Presentation summaries

Approximation and Optimization in Image Restoration and Reconstruction
Ile de Porquerolles, France
June 8–12, 2009

COMPUTING MINIMAL PARTITIONS

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We will present a convex representation for the minimal partition problem, based on the notion of "paired calibrations" [2], [3]. We propose an efficient algorithm for minimizing this problem, and discuss the practical implementation with "many" labels. It can be applied to a number of problems, from segmentation to stereo reconstruction. The quality of the results is substantially better than with the approaches based on Pott's model, although the computational cost can be quite high. We also discuss the possibility of minimizing energies with a bulk term, such as the Mumford-Shah functional, following the "calibration" approach of [1].

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HYBRID REGULARIZATION FOR DATA RESTORATION

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During the last five years, several convex optimization algorithms have been proposed for solving inverse problems (e.g. [1,2]). Most of the time, they allow us to minimize a criterion composed of two terms one of which permits to "stabilize" the solution. Different choices are possible for the so-called regularization term, which plays a prominent role for solving ill-posed problems.
Much interest has been gained in introducing a priori information about the target image in a transformed domain. In this respect, redundant frames constitute more flexible tools than orthonormal bases for building linear representations of images. One of the drawbacks of the approaches based on wavelet representations is that they may introduce visual artefacts. Alternative solutions based on the use of the total variation can be employed but they often lead to so-called staircase effects. A compromise can be envisaged by combining these regularization functions.
We are interested in image deconvolution in the presence of non necessarily additive noise. We propose an algorithm based on [2,3] to achieve the minimization of the associated (possibly constrained) convex optimization problem when the proximity operator associated with the data fidelity term can not be explicitly expressed.

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UNSUPERVISED WAVELET-BASED REGULARIZATION IN PARALLEL MRI RECONSTRUCTION

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Since the 1990-s, parallel Magnetic Resonance Imaging (pMRI) has emerged as a powerful 3D imaging technique for reducing scanning time. To speed up acquisition, the acquired k-space is sampled R times under the Nyquist rate. Full Field of View images are then reconstructed from the aliased data acquired along complementary coils, by applying for instance the Sensitivity Encoding (SENSE) algorithm [1]. However, SENSE-based reconstructed images suffer from several artifacts because of noise and inaccurate coil sensitivity profiles. The inverse pMRI reconstruction problem being ill-conditioned, regularization tools allowed us to obtain a significant enhancement of the reconstructed image quality even at high reduction factors (e.g. $R = 4$) and low magnetic field (1.5 Tesla) [2,3]. In this talk, we summarize our recent advances for regularizing the pMRI reconstruction in the Wavelet Transform (WT) domain, which gives access to sparse image representations. Here, a special attention has to be paid about the regularization model since the data and the unknown image are complex-valued. Several penalty functions, which assume independence or not between real and imaginary parts of the wavelet coefficients, have been successively tested. On human brain data, we illustrate how the proposed regularization scheme enhances image reconstruction in comparison to other existing techniques.

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THEORY AND ALGORITHMS FOR ANISOTROPIC TRIANGULATIONS WITH APPLICATIONS TO IMAGE REPRESENTATIONS.

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We present the first results of an ongoing project revolving around approximation by finite element functions on adaptive and anisotropic triangulations, with application to image processing.
We first recall the available theory for isotropic triangulations which involves Besov-Sobolev spaces. For anisotropic triangulations, we present an analytic criterion that governs the rate of convergence in L^p norms for optimally built triangulations. We propose a greedy algorithm which has the ability to generate triangulations that exhibit a locally optimal aspect ratio and prove that the optimal convergence rate is met by the algorithm. We also present applications to image representation and compression.

FENCHEL-ROCKAFELLAR DUALIZATION OF SIGNAL RECOVERY PROBLEMS

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In [1,2] we have shown that many problems in signal recovery can be reduced to the minimization of the sum $f_1 + f_2$ of two proper lower semicontinuous convex functions in a Hilbert space $\mathcal{H}$ and provided reliable algorithms to solve this variational problem. To be numerically implementable, these algorithms require that the proximity operator of f_1 be tractable and that either f_2 be Lipschitz differentiable on $\mathcal{H}$ [1] or that its proximity operator be tractable [2]. Important formulations do not fall within these frameworks, which leaves their numerical solution an open problem. We show that a such problems can often be approached via Fenchel-Rockafellar duality. More specifically, we propose a duality framework that leads to a numerically implementable algorithm for solving both the primal problem and its dual. Convergence to primal and dual solutions is formally established. This framework captures and extends several existing duality schemes that were often devised without formal convergence proofs, and makes it possible to solve a variety of new structured signal recovery problems outside the scope of [1,2].

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THE ROLE OF IMAGE RECONSTRUCTION ALGORITHMS IN POSITRON EMISSION TOMOGRAPHY

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Positron emission tomography (PET) is a biomedical imaging technique that allows for the visualization and the measurement of biological processes. PET is based on the injection to the patient of a molecular probe for the process of interest. The probe is labeled with radio-isotopes that decay by emitting a positron. The emitted positron annihilates almost immediately with an electron, resulting in the emission of two photons that are detected in detector rings surrounding the patient. In a first approximation, data collected in such a way can be modeled as a Poisson inhomogeneous process, with mean given by the X-ray transform of the probe spatial distribution. PET reconstruction is the inverse problem associated : finding the spatial distribution of the probe from a noisy realization.
The reconstructed images suffer from two main drawbacks : limited spatial resolution and high level of statistical noise due to the low number of photons detected. Spatial regularization during reconstruction is often used, which further degrades spatial resolution. To obtain an optimal trade-off between resolution and statistical noise, it is necessary to accurately model during reconstruction the detection process and the noise in the data. Therefore, the choice of the reconstruction algorithm plays an important role in the quality of the reconstructed images, hence in their diagnostic utility. The development of new reconstruction algorithms for PET is an active field of research.
The various strategies that are used for PET image reconstruction will be presented, with emphasis on iterative techniques. The impact of the modeling approximations will be discussed in regard to the use of the reconstructed images by the physician.

ALGORITHMS FOR SUPERIORIZATION AND THEIR APPLICATIONS TO IMAGE RECONSTRUCTION

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Constrained optimization and feasibility seeking are fundamental modeling approaches in a variety of significant problems and applications. The first is sometimes too demanding conceptually and needs computationally too demanding algorithms. We are proposing a new methodology called *superiorization* that lies in between these two tasks. Our recent discovery that some (computationally less demanding) feasibility-seeking iterative algorithms can be perturbed without loosing their convergence properties (in the sense that convergence to a feasible point is retained even if such perturbations are introduced during each iterative step of the algorithm) enables us to devise useful perturbation schemes. Superiorization uses the resiliency of these algorithms to steer the iterative process towards a feasible point that is *superior* in the sense of some functional having a small value. In particular, we discuss the resiliency of two classes of projection methods (amalgamated [1] and block-iterative [2]) and illustrate the potential usefulness of them on a problem in image reconstruction from projections using total variation as the functional.

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A FAST GRADIENT PROJECTION ALGORITHM FOR ℓ_1-CONSTRAINED SIGNAL RECOVERY

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In this talk I will consider the problem of recovering a sparse object from noisy linear measurements both in the compressed sensing and inverse problem frameworks. The most common approach is to reduce the problem to a convex optimization one, involving a penalty or constraint on the ℓ_1 norm of the sequence of coefficients describing the object. Several iterative and non-iterative algorithms have been proposed for computing the optimal solution. Some of them, like iterative soft-thresholding, exhibit slow convergence and a lot of recent literature has been dedicated to devise accelerated schemes. I will present a new gradient projection algorithm, proposed in [1], which compares favorably with the fastest of these algorithms. The method exploits a line-search along the feasible direction and an adaptive steplength selection based on recent strategies for the alternation of the well-known Barzilai-Borwein rules. The performances of this algorithm will be compared with those of other iterative methods such as ISTA, FISTA, GPSR, SpaRSA and Projected Steepest Descent.

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FROM PROJECTION METHODS FOR FEASIBILITY TO THEIR EXTENSION FOR GENERAL CONVEX NONDIFFERENTIABLE CONSTRAINED OPTIMIZATION : APPLICATIONS IN EMISSION COMPUTED TOMOGRAPHY

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Sequential orthogonal projections onto convex sets (POCS) is a very well known method for solving convex feasibility problems arising in many important areas of application, especially in signal processing, computed tomography and many other large scale inverse problems as well. It often occurs that the problem to be solved is not feasible, and, in that case, POCS could be conveniently underrelaxed, in order to get a least squares solution. This underrelaxation that overcomes nonfeasibility is a clue for the natural extension of POCS to more general methods for solving non-differentiable constrained convex optimization problems. In this talk, we present a general framework that contains many old and new block projection based methods for convex optimization in their sequential and parallel versions. The new family of methods is essentially based on the separation in two steps. One step is defined by a projection-like operator that enforces feasibility and combines subgradient projections onto the sets which have a nonempty intersection (constraints set). The other step makes use of underrelaxed negative subgradient directions of the function to be minimized. Also, the approach allows the decomposition of the objective function as a sum of functions in such a way that each one can be considered separately. We present convergence results and we describe an application aiming at improving data acquisition in SPECT (Single Photon Emission Computed Tomography). This is illustrated with numerical experiments using simulated and real data.

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DECODING IN COMPRESSED SENSING

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Compressed sensing is a new paradigm for signal and image processing. In discrete compressed sensing, we want to capture a vector $x \in \mathbb{R}^N$ (where N is very large) by using a much smaller number n of measurements. These new measurements are to take the form of inner products of x with prescribed vectors $r_1, \ldots, r_n \in \mathbb{R}^N$. Thus the information we extract from x consists of the n numbers $r_j \cdot x$. This can be represented in matrix form as $y = \Phi x$ where the rows of Φ are the vectors $r_1, \ldots, r_n$ and the output y is the information we record about x.
Compressed sensing is successful if the signal x is either sparse (i.e. has relatively few nonzero components) or compressible (can be well approximated by sparse vectors). The two main questions in compressed sensing are : (i) what are the best sensing matrices Φ, (ii) how do we decode (approximate) x from the information y that we have about x.
This talk will say a little about (i) but will concentrate on item (ii) since it entails optimization techniques. We shall discuss the three main methods used for decoding in (ii), namely, ℓ_1 minimization, greedy algorithms, and iterative reweighted least squares. We shall formulate a criteria called 'instance optimality' for measuring performance of the encoding-decoding and then discuss which encoder-decoder pairs achieve the highest range of instance optimality.

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GAUSS-MARKOV-POTTS PRIORS FOR IMAGES IN COMPUTER TOMOGRAPHY RESULTING TO JOINT RECONSTRUCTION AND SEGMENTATION

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In many applications of Computed Tomography (CT), we may know that the object under the test is composed of a finite number of materials meaning that the images to be reconstructed are composed of a finite number of homogeneous area. To account for this prior knowledge, we propose a family of Gauss-Markov fields with hidden Potts label fields. Then, using these models in a Bayesian inference framework, we are able to jointly reconstruct the images and segment them in an optimal way. In this paper, we first present these prior models, then propose appropriate MCMC or Variational Bayesian methods to compute the mean posterior estimators. We finally show a few results showing the efficiency of the proposed methods for CT with limited angle and number of projections.

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DEQUANTIZING COMPRESSED SENSING : SPARSE RECOVERY WITH NON-GAUSSIAN CONSTRAINTS

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In this paper, following the Compressed Sensing paradigm, we study the problem of recovering sparse or compressible signals from uniformly quantized measurements. We present a new class of convex optimization programs, or decoders, coined Basis Pursuit DeQuantizer of moment p (BPDQ$_p$), that model the quantization distortion more faithfully than the commonly used Basis Pursuit DeNoise (BPDN) program. Our decoders proceed by minimizing the sparsity of the signal to be reconstructed while enforcing a data fidelity term of bounded ℓ_p-norm, for $2 < p \leq \infty$.
We show that in oversampled situations, i.e. when the ratio between the number of measurements and the sparsity of the signal becomes large, the performance of the BPDQ$_p$ decoders are significantly better than that of BPDN, with reconstruction error due to quantization divided by $\sqrt{p+1}$. This reduction relies on a modified Restricted Isometry Property of the sensing matrix expressed in the ℓ_p-norm (RIP$_p$) ; a property satisfied by Gaussian random matrices with high probability.
The challenging BPDQ$_p$ optimization programs are solved by monotone operator splitting methods (Douglas-Rachford and Forward-Backward). In particular, the computation of the projector onto the ℓ_p-norm data fidelity constraint relies on a dual formulation and Newton method. To demonstrate the power of BPDQ$_p$, we report numerical simulations that support our theoretical findings and compare BPDQ$_p$ and BPDN for various signal and image reconstruction problems.

RECENT ADVANCES IN ITERATIVE SHRINKAGE/THRESHOLDING

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Iterative shrinkage/thresholding (IST) algorithms are an important component of the computational toolbox used in image restoration/reconstruction problems under non-quadratic regularization (e.g., total-variation, wavelet-based). IST algorithms are typically used to address unconstrained minimization formulations, where the objective function includes a linear-quadratic data term (corresponding to a linear observation operator followed by additive Gaussian noise) and a sparsity-inducing regularizer (typically the p-th power of an ℓ_p norm). In the first part of this talk, we will briefly review the several ways in which IST algorithms can be derived (expectation-maximization [1], majorization-minimization [2], forward-backward splitting [3], separable approximation [4]) as well as several convergence results. We will then present some recent advances : (a) new ways to derive fast IST-like algorithms for deconvolution, (b) new IST-type algorithms for non-Gaussian noise (namely Gamma and Poisson, which are common in remote sensing and medical imaging).

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ITERATIVELY RE-WEIGHTED LEAST SQUARE METHODS FOR SPARSE OPTIMIZATION

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Iteratively Re-weighted Least Squares minimization (IRLS) appears for the first time in the approximation practice in the Ph.D. thesis of C. L. Lawson in 1961 for L_∞ minimization. In the 1970s extensions of Lawson's algorithm for ℓ_p-minimization were proposed, as reported in the work of M. R. Osborne. IRLS has been proposed for sparse recovery in signal processing in [5] and for total variation minimization in [3]. We would like to present the analysis of IRLS as provided in [1], and an application in color image restoration [4]. We conclude our talk by illustrating recent joint results with H. Rauhut and R. Ward on the use of IRLS for nuclear norm minimization in low-rank matrix completion [2].

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SELF-SIMILARITY OF IMAGES IN THE CONTEXT OF THE NON-LOCAL MEANS FILTER

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The non-local means filter (NL-means) is very efficient in restoring images degraded by additive noise, thanks to its ability to exploit non local similarities between patches. In this talk we study the possibility to extend the number of similar patches by considering the self-similarity property of natural images. Although natural images are not really self-similar, the success of algorithms based on scale-invariance properties, such as fractal coding, let us expect that the NL-means could be enhanced by comparing patches at different scales. To our knowledge, the first investigation in this area has been done by Ebrahimi and Vrscay in 2008 [1], where they underlined that the distribution of similarities between patches at the same scale and patches across two scales are very close. Therefore, the use of patches at a coarser scale gives comparable results to the classical NL-means. In [2], a novel IFS-type fractal coding scheme with overlapping blocks is presented and its denoising performance is also comparable to NL-means. In [3], a model of self-similarity of images is proposed in order to describe both same-scale as well as cross-scale similarity. Despite these works, efficiently mixing same-scale and cross-scale similarities in a NL-means scheme remains an open problem.

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NON-LINEAR X-RAY CT RECONSTRUCTION FROM REAL DATA

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X-ray computed tomography (CT) performs adequately in the vast majority of clinical situations. However, in specific cases, it falls short of the required precision (e.g., for characterization of very small structures or in the presence of highly absorbing materials). In order to overcome these difficulties, it is generally necessary to cast the reconstruction problem in an estimation framework which presents significant challenges, among which : (*i*) the complexity of the geometry of modern scanners and the very large size of the corresponding data sets, and (*ii*) the non-linearity of the data formation model in the presence of highly absorbing materials.
These challenges had to be met for the development of X-ray CT vascular imaging for follow-up of patients having undergone stent placement. The key elements for the derivation of an efficient method were : (*i*) minimal approximations to the geometry of the data formation model, associated with a simplified yet explicit model of the nonlinear interactions between X-rays and highly absorbing materials ; (*ii*) development of efficient structures for data storage (observed data, object, projection matrix), suited for fast implementation of elementary matrix operations involved in most reconstruction methods (e.g., projections and backprojections) ; (*iii*) representation of the unknown object on a non-homogeneous grid in order to reduce the computation load ; (*iv*) determination of the major sources of uncertainty and selection of appropriate stochastic models thereof, the end result being a simple yet robust additive Gaussian model ; (*v*) choice of a MAP estimator with edge-preserving priors along with an estimation scheme which overcomes the numerical ill-conditioning that results from the non-homogeneous representation of the object ; (*vi*) selection of an efficient minimization algorithm, the somewhat surprising finding being that general purpose procedures, such as the bound-constrained limited-memory quasi-Newton solver l-BFGS-B, generally outperform methods commonly used for CT reconstruction.
The impact of several of the above elements will be illustrated on real-data examples.

FINITELY CONVERGENT SEQUENTIAL ALGORITHMS AND THEIR APPLICATIONS TO INTENSITY-MODULATED RADIATION THERAPY

Gabor T. Herman and **Wei Chen**

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Finding a point that satisfies a set of constraints is a common task in scientific computing, examples are the linear feasibility problem and the convex feasibility problem. One of the many application areas is Intensity-Modulated Radiation Therapy (IMRT) planning. Finitely convergent sequential algorithms can be used for solving such problems; an example of such an algorithm is ART3 [1], which is defined in such a way that its control is cyclic in the sense that during its execution it repeatedly cycles through the given constraints. We found a variant of ART3 whose control is no longer cyclic, but which is still finitely convergent and in practice it usually converges faster than ART3 does [2]. We discuss a general methodology for automatic transformation of finitely convergent sequential algorithms, in such a way that (i) finite convergence is retained and (ii) the speed of convergence is improved. The first of these two properties is proven by mathematical theorems, the second is illustrated by applying the algorithms in IMRT planning.

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ALGORITHMS FOR IMAGE RESTORATION AND RECONSTRUCTION MAKING USE OF THE MM PRINCIPLE

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In the years 1990, Half-Quadratic (HQ) algorithms have been proposed to minimize penalized least square criteria for edge-preserving image restoration purposes. They have been extensively used and studied since then. More recently, it has become clear that HQ algorithms are particular instances of a larger family of minimization algorithms that proceed along the Maximize-Minimize (MM) principle, also known as *iterative majorization*. More precisely, HQ algorithms rely on *quadratic* tangent majorant functions. However, the cost per iteration of HQ algorithms is often prohibitive for large scale problems, since each iteration involves the solving of a linear system of the same size as the image. Consequently, *truncated versions* are usually preferred in practice, at the price of leaving the MM framework.
In this talk, recent results will be presented, showing that truncated HQ algorithms possess the same convergence properties as the true HQ versions, even in the case of an arbitrarily strong truncation. Another family of algorithms will be introduced, where the MM principle is only used as a unidimensional minimization tool to choose the stepsize in given directions. In particular, convergence results will be given for non linear conjugate gradient methods using such an MM line search strategy.
The latter algorithms only apply to gradient-Lipschitz criteria. Therefore, they are not suited to minimize criteria containing a barrier function, such as those encountered in maximum entropy reconstruction or positron emission tomography. In particular, a barrier function admits no quadratic majorant function. Recent results rely on a scalar, nonquadratic majorant function to produce a new line search strategy for barrier function minimization. These results include the convergence of several descent methods suited to large scale problems, among which non linear conjugate gradient methods.

FIRST ORDER METHODS FOR NON-SMOOTH CONVEX OPTIMIZATION : PROXIMAL FORWARD-BACKWARD SPLITTING AND GENERALIZED GRADIENT METHODS

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Several problems in signal and image processing involve non-smooth convex optimization problems. Popular examples are TV methods in image processing or decoding by ℓ^1 minimiziation in compressed sensing and regularization of inverse problems [3]. In this talk we study first order methods for the minimization of functionals of the form $S + R$ where S is differentiable with Lipschitz continuous derivative and R is convex. In particular we deal with forward-backward splitting methods of the form

$$u^{n+1} = (I + \partial R)^{-1}(u^n - s_n S'(u^n))$$

which exploit that the operator $(I + \partial R)$ may be easily invertible [2]. We derive a resembling method as a generalized gradient projection method. With the help of this reformulation we are able to prove strong convergence of the iterates. Moreover, we will give conditions under which the algorithm converges with linear speed [1]. These conditons are fulfilled, for example, for the case of ℓ^1 minimization problems. The viewpoint as a generalized gradient projection method will enable us to extend some parts of the theory to non-convex functionals R.

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ITERATED PROXIMAL OPERATORS : NOVEL PERSPECTIVES AND APPLICATIONS

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Steepest descent and alternating projections are perhaps the most widely used iterative techniques for solving both convex and nonconvex problems. We unify these and many other lesser-known iterative techniques through the use of proximal operators. We update these algorithms in a proximal framework and demonstrate a surprizing range of applications, from limited memory matrix secant methods for large-scale optimization, to phase retrieval in crystallography, to $\ell 0$ minimization and combinatorial optimization.

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AVERAGE PERFORMANCE OF THE SPARSEST APPROXIMATION USING A DICTIONARY

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We consider the minimization of the number of non-zero coefficients (the ℓ_0 "norm") of the representation of a data set in a general dictionary under a fidelity constraint. (The dictionary and the norm defining the constraint are arbitrary.) This (nonconvex) optimization problem leads to the sparsest approximation.
Our goal is to assess the average performance of this approximation (APA). To this end, we describe the geometry of the sets of data yielding a K sparse solution and then evaluate their Lebesgue measure. Under the assumption that data are uniformly distributed on a domain defined using a norm ν, we derive the probability to obtain a K-sparse solution. These probabilities are expressed in terms of the parameters of the model and the accuracy of the approximation. We comment the obtained formulas and give a simulation.

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ISSUES IN THERMOACOUSTIC TOMOGRAPHY

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Thermoacoustic Tomography (TAT) is a non invasive imaging modality [1,2], which can be used in various domains such as medical imaging or small animal imaging. It can be described as follows : the sample to be imaged is illuminated by a pulsed electromagnetic energy, which results in a non-uniform thermoelastic expansion. The emitted pressure wave depends on the distribution of absorbed energy f, and the inverse problem one is then facing is that of recoring f from measurements of the acoustic pressure field performed outside the sample.
In this talk, we shall first review various aspects of the physical and mathematical modellings of TAT, and then consider several issues concerning the reconstruction problem. We shall discuss in particular the possibility and interest of an approach based on the notion of regularization by mollification [3,4].

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A SOLUTION TO THE NON-CRYSTALLOGRAPHIC PHASE PROBLEM

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When a plane wave illuminate a non-crystallographic object, the scattered waves in the far field represent the Fourier transform of the object. While the modulus of the Fourier transform can be directly measured, the phases are lost which constitutes the well-known phase problem. In this talk, I will present a solution to the non-crystallographic phase problem, denoted the oversampling method, whose principle can be traced back to the Shannon sampling theorem. When the modulus of the Fourier transform is sampled at a frequency sufficiently finer than Nyquist interval, the phase information in principle encodes into the modulus and can be directly recovered by using iterative algorithms. The applications of the oversampling method are anticipated to be very broad across several fields including physics, chemistry, materials science, nanoscience and biology.

AVERAGE PERFORMANCE OF THE SPARSEST APPROXIMATION IN AN ORTHOGONAL BASIS

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We study the minimization of the number of non zero coefficients (the so called ℓ_0-norm) of the decomposition of data into an orthogonal basis (e.g. wavelets or splines), under a data-fidelity constraint defined by an ℓ_p-norm and a tolerance constant. This non convex optimization problem can be calculated numerically and it leads to the sparsest approximation, compared to other objectives than ℓ_0.
Under the assumption that data are uniformly distributed on a domain defined via an ℓ_q-norm, we calculate the bounds of the Lebesgue measure of all data leading to a K-sparse solution quite precisely. We deduce easily the probability to obtain such a solution. We provide examples that exhibit the role of the choice of the norms and the constants defining the model.
This work is a specialization of the talk of François Malgouyres which allows the obtention of more precise bounds and various numerical simulations.

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NEW ALGORITHMS IN INFORMATION SCIENCE

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The past few years have seen an incredible explosion of new (or revival of old) fast and effective algorithms for various imaging and information science applications. These include : nonlocal means, compressive sensing, graph cuts, Bregman iteration, as well as relatively old favorites such as the level set method and PDE based image restoration. I'll give my view of where we are, hopefully giving credit to all the creators of these new and exciting multiscale techniques. In particular : remarkably successful algorithms for L1 type minimizations have recently been developed. These include L1, TV, B1,1, nonlocal TV and combinations. Bregman iteration, in its various incarnations, turns out to be unreasonably effective. New applications include image segmentation, shape optimization, etc. I'll discuss this, which is joint work with many people.

THE EMPIRICAL MODE DECOMPOSITION : A NEW FORMULATION BASED ON CONSTRAINED OPTIMIZATION

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The *empirical mode decomposition* (EMD) is a relatively recent method introduced by Huang *et al* [2], whose purpose is to adaptively decompose any signal into zero-mean components, called *Intrinsic Mode Functions* (IMF). These IMF depend on the signal (EMD is a data-driven technique) and are in practice computed by a geometric and iterative procedure whose study is particularly complicated [1]. From the mathematical point of view, the definition of the IMFs is somewhat unclear. Moreover, the condition of "zero local mean" suggested by Huang *et al* [2] should not be fulfilled in many instances, and was leading to the introduction of *weak-IMF* by Sharpley and Vatchev [4].
In this talk, we present another approach for the definition of *weak-IMF* based on the direct construction of the mean envelope of the signal. The definition of the mean envelope is achieved through the resolution of a quadratic programming problem with equality and inequality constraints. Some numerical experiments illustrate the validity of the approach and comparisons are carried out with the classical EMD.

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COMPRESSIVE WAVE COMPUTATION

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Joint work with Laurent Demanet, Stanford University.

In this talk, I will show how recent developpements in the field of compressive sensing data acquisition [1,2] also translate into advances in the large-scale simulations of wave propagation phenomena. It is indeed possible to accurately compute a wavefield by decomposing it onto a largely incomplete set of eigenfunctions of the Helmholtz operator, chosen at random. This provides a natural way of parallelizing wave simulations for memory-intensive applications.
The restoration of the wave equation solution from these compressive computation is obtained through ℓ_1 minimization, that promotes the sparsity of the solution. This "compressive sensing in the Helmholtz domain" is effective for wave propagation in 1D domains of bounded varations because the solution remains sparse enough and because the eigenvectors are delocalized enough, as shown in [4].
In practice, the compressive strategy makes sense because the computation of eigenfunctions can be assigned to different nodes of a cluster in an embarrassingly parallel way. Numerical examples are presented in one spatial dimension and show that as few as 10 percents of all eigenfunctions can suffice for accurate results. Finally, we argue that the compressive viewpoint suggests a competitive parallel algorithm for an adjoint-state inversion method in reflection seismology.

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ON SPATIAL PRIORS IN MULTIRESOLUTION IMAGE DENOISING : FROM LOCAL SPATIAL ACTIVITY INDICATORS TO MRF AND MPGSM MODELS

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Recent studies in image denoising show the importance of using (i) multiresolution transformations with improved orientation selectivity and (ii) the appropriate spatial context models. Regarding the first point, a number of the so-called "geometrical representations" have been introduced (e.g., curvelets, contourlets, shearlets,...) that represent image discontinuities better than the classical wavelets, which also results in better noise reduction results. It has been also proved that using better models for the statistical dependencies of the transform coefficients improves the estimation of the noise free data. A number of different contextual models have been proposed in this respect starting from the simple bivariate parent-child models or local spatial indicators to the more complex Markov Random Field (MRF) models, Gaussian Scale Mixture (GSM) models and recently mixtures of the GSM (MGSM), the mixtures of the projected GSM (MPGSM), Field of Experts (FoE) models and Gaussian Conditional Random Field (GCRF) models.
In this talk, we wish to discuss these developments from the perspective of our own research and to identify possible directions for future developments in this field. In particular, we will discuss detection and estimation of a signal of interest in the noisy image using a local spatial activity indicator, a MRF model and a GSM model. We will then compare this approach with a state-of-the art estimator using MPGSM model and link it to some other most recent developments in this field. We will also show some practical applications to different types of images with natural noise, including some medical imaging modalities and digital camera images.

SPECTRAL ANALYSIS APPROACHES TO 4D IMAGE RECONSTRUCTION ALGORITHMS FOR DYNAMIC POSITRON EMISSION TOMOGRAPHY

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Positron Emission Tomography (PET) can quantitatively image trace concentrations of labelled molecules at high sensitivity, at all locations within the human body. This enables detailed investigation and increased understanding of the physiology underlying healthy as well as diseased states in neurology, psychiatry, oncology and cardiology. PET's remarkable versatility arises from the vast and growing array of radiolabelled compounds (radiotracers) which are available : from water (for imaging blood flow), to fluoro-deoxyglucose (FDG, for imaging glucose metabolic rate) to a whole range of transmitter-specific ligands (e.g. for receptor imaging). In order to reconstruct images of these biological parameters it is necessary to perform dynamic (time-dependent) PET image reconstruction. Conventional 3D reconstruction methods are typically used followed by kinetic analysis to finally estimate the biological parameters of interest. However, this conventional approach can result in high levels of variance in the final images. This work considers the use of a set of temporal basis functions (a set of exponential functions of differing decays, each convolved with the arterial input function) to appropriately constrain the reconstruction of the radiotracer kinetics. The resulting images of biological parameters have lower levels of noise compared to the conventional approach, which in turn allows the spatial resolution of a given PET scanner to be more fully realized.

ON REGULARIZATION METHODS OF EM-KACZMARZ TYPE

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The EM (Expectation-Maximization) algorithm is a convenient tool for approximating maximum likelihood estimators in situations when available data are incomplete, as it is the case for many inverse problems. Our focus here is on the continuous version of the EM algorithm for a Poisson model, which is known to perform unstably when applied to ill-posed integral equations. We interpret and analyse the EM algorithm as a regularization procedure. In the case of perturbed data, a stopping rule of discrepancy type is provided, under boundedness assumptions on the problem data. Moreover, we consider regularization methods of Kaczmarz type in connection with the expectation-maximization (EM) algorithm for solving ill-posed equations. For noisy data, our methods are stabilized extensions of the well established ordered-subsets expectation-maximization iteration (OS-EM). We show monotonicity properties of the methods and present a numerical experiment which indicates that, in some cases, our extended OS-EM methods are faster than the standard EM algorithm.

The talk is based on joint work with

H. Engl (Radon Institute, Linz, and University of Vienna, Austria)
M. Haltmeier (University of Innsbruck, Austria)
A.N. Iusem (IMPA, Rio de Janeiro, Brazil)
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SAR IMAGE REGULARIZATION WITH GRAPH-CUTS BASED FAST APPROXIMATE DISCRETE MINIMIZATION

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Synthetic aperture radar (SAR) images, like other coherent imaging modalities, suffer from speckle noise. The presence of this noise makes the automatic interpretation of images a challenging task and noise reduction is often a prerequisite for successful use of classical image processing algorithms.
Markov Random Field (MRF) modelization provides a convenient way to express both data fidelity constraints and desirable properties of the filtered image. In this context, total variation minimization has been extensively used to constrain the oscillations in the regularized image while preserving its edges.
However, heavy-tailed speckle distributions such as Nakagami law that governs SAR amplitude lead to non-convex likelihood terms. The underlying MAP optimization problem is therefore difficult since many local energy minima are present. Moreover, we consider a non-smooth joint prior that favors the co-localization of edges in multi-channel images.
Graph-cuts algorithms offer an efficient way to handle both non-convexity and non-smoothness. Although they can theoretically reach a global minimum, they can hardly be applied in practice due to computational and memory constraints. We derive a minimization algorithm suitable for (joint) regularization of large images.
It has been applied applied to joint regularization of the amplitude and interferometric phase in urban area SAR images. A satisfying solution can be reached in few iterations by performing a graph-cut based combinatorial exploration of large trial moves. This algorithm is faster than existing graph-cut-based techniques. We also show that joint regularization can be performed with little computation overload. Last it helps preventing the loss of small objects (over-regularization) by merging all information.

REMOVING MULTIPLICATIVE NOISE BY DOUGLAS-RACHFORD SPLITTING METHODS

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Multiplicative noise appears in various image processing applications, e.g., gamma distributed (speckle) noise in synthetic aperture radar (SAR) or Poisson noise in connection with blur in electronic microscopy and positron emission tomography (PET).
We start by reviewing recently proposed denoising methods, in particular we will see that the minimizer of functional proposed in [2] based on the MAP estimator for gamma distributed noise in [1] coincides with the minimizer of an I-divergence - TV model. Then we propose to compute the minimizer of certain functionals by applying Douglas-Rachford splitting combined with an efficient algorithm to solve the involved nonlinear systems of equations. We prove the convergence of our iterative scheme. Note that for the involved functionals Douglas-Rachford splitting is equivalent to an alternating split Bregman algorithm. Finally we demonstrate the performance of our algorithm by numerical examples.
This is joint work with S. Setzer and T. Teuber (University of Mannheim).

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FAST GRADIENT-BASED SCHEMES FOR TOTAL VARIATION MINIMIZATION

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We present fast gradient-based schemes for image denoising and deblurring problems based on the discretized total variation (TV) minimization model with constraints. Our approach relies on combining a novel monotone version of the fast iterative shrinkage/thresholding algorithm (FISTA) we recently introduced in [1], with the well known dual approach to the denoising problem. We derive a fast algorithm for the constrained TV-based image deblurring problem. The proposed scheme is remarkably simple and is proven to exhibit a global rate of convergence which is significantly better than currently known gradient based methods. Our results [2] are applicable to both the anisotropic and isotropic discretized TV functionals. Initial numerical experiments confirm the predicted underlying theoretical convergence rate results, and demonstrate the viability and efficiency of the proposed algorithms on image deblurring problems with box constraints.

This talk is based on joint work with **Amir Beck**, Technion, Israel Institute of Technology.

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BUILDING LOOK-UP TABLES OF SPARSELY SAMPLED COLOR SIGNALS

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Look-up tables are frequently used in many devices for fast approximation of non-linear functions. Building the table requires sampling the physical process, which can be time-consuming and expensive. This is the motivation for sparse sampling. However, the functions are not necessarily smooth or convex. We describe some problems in detail and investigate methods to approximate some multidimensional color functions accurately from this sparse sampling. The methods require local modeling and the generation of values outside the range of the original function, since both the forward and inverse functions are required. The extension from locally defined functions to a global function is the basis for building the final tables.

FAST MULTI-LEVEL RECONSTRUCTION OF BIOMEDICAL IMAGES USING WAVELET SPARSITY CONSTRAINTS

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Wavelet-domain ℓ_1-regularization is a powerful approach for solving inverse problems. In their 2004 landmark paper, Daubechies et al. proved that one could solve such linear inverse problems by means of a "thresholded Landweber" (TL) algorithm [1]. While this iterative procedure is simple to implement, it is known to converge slowly. Here, we present a multilevel version of the algorithm that is inspired from the multigrid techniques used for solving PDEs, but with one important difference : instead of cycling through coarser versions of the problem (REDUCE part of multigrid), the multilevel algorithm cycles through the successive wavelet subspaces. The method works with arbitrary wavelet representations ; it typically yields a 10-fold speed increase over the standard TL algorithm, while providing the same restoration quality. We illustrate the applicability of the method to three biomedical image reconstruction problems : the deconvolution of 3D fluorescence micrographs [2], the global reconstruction of dynamic PET from time measurements [3], and the reconstruction of magnetic resonance images from arbitrary (non-uniform) k-space trajectories. We present experimental results with real data sets in all three cases.

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WAVELETS TAILORED TO FUNCTIONAL IMAGING : APPLICATIONS TO FMRI AND DYNAMIC PET

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Traditional wavelets have a number of vanishing moments that corresponds to their equivalent order of the derivation. They offer good energy compaction for piecewise smooth signals, but are less appropriate for more complex signals such as those originating in functional imaging ; e.g., the hemodynamic response after brain activation in functional magnetic resonance imaging (fMRI) and time activity curves (TACs) in positron emission tomography (PET). The framework of exponential-spline wavelets [1] allows us to design new wavelet bases that act like a given differential operator ; i.e., they can be tuned to the characteristics of a system and yield a sparse representation of some corresponding class of signals.
We show two examples. For fMRI, the wavelets are tuned according to the hemodynamic response of the system. The combination with a ℓ_1-regularization constraint reveals brain activation patterns without the knowledge of a stimulation paradigm [2]. For dynamic PET, the wavelets are tailored to the compartmental description of the dynamics of the tracer distribution. The ℓ_1-regularization constraint can then be incorporated in the tomographic reconstruction process [3].

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SIGNAL PROCESSING APPLICATIONS OF A PAIR OF SIMPLE FIXED POINT ALGORITHMS

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Inspired by the remarkable flexibility of the fixed point theory of (quasi-)nonexpansive mapping and its successful applications to inverse problems, we have developed a pair of extremely simple algorithms and have shown their effectiveness in many signal processing applications. The *hybrid steepest descent method* [1] can approximate iteratively the solution of the variational inequality problem formulated over the fixed point set of (quasi-)nonexpansive mapping in a real Hilbert space $\mathcal{H}$, hence realizes simple computational schemes for a certain *hierarchical constrained optimization problem* : given a pair of convex cost functions $f_i : \mathcal{H} \to \mathbb{R}$ $(i = 1, 2)$ and a closed convex set $C_1 \subset \mathcal{H}$, minimize f_2 over $C_2 := \arg\min_{x \in C_1} f_1(x) \neq \emptyset$. The *adaptive projected subgradient method* [2] minimize asymptotically *a certain sequence of nonnegative convex functions* over a closed convex set in $\mathcal{H}$. This algorithm offers a unified mathematical foundation of not only a wide range of the *projection based adaptive filtering algorithms* but also a powerful online classification algorithm.
In this talk, we introduce recent signal processing applications of these algorithms.

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