University of Notre Dame — September 29, 2007
Department of Electrical Engineering
EE 67003: Advanced Digital Signal Processing — Dr. A. Zaidi

Homework 3

Due: October 4, 2007, 12:15am (end of class)

Reading: Textbook sections 9.4-9.7

Problems from textbook:

1. Problem 9.30

Problem 1:

The ¤ow graph of a £rst-order system is shown in Fig. 1.

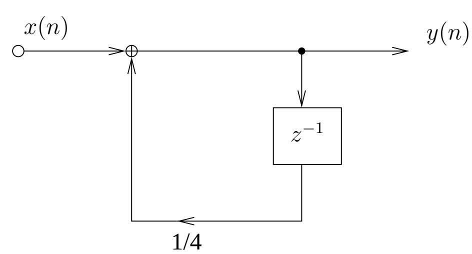

Figure 1:

(a) Assuming in£nite-precision arithmetic, £nd the response of the system to the input

$$x(n) = \begin{cases} .5 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

What is the result for large n?

Now suppose that the system is implemented with £xed-point binary arithmetic. The coef£cients and all variables in the network are represented in sign-magnitude notation with 5 bit $(b_0 b_1 b_2 b_3 b_4)$, b_0 denoting the sign. The result of a multiplication of a sequence value by a coef£cient is truncated before additions occur.

(b) Compute the response of the quantized system to the input $x(n)$ from part (a), and plot the responses of both the quantized and unquantized systems for $0 \leq n \leq 5$. How do the responses compare for large n?

(c) Now consider the system depicted in Fig. where

$$x(n) = \begin{cases} .5(-1)^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

Repeat (a) and (b) for this system and the above input.

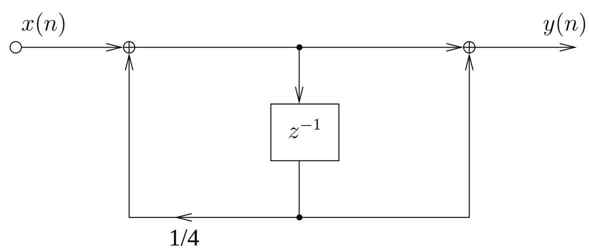

Figure 2:

Problem 2:

A causal LTI system has a system function

$$H(z) = \frac{1}{1 - 1.04\, z^{-1} + 0.98 z^{-2}}.$$

(a) Is the system stable?

(b) If the coef£cients are rounded to the nearest tenth, would the system be stable?

Problem 3:

Given is the following second-order transfer function:

$$H(z) = \frac{2 - 0.4\,z^{-1} - 0.6\,z^{-2}}{1 + 0.2\,z^{-1} - 0.15\,z^{-2}}.$$

Realize this structure in

(a) direct form II,

(b) cascade form,

(c) parallel form.

Each section in the cascade and parallel form is realized in direct form II. Sketch the corresponding ¤ow graphs. Assume now that the systems are implemented with £xed-point arithmetic, where coef£cients and all variables are represented in two's complement representation. Products are rounded before additions are performed.

1. Insert appropriate round-off noise sources into the ¤ow graph.
2. Compute the round-off noise variance at the system output for each of the three realizations, where all noise sources are assumed to have the normalized variance $\sigma_e^2 = 1$.
3. Which realization has the lowest round-off noise?

Hint: For second-order transfer functions the round-off variance at the output for an input noise process can be computed by a partial fraction expansion of $H(z)$, a subsequent inverse z-transform, and the computation of the term $\sum_n |h(n)|^2$.