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# DEQUANTIZING COMPRESSED SENSING : SPARSE RECOVERY WITH NON-GAUSSIAN CONSTRAINTS

**Jalal Fadili[1], Laurent Jacques[2,3] and David Hammond[2]**

[1]GREYC CNRS-ENSICAEN-Université de Caen, 14050 Caen France
[2]Institute of Electrical Engineering, Ecole Polytechnique Fédérale de Lausanne (EPFL), CH-1015 Lausanne, Switzerland
[3]Communications and Remote Sensing Laboratory, Université catholique de Louvain (UCL), B-1348 Louvain-la-Neuve, Belgium

In this paper, following the Compressed Sensing paradigm, we study the problem of recovering sparse or compressible signals from uniformly quantized measurements. We present a new class of convex optimization programs, or decoders, coined Basis Pursuit DeQuantizer of moment $p$ (BPDQ$_p$), that model the quantization distortion more faithfully than the commonly used Basis Pursuit DeNoise (BPDN) program. Our decoders proceed by minimizing the sparsity of the signal to be reconstructed while enforcing a data fidelity term of bounded $\ell_p$-norm, for $2 < p \leq \infty$.
We show that in oversampled situations, i.e. when the ratio between the number of measurements and the sparsity of the signal becomes large, the performance of the BPDQ$_p$ decoders are significantly better than that of BPDN, with reconstruction error due to quantization divided by $\sqrt{p+1}$. This reduction relies on a modified Restricted Isometry Property of the sensing matrix expressed in the $\ell_p$-norm (RIP$_p$) ; a property satisfied by Gaussian random matrices with high probability.
The challenging BPDQ$_p$ optimization programs are solved by monotone operator splitting methods (Douglas-Rachford and Forward-Backward). In particular, the computation of the projector onto the $\ell_p$-norm data fidelity constraint relies on a dual formulation and Newton method. To demonstrate the power of BPDQ$_p$, we report numerical simulations that support our theoretical findings and compare BPDQ$_p$ and BPDN for various signal and image reconstruction problems.