University of Notre Dame
Department of Electrical Engineering
EE 67003: Advanced Digital Signal Processing

November 9, 2007

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Homework 8

Due: December 4, 2007, 12:15am (end of class)

Reading: Textbook sections 14.1-14.3

Problem 1:

Prove that the normalization factor

$$U = \frac{1}{L}\sum_{n=0}^{L-1} |w(n)|^2$$

used in Welch's method ensures that

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} W(e^{j\omega})d\omega = 1$$

for

$$W(e^{j\omega}) = \frac{1}{UL}\left|\sum_{n=0}^{L-1} w(n)e^{-j\omega n}\right|^2 .$$

Problem 2:

A continuous-time signal $x_a(t)$ is bandlimited to $5\,$kHz,i.e., $x_a(t)$ has a spectrum $X_a(j\Omega)$ that is zero for $|\Omega| > 2\pi \cdot 5000 s^{-1}$. Only 10 seconds of the signal have been recorded and is available for processing. Bartlett's method of periodogram averaging is used to estimate the power spectrum. The required resolution of the estimation is at least $10\,$Hz.

(a) If the data is sampled at the Nyquist rate, what is the minimum section length that is needed to get the desired resolution?

(b) Using a radix-2 FFT algorithm, with 10 seconds of data, how many sections are available for averaging if the section length is not shorter than the minimum section length determined in (a) and no zero padding is performed.

(c) How does the choice of the sampling rate affect the resolution and variance of the estimate? Are there any benefits to sampling above the Nyquist rate?

Problem 3:

Bartlett's method is used to estimate the power spectrum of a process from a sequence of $N = 2000$ samples.

(a) What is the minimum length L that may be used for each sequence if we are to have a resolution of $\Delta\omega = 2\pi \cdot 0.005$?

(b) Is it advantageous to increase L beyond the value found in (a)? If yes, why? If no, why not?

(c) The quality factor Q of a spectrum estimate is defined to be the inverse of the variability. Using Bartlett's method, what is the minimum number of samples N that are necessary to achieve a resolution of $\Delta\omega = 2\pi \cdot 0.005$, and a quality factor that is five times larger than that of the periodogram?

Problem 4:

The autocorrelation sequence $\varphi_{vv}(\kappa)$ of a random process $v(n) = A \cdot \cos(\omega_0 n + \theta(n)) + \eta(n)$ is given by

$$\varphi_{vv}(\kappa) = \frac{1}{2}A^2 \cdot \cos(\kappa\omega_0) + \sigma_\eta^2 \delta(\kappa).$$

$\eta(n)$ is white noise with variance $\sigma_\eta^2 = 1$. The phase $\theta(n)$ is uniformly distributed in the range of $-\pi ... \pi$.

(a) If $\omega_0 = 0.25 \cdot \pi$ and $A = \sqrt{2}$, find an estimate for the power spectrum $\Phi_{vv}^{(\mathrm{AR})}(e^{j\omega})$ using a second order AR model.

(b) Find the maximum of the $\Phi_{vv}^{(\mathrm{AR})}(e^{j\omega})$ in the range of $-\pi ... \pi$.