University of Notre Dame — Aug. 28, 2007
Department of Electrical Engineering
EE 67003: Advanced Digital Signal Processing — Dr. A. Zaidi

Homework 1

Due: September 13, 2007, 12:15am (end of class)

Reading: Textbook sections 6.1-6.3, 6.4.1, 6.5

Problems from textbook:

1. Problem 6.1
2. Problem 6.9

Problem 1:

Consider the LTI system in Fig. 1,

$$H(e^{j\omega}) = e^{-j\omega/2}, \quad |\omega| \leq \pi, \quad \text{(half-sample delay).}$$

(a) Determine a choice for T and $H_a(j\Omega)$ in the system of Fig. 1(b) so that the system in Fig. 1(a) with $H(e^{j\omega})$ as specified is equivalent to the system in Fig. 1(b).

(b) Determine and sketch $y(n)$ when the input sequence is

$$x(n) = \cos\left(\frac{5\pi}{2}n - \frac{\pi}{4}\right)$$

as sketched in Fig. 2.

Problem 2:

Consider the two signal processing systems in Fig. 3, where the A/D and D/A converters are ideal. The mapping $g[x] = x^2$ represents a nonlinear memoryless operation.

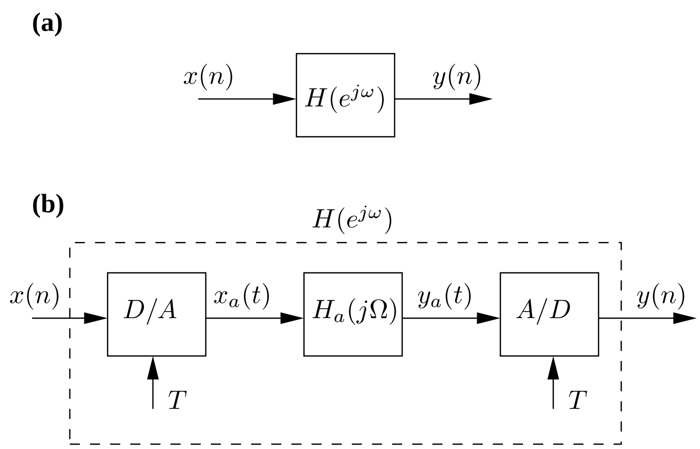

Figure 1:

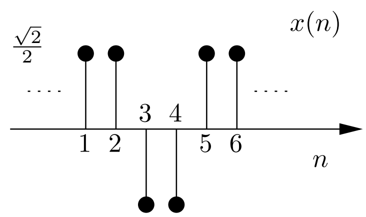

Figure 2:

(a) For the two systems in Fig. 3, sketch the signal spectra at points 1, 2, and 3 when the sampling rate is selected to be $1/T = 2f_m$ and $x_a(t)$ has the Fourier transform shown in Fig. 3. Is $y_{a1}(t) = y_{a2}(t)$? If not, why not? Is $y_{a1}(t) = x_a^2(t)$?

(b) Consider System 1 and let $x_a(t) = A \cdot \cos(30\pi t)$. Let the sampling rate be $1/T = 40\,\text{Hz}$. Is $y_{a1}(t) = x_a^2(t)$?

(c) Consider the signal-processing system shown in Fig. 4 where $g[x] = x^3$ and $g^{-1}[x]$ is the inverse, i.e., $g^{-1}[g(x)] = x$. Let $x_a(t) = A \cdot \cos(30\pi t)$ and $1/T = 40\,\text{Hz}$. Express $y(n)$ in terms of $x(n)$. Is there spectral aliasing? Express $y_1(n)$ in terms of $y(n)$. What conclusion can you reach from this example? You may find the following identity helpful: $\cos^3(\Omega_0 t) = 3/4 \cdot \cos(\Omega_0 t) + 1/4 \cdot \cos(3\Omega_0 t)$.

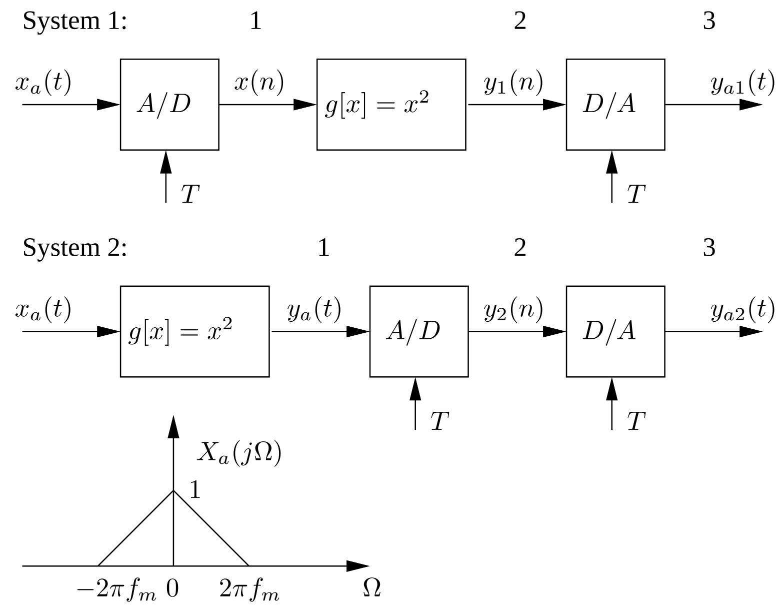

Figure 3:

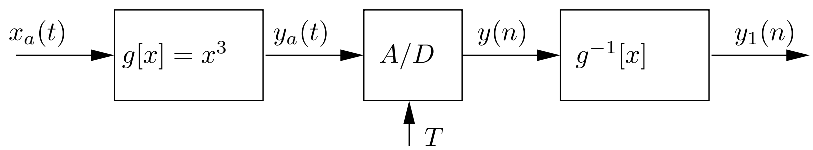

Figure 4:

(d) One practical problem is that of digitizing a signal having a large dynamic range. Suppose we compress the dynamic range by passing the signal through a memoryless nonlinear device prior to A/D conversion and then expand it back after A/D conversion. What is the impact of the nonlinear operation prior to the A/D converter in our choice of the sampling rate?