Texture Synthesis Guided by a Low-Resolution Image

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Introduction

Texture synthesis: creating a large, coherent, and non-periodic texture image from a given sample.

➤ A low-resolution version of the sought texture is available in addition to the given high-resolution sample.

Notations

- $\Gamma_0(\mathbb{R}^N)$: the class of lower semi-continuous convex functions from $\mathbb{R}^N$ to $]-\infty, +\infty]$ such that $\operatorname{dom} f \neq \emptyset$.
- f^*: the conjugate of the function f.
- β-Lipschitz continuous gradient of a differentiable convex function f: if $(\forall (x,y) \in \mathbb{R}^N \times \mathbb{R}^N) \quad \|\nabla f(x) - \nabla f(y)\| \leq \beta \|x-y\|$, where $\beta \in]0, +\infty[$.
- ι_C: is the indicator function of C, equal to 0 on C and $+\infty$ otherwise (C be a nonempty subset of $\mathbb{R}^N$).

Problem formulation

$\bar{x} \in \mathbb{R}^N$: the unknown signal to be recovered ($N = N_1 \times N_2$)
$z^{(1)} \in \mathbb{R}^Q$: the complete low-resolution image
$z^{(2)} \in \mathbb{R}^M$: the given sample

$$z^{(1)} = D\bar{x}, \quad z^{(2)} = M\bar{x},$$

$\mathrm{D} \in \mathbb{R}^{Q \times N}$: spatial down-sampling operator,
$\mathrm{M} \in \mathbb{R}^{M \times N}$: a selection operator that extracts the patch from $\bar{x}$.

To recover $\bar{x}$ from the observations $z^{(1)}$ and $z^{(2)}$, We propose a variational approach that leads to solving the following optimization problem:

$$\underset{x \in \mathbb{R}^N}{\text{minimize}} \; \underbrace{\boxed{\|\mathrm{D}x - z^{(1)}\|^2 + \iota_{C_1}(x)}}_{\text{Data fidelity term}} + \underbrace{\boxed{H(x, z^{(2)})}}_{\text{Statistical prior}} + \underbrace{\boxed{\iota_{C_2}(x)}}_{\text{Frequency constraint}},$$

where $C_1 = \{x \in \mathbb{R}^N | \; Mx = z^{(2)}\}$.

Statistical prior

The Wasserstein distance between the histograms of two images $u \in \mathbb{R}^N$ and $v \in \mathbb{R}^N$ is defined as follows [1]:

$$\mathcal{W}_2^2(\nu_u, \nu_v) = \min_{\sigma \in \Sigma_N} \|u - v \circ \sigma\|^2,$$

ν_u and ν_v: normalized histograms of u and v, $v \circ \sigma$: permutation of the vector v, Σ_N: the set of all the permutations of N-length vectors. For grayscale images, the optimal permutation σ^* is computed as $\sigma^* = \sigma_v \circ \sigma_u^{-1}$.

✗ nonconvex (due to the histogram transformation)
✓ gradient is Lipschitz-continuous, $\quad \nabla_u \mathcal{W}_2^2(\nu_u, \nu_{\tilde{v}}) = 2(u - \tilde{v} \circ \sigma_{\tilde{v}} \circ \sigma_u^{-1})$.

We define the term H as $\quad H(x, z^{(2)}) = \sum_{s=1}^4 \alpha_s \mathcal{W}_2^2(\nu_{L_s x}, \nu_{\tilde{z}_s^{(2)}})$

where, for every $\forall s \in \{1, \ldots, 4\}$, $\alpha_s > 0$, $z_s^{(2)} = L_s z^{(2)}$, $\tilde{z}_s^{(2)}$ is the extension of $z_s^{(2)}$ (after the linear transformation), and for every $s \in \{1, \ldots, 4\}$, $L_s \in \mathbb{R}^{N_s \times N}$ is defined as follows: L_1: the identity matrix ($N_1 = N$), L_2: the concatenation of the horizontal and vertical difference operators ($N_2 = 2N$), L_3: the concatenation of the diagonal difference operators ($N_3 = 2N$), L_4: the isotropic Laplacian operator ($N_4 = N$).

Frequency constraint

Textures having the same second-order statistics share a common auto-covariance and, therefore, a common Fourier magnitude ➜ we search for an image $x \in \mathbb{R}^N$ such that

$$\forall m, \quad |\hat{x}(m)| = |\hat{z}^{(2)}(m)|.$$

$\hat{x}$ (resp. $\hat{z}^{(2)}$) is the orthogonal discrete Fourier transform of x (resp. $z^{(2)}$)

We rewrite the fourier spectrum constraint as

$$C_2 = \{x \in \mathbb{R}^N | \; \forall m, \; \exists \, \varphi(m) : \hat{x}(m) = e^{i\varphi(m)} \hat{z}^{(2)}(m)\}$$

Since x and $z^{(2)}$ are real images, $\varphi(m)$ must be antisymmetric modulo 2π.

Proximity operator

Let f be a proper l.s.c. convex function. For every $\overline{x} \in \mathcal{H}$, there exists a unique minimizer of the function

$$f + \frac{1}{2}\| \cdot - \overline{x}\|^2$$

This minimizer is called the *proximity operator* of f at $\overline{x}$ and is denoted by $\operatorname{prox}_f \overline{x}$.

Property: $\operatorname{prox}_{\iota_C} = \mathrm{P}_C$.

Algorithm 1 FBPD [2]

INITIALIZATION

- Choose $\left(x^{[0]}, y^{[0]}\right) \in \mathbb{R}^n \times \mathbb{R}^{Kn}$
- set $\tau > 0$ and $\omega > 0$ such that $\tau(\beta/2 + \omega) < 1$

FOR $l = 0, 1, \ldots$

$$\begin{aligned}
&\widehat{x}^{[l]} = \nabla f(x^{[l]}) + y^{[l]} \\
&x^{[l+1]} = \mathrm{P}_{\{\mathbf{M}\cdot = \mathbf{z}^{(2)}\}}\left(x^{[l]} - \tau \widehat{x}^{[l]}\right) \\
&\widehat{y}^{[l]} = \left(2x^{[l+1]} - x^{[l]}\right) \\
&y^{[l+1]} = y^{[l]} + \omega \widehat{y}^{[l]} - \mathrm{P}_{C_2}\left(y^{[l]} + \omega \widehat{y}^{[l]}\right)
\end{aligned}$$

Tools

✓
$$\mathrm{P}_{C_2}(\hat{x}(m)) = \frac{\hat{x}(m) \cdot \hat{z}^{(2)}(m)}{|\hat{x}(m) \cdot \hat{z}^{(2)}(m)|} \hat{z}^{(2)}(m),$$
where $x \cdot y$ denotes the hermitian product.

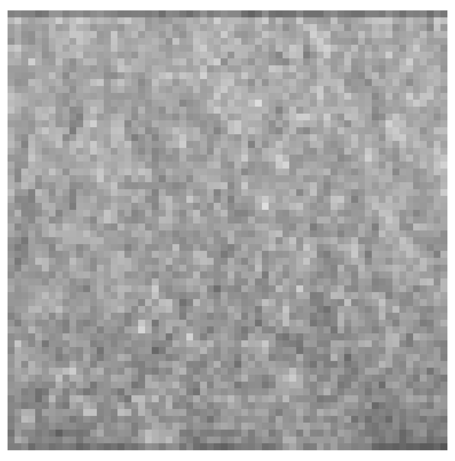
✓
$$\mathrm{P}_{\{\mathrm{M}\cdot = z^{(2)}\}}(x) = x + \mathrm{M}^\top(z^{(2)} - \mathrm{M}x).$$

✓ The gradient of the sum of the remaining terms, that is $f(x) = \|\mathrm{D}x - z^{(1)}\|^2 + \sum_{s=1}^4 \alpha_s \mathcal{W}_2^2(\nu_{L_s x}, \nu_{\tilde{z}_s^{(2)}})$, reads

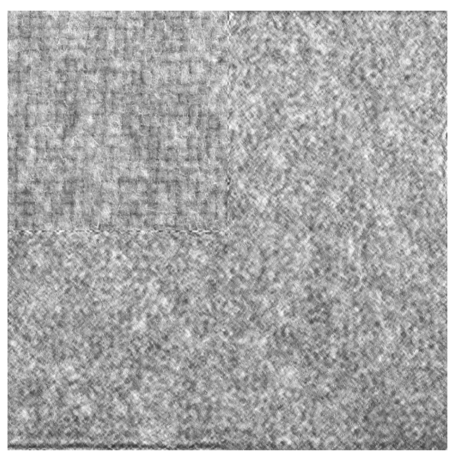
$$\begin{aligned}
\nabla f &= 2\,\mathrm{D}^\top(\mathrm{D}x - z^{(1)}) \\
&+ 2\sum_{s=1}^4 \alpha_s L_s^\top (L_s x - \tilde{z}_s^{(2)} \circ \sigma_{\tilde{z}_s^{(2)}} \circ \sigma_{L_s x}^{-1}),
\end{aligned}$$

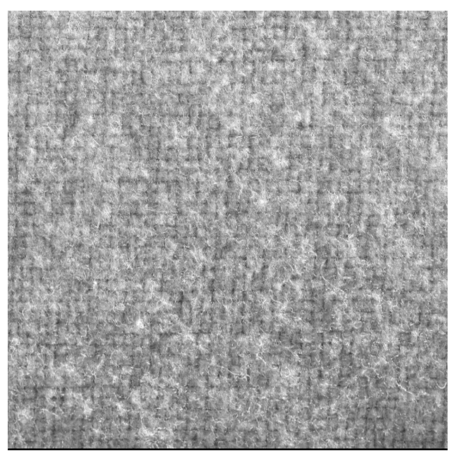
where ∇f is β-Lipschitz with $\beta = 2(1 + \sum_1^4 \alpha_s \|L_s\|^2)$.

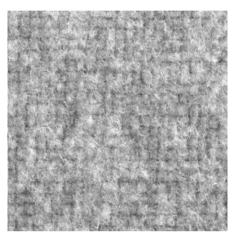
Results

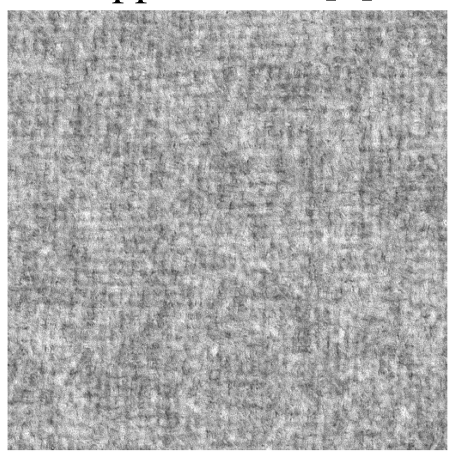

Low-Resolution texture

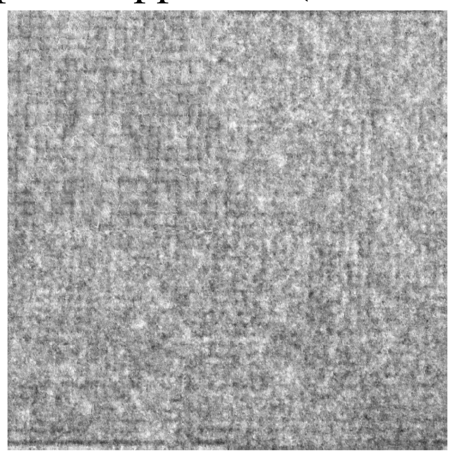

Approach of [3]

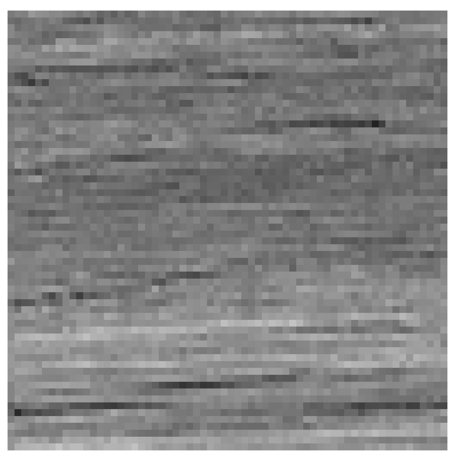

Proposed approach (without C_2)

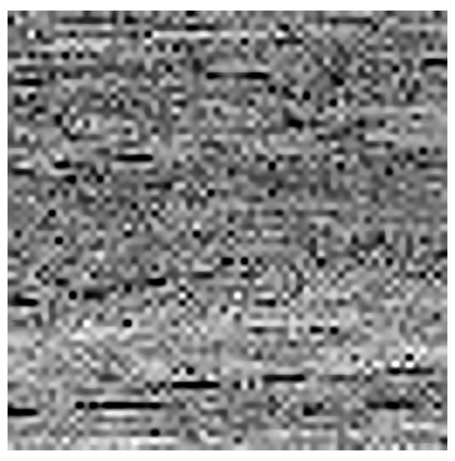

True image

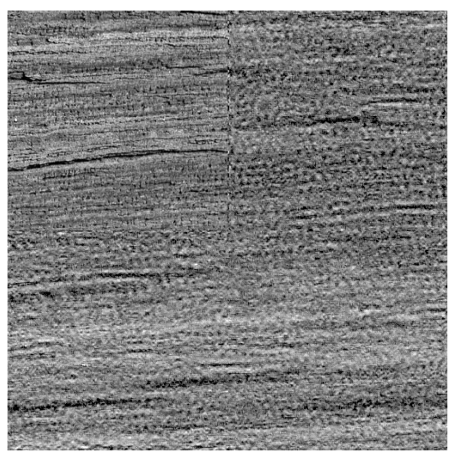

High-Resolution patch


Approach of [4]

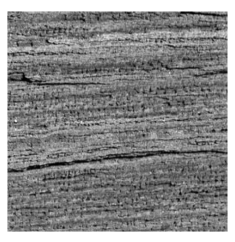

Proposed approach

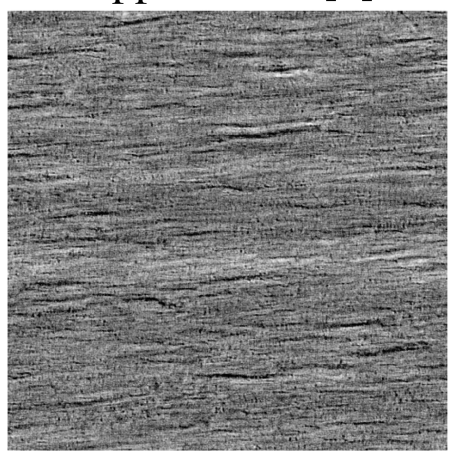

Low-Resolution texture

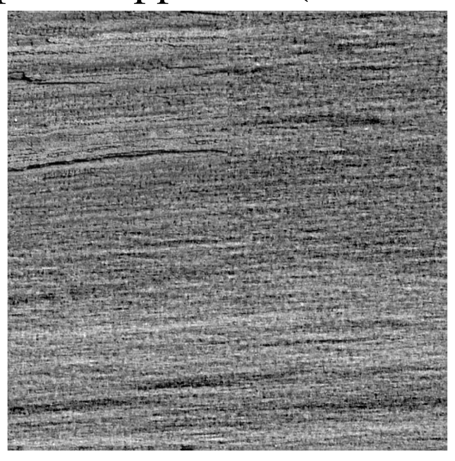

Approach of [3]


Proposed approach (without C_2)


True image


High-Resolution patch


Approach of [4]


Proposed approach

References

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[2] Laurent Condat, "A primal-dual splitting method for convex optimization involving lipschitzian, proximable and linear composite terms," *J. of Optim. Theory and Appl.*, vol. 158, no. 2, pp. 460–479, Aug. 2013.

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[4] J. Portilla and E. P. Simoncelli, "A parametric texture model based on joint statistics of complex wavelet coefficients," *Int. J. Comput. Vision*, vol. 40, no. 1, pp. 49–71, Oct. 2000.