless channels and the received signals at node $\mathrm{T}_i$ are $\mathbf{Y}_{i,k}$, $i = 1,2$ for $k \in \{1, 2, \cdots, K\}$

$$\mathbf{Y}_{i,k} = h_{i,k}\mathbf{X}_{r,k} + \mathbf{Z}_{i,k}, \tag{3}$$

$Z_{i,k} \sim \mathcal{N}(0, \sigma_{i,k}^2)$ is an AWGN. We assume that perfect CSI is available for all nodes and that noise components are independent of each other and from the channel inputs. In the sequel, we investigate the achievable rates and present the design of our scheme.

## III. Achievable rate regions

*Theorem 1:* For parallel Gaussian TWRC, the convex hull of the following end-to-end rates $(R_{12}, R_{21})$ is achievable:

$$R_{12} \leq \frac{\alpha}{2}\sum_{k=1}^{K}\log_2\left(1+\frac{|h_{1,k}|^2 P_{1,k}}{\sigma_{r,k}^2+\cfrac{1}{D^*-\cfrac{1}{\max\limits_{i\in\{1,2\}}\sum_{k=1}^{K}|h_{i,k}|^2 P_{i,k}+\sigma_{r,k}^2}}}\right) \tag{4}$$

$$R_{21} \leq \frac{\alpha}{2}\sum_{k=1}^{K}\log_2\left(1+\frac{|h_{2,k}|^2 P_{2,k}}{\sigma_{r,k}^2+\cfrac{1}{D^*-\cfrac{1}{\max\limits_{i\in\{1,2\}}\sum_{k=1}^{K}|h_{i,k}|^2 P_{i,k}+\sigma_{r,k}^2}}}\right) \tag{5}$$

where $D^*$ satisfies:

$$\alpha\log_2\left(\frac{\max\limits_{i\in\{1,2\}}\sum_{k=1}^{K}|h_{i,k}|^2 P_{i,k}+\sigma_{r,k}^2}{D^*}\right) \leq (1-\alpha)\min_{i\in\{1,2\}}\sum_{k=1}^{K}\log_2\left(1+\frac{|h_{i,k}|^2 P_{r,k}}{\sigma_{i,k}^2}\right) \tag{6}$$

with $\alpha \in [0,1]$.

## IV. Proof of Theorem 1

In this section, we present a detailed proof of Theorem 1. The main idea of the proposed scheme is the following: during the BC phase, the relay station sends a compressed version of the collection of signals received during the MAC phase. The relay employs a lossy compression Wyner-Ziv scheme using nested lattices that is tuned to the user with the global weakest side information. The proof is divided into three paragraphs: in section IV-A, we present the WZ strategy which is tailored to the weakest side information at the receivers. In section IV-B, we introduce the lattice coding scheme for the WZ model and finally in IV-C, we derive the achievable rates of the proposed scheme.

### A. Wyner-Ziv using the weakest global side information

Let $\mathbf{S}_i = [h_{i,1}\mathbf{X}_{i,1}, h_{i,2}\mathbf{X}_{i,2}, \cdots, h_{i,K}\mathbf{X}_{i,K}]$, be the collection of side information available at terminal $\mathrm{T}_i$, $i = 1,2$.
We will denote by $\mathrm{T}^*$ (resp. $\mathrm{T}^+$) the user that has lowest (resp. largest) side information variance given by

$\min_{i\in\{1,2\}} \sum_{k=1}^{K} |h_{i,k}|^2 P_{i,k}$ (resp. $\max_{i\in\{1,2\}} \sum_{k=1}^{K} |h_{i,k}|^2 P_{i,k}$). We will refer to their corresponding variables by $(.)^*$ and $()^+$ respectively.

The quantization performed by the relay is tuned so that $T^*$ reconstructs a local version $\hat{\mathbf{Y}}_r^*$ of $\mathbf{Y}_r$ with a distortion $D^*$ such as $\frac{1}{n_1 K}\mathbb{E}\|\mathbf{Y}_r - \hat{\mathbf{Y}}_r^*\|^2 \leq D^*$. The other terminal $T^+$ will undergo this choice on its decoded signal at the end of the transmission.

The source $\mathbf{Y}_r$, of dimension $n_1 K$, can be written as the sum of two independent Gaussian random vectors: the side information $\mathbf{S}^*$ and the unknown part $\mathbf{U}^* = \mathbf{Y}_r|\mathbf{S}^*$ that will be decoded at the end. The variance per dimension of $\mathbf{U}^*$ is

$$\sigma_{U^*}^2 = VAR(Y_r|S^*) = \max_{i\in\{1,2\}} \sum_{k=1}^{K} |h_{i,k}|^2 P_{i,k} + \sigma_{r,k}^2 \quad (7)$$

### B. Lattice based source coding

We use a pair of $(n_1 K)$-dimensional nested lattices $(\Lambda_1, \Lambda_2)$ chosen: the fine lattice $\Lambda_1$ is good for quantization with basic Voronoi region $\mathcal{V}_1$ of volume $V_1$ and second moment per dimension $\sigma^2(\Lambda_1) = D^*$ and the coarse lattice $\Lambda_2$ is good for channel coding with basic Voronoi region $\mathcal{V}_2$ of volume $V_2$ and second moment $\sigma^2(\Lambda_1) = \sigma_{U^*}^2$. The encoding operation is performed in a WZ Lattice Coder (WZLC) with four successive operations: first, the input signal $\mathbf{Y}_r$ is scaled with a factor $\beta$. Then, a random dither which is uniformly distributed over $\mathcal{V}_1$ is added. This dither is known by all nodes. The dithered scaled version of $\mathbf{Y}_r$, $\beta\mathbf{y}_r + \mathbf{t}$ is quantized to the nearest point in $\Lambda_1$. The outcome of this operation is processed with a modulo-lattice operation in order to generate a vector $\mathbf{v}_r$ of size $n_1 K$ as shown in Fig.2.

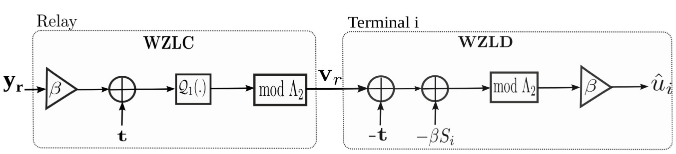


Fig. 2. WZ lattice encoding and decoding at $T_i$, $i = 1, 2$

$$\mathbf{v}_r = Q_1(\beta\mathbf{y}_r + \mathbf{t}) \mod \Lambda_2 \quad (8)$$

The relay recovers the index of $\mathbf{v}_r$ that identifies the coset of $\Lambda_2$ relative to $\Lambda_1$ that contains $Q_1(\beta\mathbf{y}_r + \mathbf{t})$. The coset leader $\mathbf{v}_r$ is represented with $\frac{V_2}{V_1}$ bits. Thus, the global source coding rate of the scheme is

$$R(D^*) = \frac{1}{n_1 K}\log_2 |\Lambda_1 \cap \mathcal{V}_2| = \frac{1}{n_1 K}\log_2 \frac{V_2}{V_1} \quad \text{(bits/dim.)} \quad (9)$$

The relay sends $K$ $n_2$-sized codewords to represent $\mathbf{v}_r$. At both users, $\mathbf{v}_r$ is generated from the subchannel-wise received codewords. $\hat{\mathbf{Y}}_{r,i}|\hat{\mathbf{S}}_i = \hat{\mathbf{U}}_i$ is reconstructed with a WZ Lattice Decoder (WZLD) using the side information $\mathbf{S}_i$ as

$$\hat{\mathbf{u}}_i = \beta((\mathbf{v}_r - \mathbf{t} - \beta\mathbf{s}_i) \mod \Lambda_2),\ i = 1, 2 \quad (10)$$

### C. Rate analysis

At the relay, the coset leader $\mathbf{v}_r$ is represented with a message $m_r$ which is mapped to $K$ $n_2$-sized codewords. Let $R_r$ be the common broadcast rate. This rate is bounded by:

$$n_1 K R(D^*) \leq n_2 R_r \quad (11)$$

On the other hand,

$$R_r \leq \min\left\{\sum_{k=1}^{K} I\left(X_{r,k}; Y_{1,k}\right), \sum_{k=1}^{K} I\left(X_{r,k}; Y_{2,k}\right)\right\} \quad (12)$$

Since real Gaussian codebooks are used for all transmissions, we have: $I(X_{r,k}; Y_{i,k}) = \frac{1}{2}\log_2\left(1 + \frac{|h_{i,k}|^2 P_{r,k}}{\sigma_{i,k}^2}\right)$, $i = 1, 2$. This constraint ensures that the message $m_r$ is transmitted reliably to both terminals over all sub-channels and $\mathbf{v}_r$ is available at the input of WZLD at both receivers. At terminal $T_i$, $\hat{\mathbf{u}}_i$ in (10) can be written as:

$$\hat{\mathbf{u}}_i = \beta((\beta\mathbf{u}_i + \mathbf{e}_q) \bmod \Lambda_2) \quad (13)$$
$$\equiv \beta(\beta\mathbf{u}_i + \mathbf{e}_q) \quad (14)$$

where $\mathbf{e}_q = Q_1(\beta\mathbf{y}_r + \mathbf{t}) - (\beta\mathbf{y}_r + \mathbf{t}) = -(\beta\mathbf{y}_r + \mathbf{t}) \mod \Lambda_1$, is the quantization error. By the Crypto Lemma, $\mathbf{E}_q$ is independent from $\mathbf{Y}_r$, thus $\mathbf{U}_i$, and it is uniformly distributed over $\mathcal{V}_1$ i.e $\text{VAR}(E_q) = \sigma^2(\Lambda_1) = D^*$. The equivalence between (13) and (14) is valid only if $\beta\mathbf{u}_i + \mathbf{e}_q \in \mathcal{V}_2$. According to [9], with good channel coding lattices, the probability $\Pr(\beta\mathbf{U}_i + \mathbf{E}_q \notin \mathcal{V}_2)$ vanishes asymptotically provided that:

$$\frac{1}{n_1 K}\mathbb{E}\|\beta\mathbf{U}_i + \mathbf{E}_q\|^2 \leq \sigma^2(\Lambda_2) \quad (15)$$

According to the properties of good lattices, we have $\frac{1}{n_1 K}\log_2(V_i) \approx \frac{1}{2}\log_2(2\pi e \sigma^2(\Lambda_i))$, $i \in \{1, 2\}$. Thus the coding rate in (9) reads:

$$R(D^*) = \frac{1}{2}\log_2\left(\frac{\sigma_{U^*}^2}{D^*}\right) \quad (16)$$

Finally, (6) is obtained by combining equations (11), (12) and (16).

The parameter $\beta$ has to be chosen so that to verify (15) and (17).

$$\frac{1}{n_1 K}\mathbb{E}\|\mathbf{Y}_r - \hat{\mathbf{Y}}_r^*\|^2 = (1-\beta^2)^2\sigma_{U^*}^2 + \beta^2 D^* \leq D^* \quad (17)$$

Taking into account that

$$\frac{1}{n_1 K}\mathbb{E}||\beta\mathbf{U}^* + \mathbf{E}_q||^2 = \frac{1}{n_1 K}\mathbb{E}||\beta\mathbf{U}^*||^2 + \frac{1}{n_1 K}\mathbb{E}||\mathbf{E}_q||^2 = \beta^2\sigma_{U^*}^2 + D^*$$

The optimal scaling factor $\beta$ is $\beta = \sqrt{1 - \frac{D^*}{\sigma_{U^*}^2}}$ as shown in [8] and [9].

The decoder at each terminal is tailored to its side information

$\sigma^2_{S_i}$. Since for $i \in \{1,2\}$, $\sigma^2_{S_i} \geq \sigma^2_{S^*}$, the following inequalities are verified at each node $\mathrm{T}_i$, $i = 1, 2$:

$$\begin{aligned} &\sigma^2_{U_i} \leq \sigma^2_{U^*}, \\ &\Pr(\beta \mathbf{U}_i + \mathbf{E}_q \notin \mathcal{V}_2) \leq \Pr(\beta \mathbf{U}^* + \mathbf{E}_q \notin \mathcal{V}_2) \underset{n_1 \to \infty}{\longrightarrow} 0, \\ &\frac{1}{n_1 K} \mathbb{E} \|\mathbf{Y}_r - \hat{\mathbf{Y}}_{r,i}\|^2 \leq D^* \end{aligned}$$

where $\hat{\mathbf{Y}}_{r,i}$ is the reconstructed source at $\mathrm{T}_i$. Thus by replacing $\mathbf{U}_i$ by its value, we conclude that:

$$\hat{\mathbf{U}}_i = \begin{bmatrix} \beta^2 h_{i,1} \mathbf{X}_{i,1} + \beta^2 \mathbf{Z}_{r,1} \\ \cdots \\ \beta^2 h_{i,K} \mathbf{X}_{i,K} + \beta^2 \mathbf{Z}_{r,K} \end{bmatrix}^{\mathrm{T}} + \beta \mathbf{E}_q \tag{18}$$

Let $\mathbf{Z}_{eq,k} = \beta^2 \mathbf{Z}_{r,k} + \beta \mathbf{E}_q(n_1(k-1) : n_1 k)$ be the effective global additive noise. The communication between $\mathrm{T}_1$ and $\mathrm{T}_2$ (resp. $\mathrm{T}_2$ and $\mathrm{T}_1$) is equivalent to virtual additive parallel Gaussian channels where the noise components are given by $\mathbf{Z}_{eq,k}$ for $k \in \{1, \cdots, K\}$. We approximate $\mathbf{E}_q$ by a Gaussian variable $\mathbf{Z}_q$ with the same variance. The equivalence is valid for asymptotic regime as $n_1 \to \infty$ [17]. Thus the achievable rate of both links satisfy:

$$\begin{aligned} nR_{12} &\leq \frac{n_1}{2} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\beta^2 |h_{1,k}|^2 P_{1,k}}{\beta^2 \sigma^2_{r,k} + D^*}\right) \\ nR_{21} &\leq \frac{n_1}{2} \sum_{k=1}^{K} \log_2 \left(1 + \frac{\beta^2 |h_{2,k}|^2 P_{2,k}}{\beta^2 \sigma^2_{r,k} + D^*}\right) \end{aligned}$$

by replacing $\frac{n_1}{n} = \alpha$ and $\beta$ by its value, (4) and (5) are verified and the proof is concluded. We refer to the proposed coding scheme as joint-WZLC.

*Remark 1:* It is possible to use the best side information $\mathbf{S}^+$ as the side information for the WZ lattice coding scheme to achieve a controlled distortion $D^+$ at terminal $\mathrm{T}^+$. For this purpose, we need two coding layers: a common layer to be sent to both nodes and a refinement layer to be decoded only by $\mathrm{T}^+$. In this case, the achievable rates can be ameliorated. This study is under investigation.

*Remark 2:* When the WZ lattice coding scheme in [8] is performed at each sub-channel separately, different distortion constraints should be verified in each sub-channel. We refer to this scheme as Separate-WZLC. The achievable rate region of this scheme is given in Theorem 2.

*Theorem 2:* For parallel Gaussian TWRC, the convex hull of the following end-to-end rates $(R_{12}, R_{21})$ is achievable:

$$R_{12} \leq \frac{\alpha}{2} \sum_{k=1}^{K} \log_2 \left(1 + \frac{|h_{1,k}|^2 P_{1,k}}{\sigma^2_{r,k} + \frac{1}{D^*_k - \frac{1}{\max\{|h_{1,k}|^2 P_{1,k}, |h_{2,k}|^2 P_{2,k}\} + \sigma^2_{r,k}}}}\right) \tag{19}$$

$$R_{21} \leq \frac{\alpha}{2} \sum_{k=1}^{K} \log_2 \left(1 + \frac{|h_{1,k}|^2 P_{1,k}}{\sigma^2_{r,k} + \frac{1}{D^*_k - \frac{1}{\max\{|h_{1,k}|^2 P_{1,k}, |h_{2,k}|^2 P_{2,k}\} + \sigma^2_{r,k}}}}\right) \tag{20}$$

where $D^*_k$ satisfies for $k \in \{1, 2, \cdots, K\}$:

$$\begin{aligned} &\alpha \log_2 \left( \frac{\max_{i \in \{1,2\}} |h_{i,k}|^2 P_{i,k} + \sigma^2_{r,k}}{D^*_k} \right) \leq \\ &(1-\alpha) \min_{i \in \{1,2\}} \left\{ \log_2 \left(1 + \frac{|h_{i,k}|^2 P_{r,k}}{\sigma^2_{i,k}}\right) \right\} \end{aligned} \tag{21}$$

with $\alpha \in [0, 1]$.

## V. Numerical results

### A. Optimization problems

We characterize the whole region of achievable rates $(R_{12}, R_{21})$ by considering all possible values for the time division coefficient $\alpha \in [0, 1]$, and by optimizing power allocation and distortions under the constraint presented in (6). The boundary points are determined by maximizing the weighted sum of both rates $R_{12}$ and $R_{21}$. The rate region can also be calculated as follows: we calculate the intersection of the achievable rate region with the line $y = \tan(\theta)$ for all $\theta \in [0, \pi/2]$. For each value of $\theta$, the limit of this intersection is found by solving the following optimization problem **OP1**:

$$\begin{aligned} \max \quad & R_{12}(\theta) && \text{(22a)} \\ \text{s.t.} \quad & 0 \leq \alpha \leq 1, && \text{(22b)} \\ & \sum_{k=1}^{K} P_{i,k} \leq P_i \;,\; i = 1, 2, r && \text{(22c)} \\ & P_{i,k} \geq 0, && \text{(22d)} \\ & D^* \text{ satisfies (6)}, && \text{(22e)} \\ & D^* \geq 0 && \text{(22f)} \end{aligned}$$

with $R_{21}(\theta) = \tan(\theta) R_{12}(\theta)$.

The optimization problem that corresponds to the achievable rates presented in Theorem 2 is different from **OP1** since it requires to optimize $K$ distortion variables, $D^*_k$, $k \in \{1, 2, \cdots, K\}$. We denote it **OP2**.

The considered optimization problems are not convex in general. Thus, the optimal joint power/time/distortion allocation policy for this problem cannot be expressed in a simple form. Hence, we are not able to provide an intuitive graphical interpretation for the optimal allocation in the general case. We provide in the sequel numerical results obtained by via Monte Carlo simulations.

### B. Simulations

We consider $K = 8$ sub-channels. The channel gains are generated using 4 i.i.d Rayleigh distributed time-domain taps. The noise variances are equal to one at each sub-channel. We present the Monte Carlo simulations for this setting where the optimization problems **OP1** and **OP2** are solved.

In Fig. 3, we consider the same power at all receivers, i.e., $P_1 = P_2 = P_r = P$. We can observe that Separate-WZLC is only 0.1 bit/channel use better than the joint-WZLC rate region.

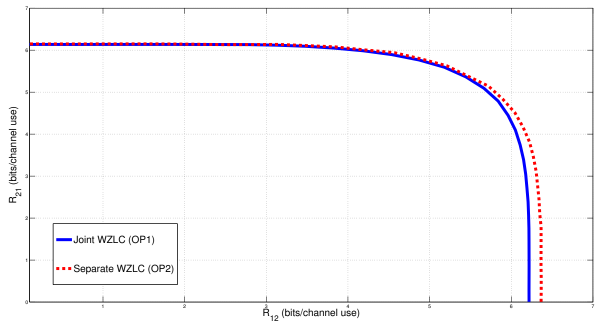


Fig. 3. Comparison between achievable rates for equal powers, P = 10 dB

The simulation results corresponding to the case where power constraints are different is represented in Fig. 4, we can notice that Joint-WZLC improves compared to Separate-WZLC with achievable rates $R_{21} > R_{12}$ for $P_1 > P_2$.

In addition, it is worth noting that the proposed scheme offers a significant complexity gain compared to Separate-WZLC scheme.

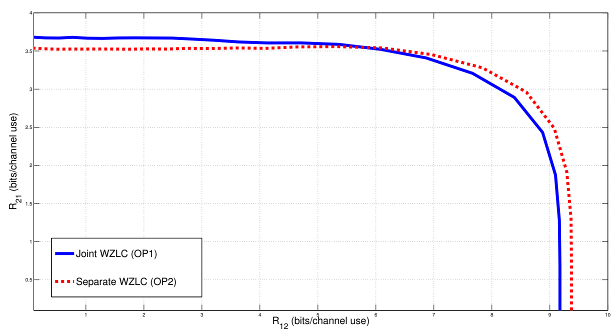


Fig. 4. Comparison between achievable rates for $P_1$ = 15 dB, $P_2$ = 5 dB and $P_r$ = 10 dB

## VI. Conclusion

In this paper, we have derived a new achievable rate region for parallel Gaussian TWRC. For this purpose, we have proposed a new practical lattice-based physical layer network coding scheme. The scheme is based on joint Wyner-Ziv source coding strategy and nested lattice codes for all sub-channels. We have shown that it achieves rates close to separate Wyner-Ziv coding at each sub-channel with reduced complexity.

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