

Homework 6

Due: November 20, 2007, 12:15am (end of class)

Reading: Textbook sections 11.6-11.9

Problem 1:

Let $h(n)$ denote the impulse response of a lowpass half-band filter with a zero at $z = -1$. Show that

$$h(0) = \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} h(n).$$

Problem 2:

Show that the two possible cascade configurations of a factor-of- L up-sampler and a factor-of- M down-sampler shown in Fig. 1 are equivalent if and only if the L and M are coprime.

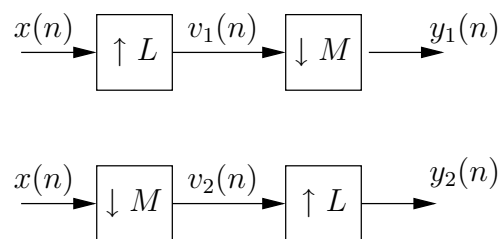


Figure 1:

Problem 3:

Consider the systems shown in Fig. 2. Suppose that $H_1(e^{j\omega})$ is fixed and known. Find $H_2(e^{j\omega})$, the frequency response of an LTI system, such that $y_2(n) = y_1(n)$ if the inputs to the systems are the same.

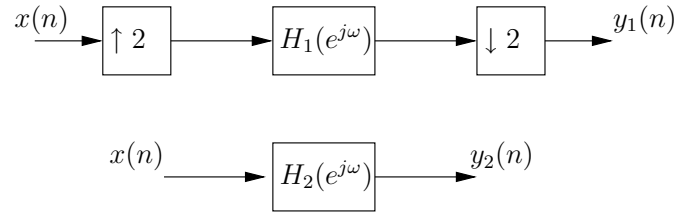


Figure 2:

Problem 4:

Consider a sequence $x(n)$ with $X(e^{j\omega})$ as shown in Fig. 3(a). Suppose we generate the sequences $v(n)$ and $y(n)$ as in Fig. 3(b), where

$$H(e^{j\omega}) = \begin{cases} 1, & \text{for } |\omega| < \pi/2 \\ 0, & \text{for } \pi/2 \leq |\omega| \leq \pi \end{cases}$$

Plot the quantities $V(e^{j\omega})$ and $Y(e^{j\omega})$.

Consider the structure shown in Fig. 4(a) with input transforms and filter responses as indicated in Fig. 4(a). Sketch the quantities $Y_0(e^{j\omega})$ and $Y_1(e^{j\omega})$.

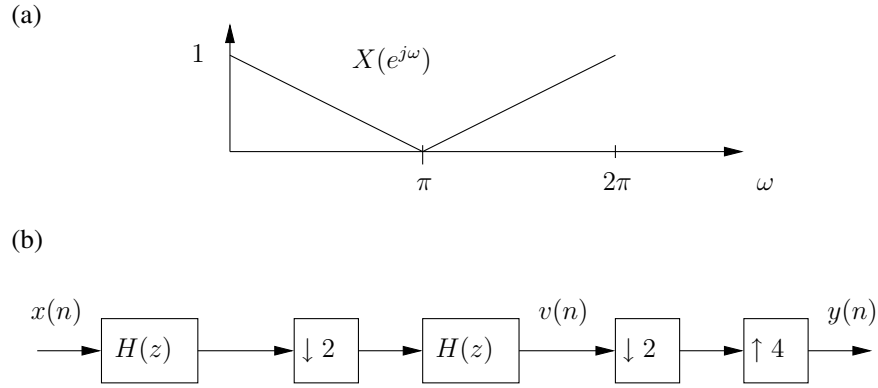
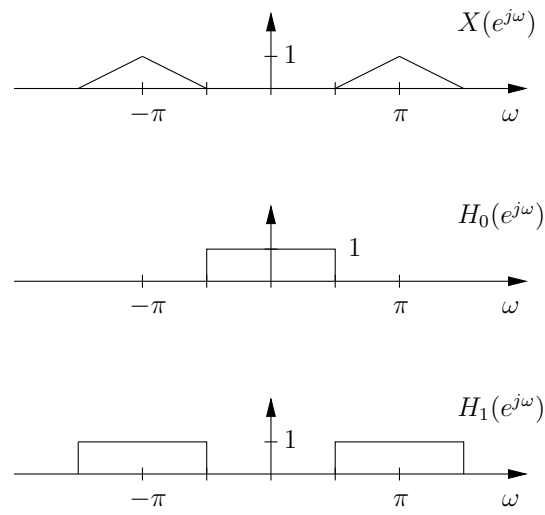


Figure 3:

Problem 5:

The multirate system of Fig. 5 implements a fixed delay of L/M samples where L and M are relatively prime integers. Let $H(z)$ be a Type 1 length N linear-phase lowpass filter with a cutoff at π/M and a passband magnitude approximately equal to M . Develop the relation between the discrete-time Fourier transforms $Y(e^{j\omega})$ and $X(e^{j\omega})$ of the output $y(n)$ and the input $x(n)$, respectively, assuming $N = 2KM + 1$, where K is a positive integer.

(a)



(b)

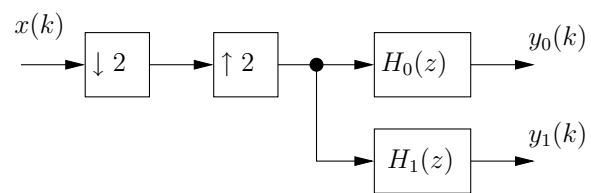


Figure 4:

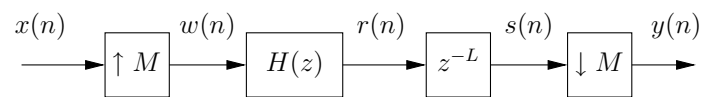


Figure 5: