University of Notre Dame — November 9, 2007
Department of Electrical Engineering
EE 67003: Advanced Digital Signal Processing — Dr. A. Zaidi

Homework 7

Due: November 27, 2007, 12:15am (end of class)

Reading: Textbook sections 11.10-11.12

Problem 1:

Develop an expression for the output $y(n)$ as a function of the input $x(n)$ for the multirate structure in Fig. 1.

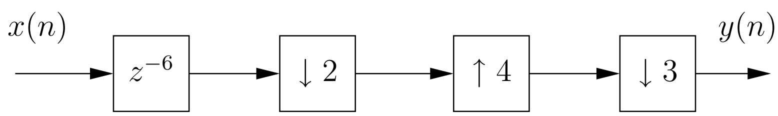

Figure 1:

Problem 2:

Consider the analysis-synthesis system shown in Fig. 2. The lowpass filter $h_0(n)$ is identical in the analysis and synthesis bank, and the highpass filter $h_1(n)$ is identical in the analysis and synthesis bank. The Fourier transforms of $h_0(n)$ and $h_1(n)$ are related by

$$H_1(e^{j\omega}) = H_0(e^{j(\omega+\pi)})$$

(a) If $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are shown as in Fig. 3, sketch (to within a scaling factor) $X_0(e^{j\omega})$, $G_0(e^{j\omega})$, and $Y_0(e^{j\omega})$.

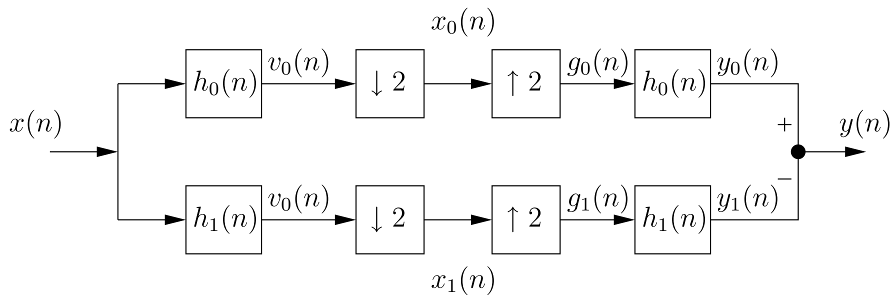

Figure 2:

(b) State a general expression for $G_0(e^{j\omega})$ in terms of $X(e^{j\omega})$ and $H_0(e^{j\omega})$. Do not assume that $X(e^{j\omega})$ and $H_0(e^{j\omega})$ are as shown in Fig. 3.

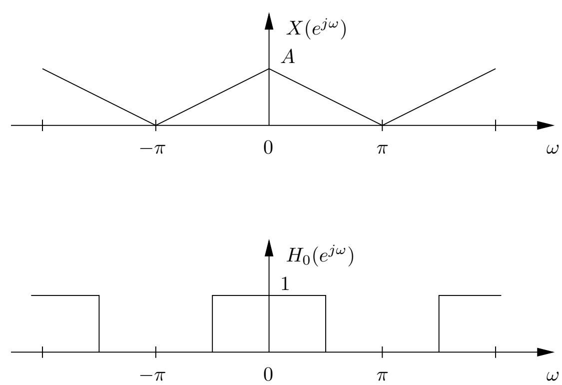

Figure 3:

(c) Determine a set of conditions on $H_0(e^{j\omega})$ that is as general as possible and that will guarantee that $y(n)$ is proportional to $x(n-D)$ for any stable input $x(n)$.

Problem 3:

(a) Show that the two-channel QMF bank, in general, is a linear, time-varying system with a period of 2.

(b) The four filters of the two-channel QMF bank are given by $H_0(z) = 3 + 4z^{-1}$, $H_1(z) = 1 + 2z^{-1}$, $F_0(z) = -0.5 + z^{-1}$, and $F_1(z) = 1.5 - 2z^{-1}$. Show that the QMF bank is a perfect reconstruction system.

(c) The highpass filter $H_1(z)$ of a two-channel perfect reconstruction orthogonal filter bank is given by $H_1(z) = a + bz^{-1} + cz^{-2} + dz^{-3}$. Determine $H_0(z)$, $F_0(z)$, and $F_1(z)$.