University of Notre Dame — September 5, 2007
Department of Electrical Engineering
EE 67003: Advanced Digital Signal Processing — Dr. A. ZAIDI

Homework 2

Due: September 20, 2007, 12:15am (end of class)

Reading: Textbook sections 9.1-9.3 (except Section 9.2.3)

Problems from textbook:

1. Problem 9.2
2. Problem 9.3
3. Problem 9.9 (a)
4. Problem 9.10

Problem 1:

Let $x(n)$ be a stationary white noise process with zero mean and variance σ_x^2 that is passed through a quantizer. We obtain the quantized signal-sequence $x_q(n)$ that can be modeled as $x_q(n) = x(n) + e_q(n)$ where $e_q(n)$ is the quantization error (see assumptions for the quantization error $e_q(n)$ in the class).

(a) Find the mean, the variance, and autocorrrelation sequence of $e_q(n)$. *Hint:* The autocorrelation sequence is de£ned as $\varphi(\kappa) = E\{e_q(n) \cdot e_q(n-\kappa)\}$.

(b) Compute the signal-to-quantization noise ratio $\frac{\sigma_x^2}{\sigma_e^2}$.

(c) The quantized signal $x_q(n)$ is £ltered by a causal digital £lter with impulse response $h(n) = 1/2 \cdot (a^n + (-a)^n)\, u(n)$, with the discrete step function

$u(n) = 1$ for $n \geq 0$ and zero otherwise. Determine the variance $\sigma_{e_y}^2$ of the noise produced at the output due to the input quantization noise. Calculate the output signal-to-quantization noise ratio SNR_{out}.

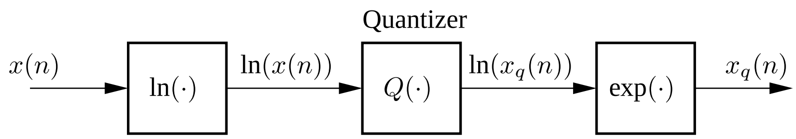

Figure 1:

In Fig. 1 the input sequence $x(n)$ is compressed before applying the uniform quantization as in (a). The output of the quantizer can be modeled as

$$\begin{aligned} \ln(x_q(n)) &= \ln(x(n)) + e_q(n) \\ \Rightarrow x_q(n) &= x(n) \cdot \exp(e_q(n)). \end{aligned}$$

For small e_q, we can approximate $\exp(e_q(n))$ by $1 + e_q(n)$, so that

$$x_q(n) \approx x(n) \cdot (1 + e_q(n)) = x(n) + f_q(n).$$

This equation will be used to describe the effect of logarithmic quantization. For $e_q(n)$ the same assumptions as in (a) are made.

(d) Determine the mean, the variance, and autocorrelation of the additive noise $f_q(n)$.

(e) What is the signal-to-quantization noise ratio $\frac{\sigma_x^2}{\sigma_f^2}$?

(f) The quantized signal is $x_q(n)$ is £ltered by a digital £lter with impulse response $h(n) = 1/2 \cdot (a^n + (-a)^n)\, u(n)$. Determine the variance $\sigma_{e_y}^2$ of the noise produced at the output due to the input quantization noise (use the result from part (c)). Calculate the output signal-to-quantization noise ratio SNR_{out}.