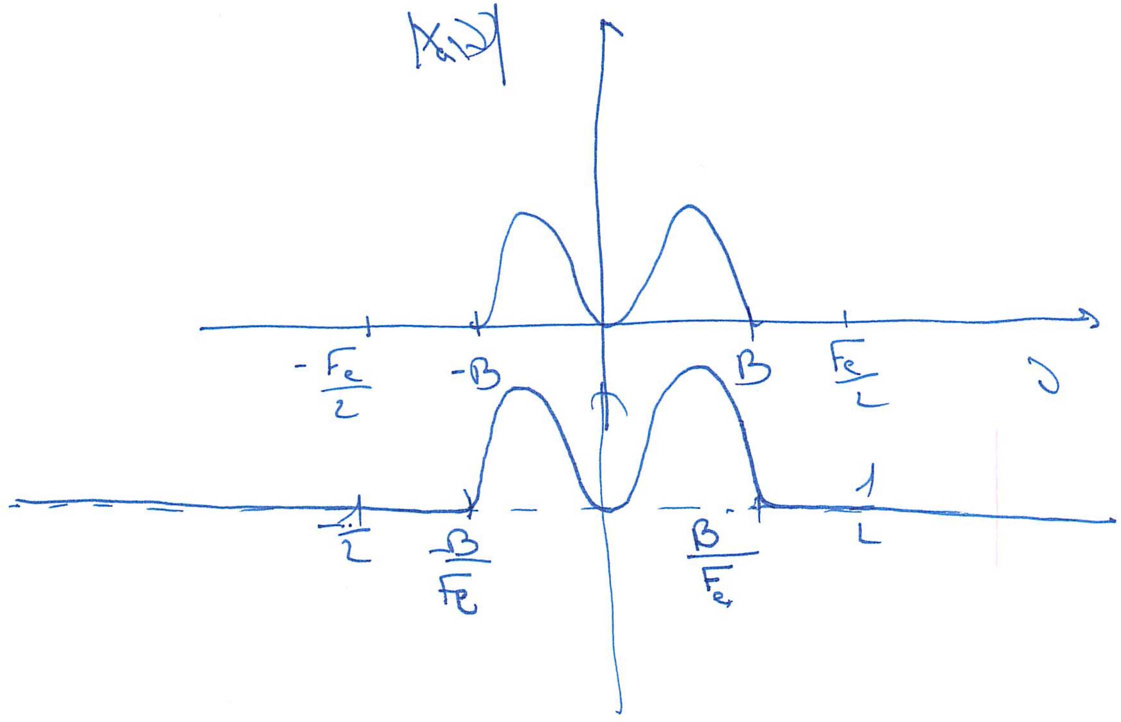
1



$$\boxed{X(f) = F_e X_a(f F_e)}$$

$$f = \nu T_e = \frac{\nu}{F_e}$$

$$\boxed{X_a(\nu) = T_e X(\nu T_e)}$$

$$\nu \in \left[-\frac{1}{2T_e}, \frac{1}{2T_e}\right]$$

$$X_a(\nu) = T_e \underbrace{\sum_n x_a(nT_e)\, e^{-2i\pi n \nu T_e}}_{X(\nu T_e)}$$

$$y_n = x_{n-n_0}$$

$$Y(f) = \sum_{n\in\mathbb{Z}} y_n e^{-2i\pi n f} = \sum_{n\in\mathbb{Z}} x_{n-n_0} e^{-2i\pi n f}$$

$m = n - n_0$, $n = n_0 + m$

$$= \sum_{m\in\mathbb{Z}} x_m \underbrace{e^{-2i\pi(n_0+m)f}}_{e^{-2i\pi n_0 f}\,\cdot\, e^{-2i\pi m f}} = e^{-2i\pi n_0 f} \underbrace{\sum_m x_m e^{-2i\pi m f}}_{X(f)}$$

$x_a(t)$, $y_a(t) = x_a(t-\tau)$

$$Y_a(\nu) = e^{-2i\pi\nu\tau} X_a(\nu)$$

2

Si $x_n \in \mathbb{R}$ pour tout n, $X(-f) = (X(f))^*$ $\forall f \in [-\frac{1}{2}, \frac{1}{2}]$

$$X(f) = \sum_n x_n e^{-2i\pi n f}$$

$$(X(f))^* = \sum_n x_n e^{2i\pi n f} = X(-f)$$

$$\frac{d}{df} X(f) = \sum_n x_n \underbrace{\frac{d}{df}\left(e^{-2i\pi n f}\right)}_{-2i\pi n \, e^{-2i\pi n f}}$$

$$= -2i\pi \sum_n n x_n e^{-2i\pi n f}$$

Valide si $\sum_n |n x_n| < +\infty$

$$X_a(\nu) = \int x(t) e^{-2i\pi\nu t} dt$$

$$\frac{d}{d\nu} X_a(\nu) = \int \frac{d}{d\nu}\left(x(t) e^{-2i\pi\nu t}\right) dt$$

$$= \int -2i\pi t \, x(t) e^{-2i\pi\nu t} dt$$

$$= -2i\pi \int t \, x(t) e^{-2i\pi\nu t} dt$$

3

$$z_n = (x * y)_n = \sum_{l} x_l \, y_{n-l}$$

$$Z(f) = \sum_{n} z_n \, e^{-2i\pi n f} = \sum_{n} \left(\sum_{l} x_l \, y_{n-l} \right) e^{-2i\pi n f}$$

$$= \sum_{n,l} x_l \, y_{n-l} \, \underbrace{e^{-2i\pi n f}}$$

$$e^{-2i\pi n f} = e^{-2i\pi l f} \times \underbrace{e^{-2i\pi (n-l) f}}_{e^{-2i\pi n f} \cdot e^{2i\pi l f}}$$

$$Z(f) = \sum_{n,l} x_l \, y_{n-l} \, e^{-2i\pi l f} \, e^{-2i\pi (n-l) f} = \sum_{m,l} \underbrace{x_l \, e^{-2i\pi l f}} \cdot \underbrace{y_m \, e^{-2i\pi m f}}$$

$$m = n - l$$

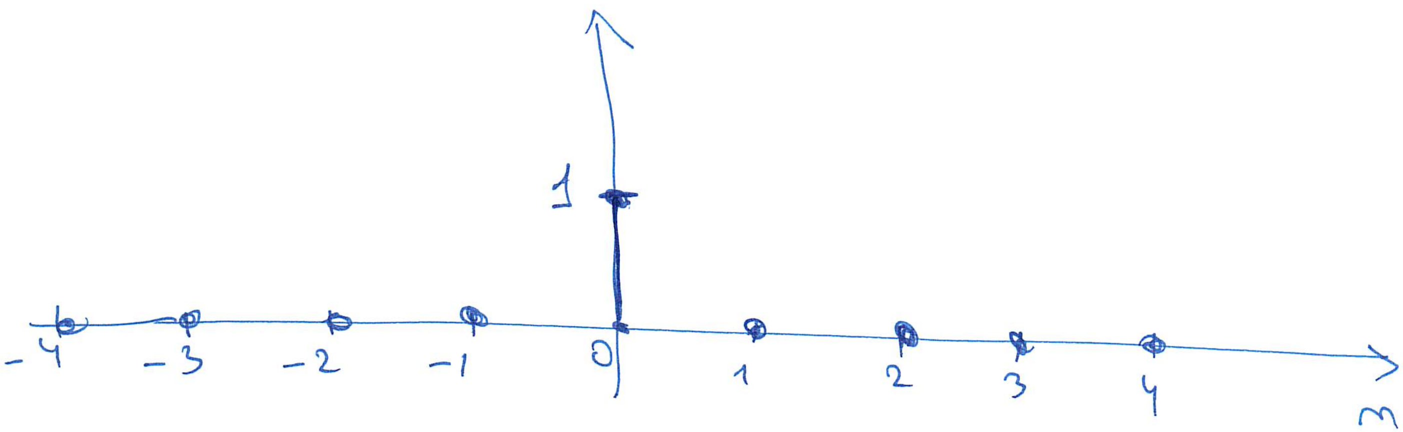
$$= \underbrace{\left(\sum_{l} x_l \, e^{-2i\pi l f} \right)} \underbrace{\left(\sum_{m} y_m \, e^{-2i\pi m f} \right)}$$

$$Z(f) = X(f) \cdot Y(f)$$

$$y_m = (h * x)_m = \sum_{\ell} h_\ell \, x_{m-\ell}$$

$(h_\ell)_{\ell \in \mathbb{Z}}$ réponse impulsionnelle du filtre.

$$x_m = \delta_m = \begin{cases} m = 0, & \delta_0 = 1 \\ m \neq 0 & \delta_m = 0 \end{cases}$$

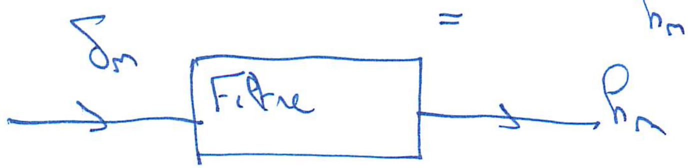


$$(h * \delta)_m = \sum_{\ell} h_\ell \underbrace{\delta_{m-\ell}}$$

$$= h_m \times 1 = h_m$$

$\delta_{m-\ell} = 0$ si $\ell \neq m$

$\delta_{m-\ell} = 1$ si $\ell = m$



5

Sat $u_n = 0$ si $n < 0$, $h_n = 0$ $n < 0$, $y_n = (h * u)_n = 0$ si $n < 0$

~~&~~ si $n \geq 0$, $y_n = \sum_{k=0}^{n} h_k u_{n-k}$

<u>$n \geq 0$</u>

$$y_n = \sum_{k \in \mathbb{Z}} h_k u_{n-k} = \sum_{k=0}^{+\infty} h_k u_{n-k} = \sum_{k=0}^{n} h_k u_{n-k} + \underbrace{\sum_{k=n+1}^{+\infty} h_k u_{n-k}}$$

$k \geq n+1$, $n - k \leq -1 \Downarrow n-k < 0 \Downarrow u_{n-k} = 0$

<u>$n < 0$</u>, si $k \geq 0$ $\cancel{u_{n-k} < 0 \Rightarrow \sum_k}$

$n - k < 0 \Rightarrow u_{n-k} = 0$ et $\sum_{k=0}^{\infty} h_k u_{n-k} = 0$

$\Rightarrow y_n = 0$.

6

$$y_n - a y_{n-1} = u_n$$

$$y_n = a y_{n-1} + u_n \qquad y_{n-1} = a y_{n-2} + u_{n-1}$$

$$y_n = a\left(a y_{n-2} + u_{n-1}\right) + u_n = a^2 y_{n-2} + a u_{n-1} + u_n$$

$$y_{n-2} = a y_{n-3} + u_{n-2}$$

$$y_n = a^2\left(a y_{n-3} + u_{n-2}\right) + a u_{n-1} + u_n = a^3 y_{n-3} + a^2 u_{n-2} + a u_{n-1} + u_n$$

$$= a^4 y_{n-4} + a^3 u_{n-3} + a^2 u_{n-2} + a u_{n-1} + u_n$$

$$\vdots$$

$$= a^n \boxed{y_0} + a^{n-1} u_1 + a^{n-2} u_2 + \dots + a^2 u_{n-2} + a u_{n-1} + u_n$$

$$y_n = a^n u_0 + a^{n-1} u_1 + \dots + a u_{n-1} + u_n = \sum_{k=0}^{n} a^k u_{n-k}$$

$$\underline{h_k = a^k \quad \forall k \geq 0}$$